Synchronization Control of a Four-wing Fractional-Order Chaotic System and Its Analog Circuit Design

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Abstract

The four-wing fractional-order chaotic system is firstly introduced in this paper, Then, its synchronization control is also discussed, and the results from numerical analysis show the chaotic synchronization control is simple and practical. An last, an analog circuit is designed to implement it, and the results are in agreement with numerical analysis, which probably provide an practical technology for application of fractional-order chaos , such as secure communication and image encryption.

Keywords: Fractional-order, chaotic system, synchronization control, analog circuit

1. Introduction

Although Fractional calculus has a history of more than 300 hundred years,[1-2] it hasn't attracted more and more attention until many systems are found to show fractional-order dynamics, such as the fractional-order Chua's circuit,[3] the fractional-order Lorenz system,[4] the fractional-order Chen system,[5-6] the fractional-order Rössler system,[7] and so on. [8-10] Recently, some study on synchronization and control of fractional-order chaotic systems are attracting more and more attention from an application point of view,such as secure communication, image encryption and control processing.[11-17] It was firstly reported in Pecora and Carrolfor's paper in 1990 that two chaotic trajectories with different initial conditions can be synchronized .[18] Subsequently, some other methods of synchronization have been represented and researched, such as the PC method, the PAD method, the one-way method and the bidirectional coupled method.[19-25]

In 2008, Chen et al. reported a integer-order four-wing chaotic system.[26] Some interesting phenomenon are found, for example, the system shows four-wing chaotic attractors, three-wing chaotic attractors, and periodic attractors in different periods when selecting different system parameters. In 2011, Jia et al. further analyzed its dynamic behaviors by computer-aid topological horseshoe analysis and analog circuit experiments.[27] In 2014, based on the theory of fractional calculus and the method of frequency domain approximation, Jia et al. study the fractional form of the integer-order four-wing system, and found that chaotic characteristic exist in it.[28] All these work show the dynamics of the four-wing chaotic system are very complex, it may be more interesting to make some application study by utilizing the system.

In this paper, the fractional-order four-wing chaotic system is firstly reported. Then, a synchronization method is discussed, and the simulation results have confirmed the effectiveness of the synchronization method. At last, an analog circuit is designed to realize the chaotic synchronization, and all of results show the synchronization method is rather simple and convenient.

2. The Four-Wing Fractional-Order Chaotic System

The four-wing fractional-order system in Ref.28 is described by

$$\begin{cases} \dfrac{d^\alpha x}{dt^\alpha} = ax + ky - yz \\ \dfrac{d^\alpha y}{dt^\alpha} = -by - z + xz \\ \dfrac{d^\alpha z}{dt^\alpha} = -x - cz + xy \end{cases} \tag{1}$$

Where $a, b, c, k \in R$; $0 < \alpha < 1$.

Based on frequency domain approximate method, the fractional operator of order "α" can be finished by the transfer function $1/s^\alpha$ in the frequency domain. Then the transfer function $1/s^\alpha$ can be represented by an approximated integer-order transfer function with errors of approximately 2 dB according to Ref.29, and thus the approximation adopted in this paper is

$$\frac{1}{s^{0.9}} \approx \frac{1.766s^2 + 38.27s + 4.914}{s^3 + 36.15s^2 + 7.789s + 0.01} \tag{2}$$

When $\alpha = 0.9$, $(a, b, c, k) = (5,12,5,1)$, the four-wing fractional -order system is chaotic as shown in Fig. 1. Its corresponding chaotic characteristics analysis has been given in the Ref..28 one can see it for the details.

