The Fractional Order Hyperchaotic Generalized Augmented Lü System and its Circuit Implementation

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Abstract

In this paper, a commensurate fractional-order hyperchaotic generalized augmented Lü system is investigated. we analyze its chaotic characteristics by drawing phase portraits, Poincaré maps, Lyapunov exponent spectra and power spectrum, and find that the system can present a four-wing hyperchaotic attractor. In addition, a circuit is designed for this system and the circuit implementation result show the existence of the four-wing hyperchaotic attractor, which verifies the correct of the theoretic analysis and provides the support for its application in engineering.

Keywords: Fractional-order; Generalized augmented Lü system; Hyperchaos; Numerical simulation; Circuit design.

1. Introduction

Since Lorenz found the first chaotic attractor in 1963,[1] the study of chaotic phenomenon have received great attention during the past forty years and many chaotic systems have been proposed to research[2-5]. Due to its complex characteristics such as extremely sensitive to initial conditions, inherent randomness, continuous broadband spectrum and so forth, chaotic system can be widely applied in engineering like secure communication, image encryption and so on[6-8]. While hyperchaos is characterized as at least two positive Lyapunov exponents, which is more complex than chaos, so it is more suitable for the engineering application.

Fractional order calculus is a kind of calculus with an arbitrary order, which has become a hot spot during the past decades for its application in engineering. It is found that the fractional-order system could reflect the physical phenomenon more accurately and fractional-order chaotic systems have more complex dynamics characteristics than integer-order chaotic systems[9].

In this paper, a new fractional order hyperchaotic system is proposed and we analyze its chaotic characteristics through Matlab numerical simulations. Furthermore, an analog circuit is designed to realized the proposed fractional order hyperchaotic system ,and the circuit experiment results agree with the theoretic analysis.

2. System Analysis

According to the generalized augmented Lü system[10], by introducing linear state feedback controllers into its second and third equation, Xue et al[11] adopt chaos anti-control to construct the hyperchaotic generalized augmented Lü system. On this basis, we will investigate the commensurate fractional

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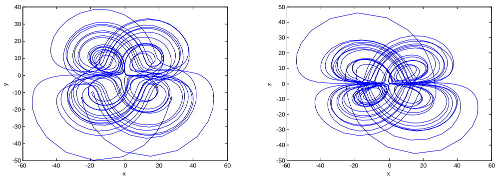
order hyperchaotic generalized augmented Lü system which is described as follow:

$$\begin{cases} \dfrac{d^q x_1}{dt^q} = -\dfrac{ab}{a+b} x_1 - x_2 x_3, \\ \dfrac{d^q x_2}{dt^q} = a x_2 + x_1 x_3 + x_4, \\ \dfrac{d^q x_3}{dt^q} = b x_3 + x_1 x_2 + c x_1 + x_1 x_4, \\ \dfrac{d^q x_4}{dt^q} = d x_2, \end{cases} \qquad (1)$$

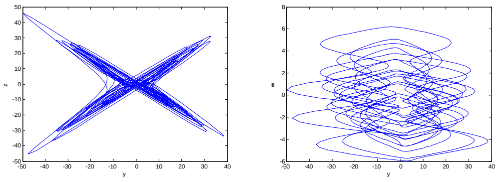
Where $0<q<1$, $a,b,d<0$ and $c \in R$ .Here when q=0.9, system(1) is characterized as a 3.6 order system(1).

3. Phase Portraits and Poincaré Maps

When q=0.9,a=-9,b=-14,c=1,d=-1,the phase portraits of system(1) is shown in Fig.1, It can be seen that the fractional-order system presents four-wing chaotic attractor.

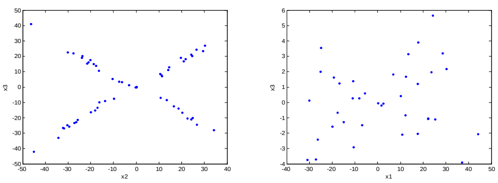


(*a*)x_1-$x_2$plan (*b*) x_1-x_3 plan



(*c*) x_2-$x_3$plan (*d*) x_2-x_4 plan

Fig.1 Phase portraits of system (1)



(*a*) x_1=1.5 plan; (*b*) x_2=0 plan

Fig.2 Poincaré maps of system (1)

Poincaré map is known as an intuitive way to determine whether a system is chaotic. From Fig. 2, we can see that Poincaré maps of system(1) are formed by points in confusion,which indicates system(1) is in chaos state. Furthermore, by calculating the Lyapunov exponents of system(1) when q=0.9,a=-9,b=-14,c=1,d=-1, we get L_1=1.6987,L_2=0.6500,L_3=-0.2817,L_4=-19.5887.For it has two positive Lyapunov exponents, system(1) presents hyperchaos and we get the dimension of system(1) as

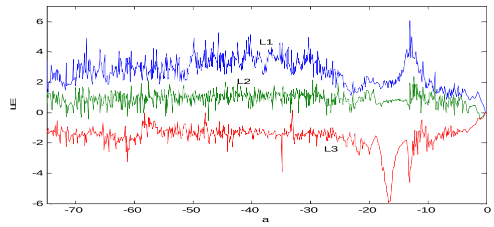
$$D_L = j + \frac{1}{|LE_{(j+1)}|}\sum_{i=1}^{j} LE_i = 2 + \frac{LE_1 + LE_2}{|LE_3 + LE_4|}$$

$$= 2 + \frac{1.6987 + 0.6500}{|-0.2817