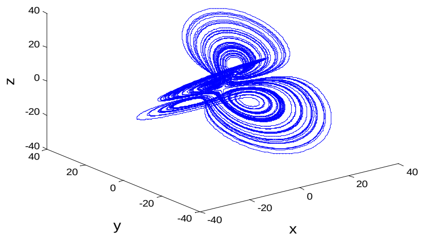

Fig. 1. Phase plots of the fractional-order system

3. Synchronization of the Four-Wing Fractional-Order Chaotic System

In this paper, based on master-slave configuration method with linear coupling, a chaotic synchronization control is briefly discussed between the two four-wing fractional-order systems whose structure are same. Here, the master four-wing fractional-order chaotic system can be written as system(1), and the coupled slave four-wing fractional-order system has the following form.

$$\begin{cases} \dfrac{d^\alpha x'}{dt^\alpha} = ax' + ky' - y'z' + k_1(x - x') \\ \dfrac{d^\alpha y'}{dt^\alpha} = -by' - z' + x'z' + k_2(y - y') \\ \dfrac{d^\alpha z'}{dt^\alpha} = -x' - cz' + x'y' + k_3(z - z') \end{cases} \tag{3}$$

The constant parameters k_1 , k_2 , k_3 are coupling strength. Now let the coupling strength $k_1 > 0$, $k_2 = 0$, $k_3 = 0$, by continuously increasing the coupling strength k_1 from zero, in step 1,a value of k_1 is obtained to make the two four-wing fractional-order systems synchronized. That is, When $k_1 = 4$, the two coupled chaotic systems can be synchronized. Simulation results are shown in Fig.2 and Fig.3, which are the time response and the phase synchronization, respectively.

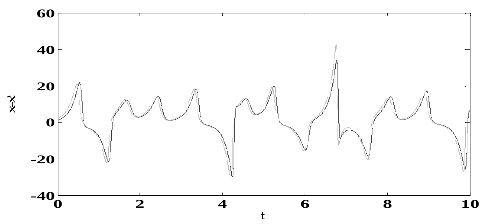

Fig. 2. time response of x (-) and x' (--);

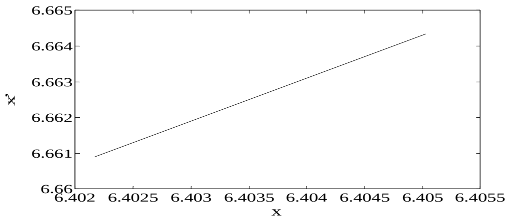

Fig. 3. Phase synchronization of x and x’

4. Analog Circuit for Synchronization of the Four-Wing Fractional-Order Chaotic System

Finally, we designed an analog circuit to realize the synchronization control for the four-wing fractional-order system by using the basic devices such as Ad633, LF347, resistors, and capacitors,as shown in Fig. 4 and Fig. 5. Fig. 4 is an analog approximate circuit for fractional-order $1/s^{\alpha}$, and Fig.5 is an analog circuit for the synchronization control. Resistors and capacitors in the circuit are $R_3 = R_3' = R_4 = R_4' = R_6 = R_6' = R_7 = R_7' = R_{10} = R_{10}' = R_{11} = R_{11}' = R_{13} = R_{13}' = R_{14} = R_{14}' = R_{17} = R_{17}' = R_{18} = R_{18}' = R_{20} = R_{20}' = R_{21} = R_{21}' = 10k\Omega$, $R_2 = R_2' = R_9 = R_9' = R_{15} = R_{15}' = 100k\Omega$, $R_8 = R_8' = 8.3k\Omega$, $R_{16} = R_{16}' = 20k\Omega$, $R_5 = R_5' = R_{12} = R_{12}' = R_{19} = R_{19}' = 1k\Omega$, $R_a = 1.55M\Omega$, $R_b = 61.54M\Omega$, $R_c = 2.5k\Omega$, $C_1 = 730nF$, $C_2 = 520nF$, $C_3 = 1.1uF$,respectively. In addition, The resistors R_1 , R_1' , and R_{22} are adjustable according to the system parameters a and k_1 . When selecting $R_1 = 20k\Omega$, $R_1' = 100k\Omega$, $R_{22} = 25k\Omega$, the two coupled chaotic systems are synchronized, and the simulation results are shown in Fig. 6.

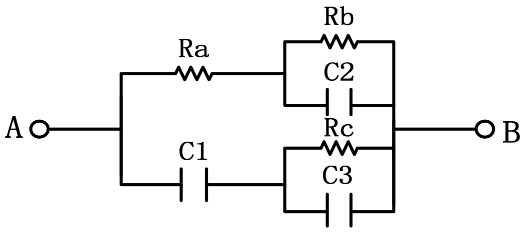

Fig. 4. Approximate Circuit for $1/s^{\alpha}$

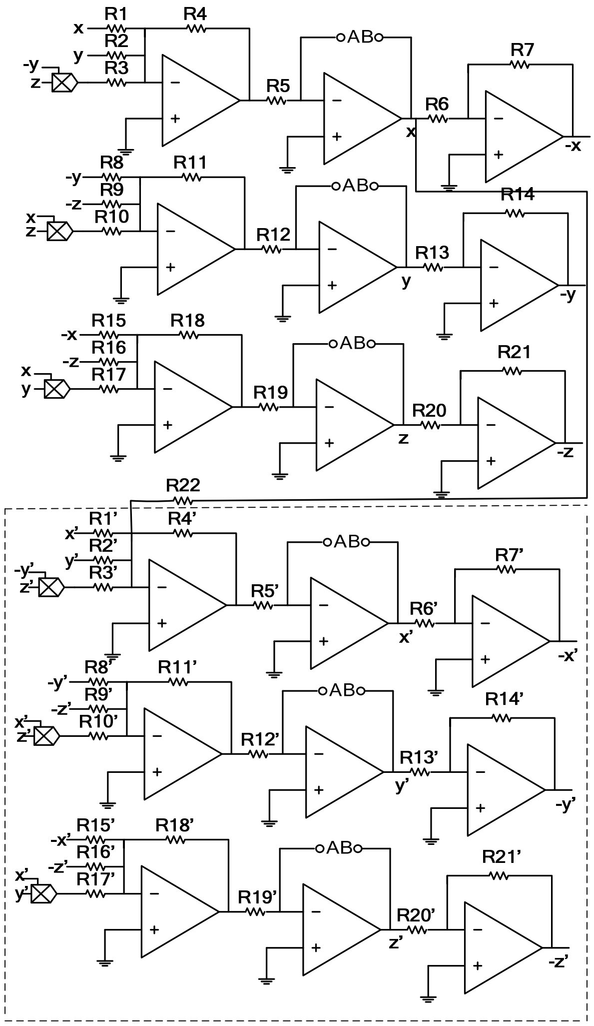

Fig. 5. Analog circuit of synchronization between system (1) and system (3) .

5. Conclusion

In this paper, by linear coupling and frequency domain approximation, a synchronization control method for a four-wing fractional-order chaotic system is studied, and the simulation results have confirmed the feasibility and effectiveness of this synchronization method. In addition, An analog circuit is also designed to implement the synchronization control. Numerical simulations and circuit experiments are given to verify the effectiveness of the proposed synchronization scheme, and it is rather simple and convenient for the synchronization scheme to be realized. Some work in this paper probably provide an practical technology for application of fractional-order chaos , such as secure communication and image encryption.

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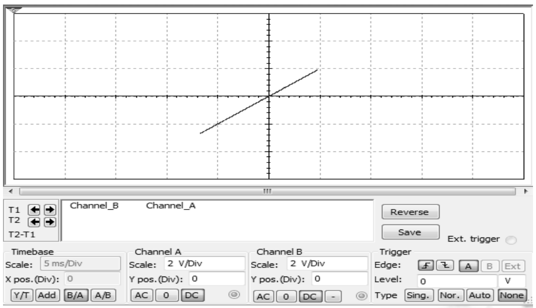
(a) Time response of $x - x'$



(b) Phase synchronization of x and x'

Fig. 6. Circuit simulation observations of synchronization

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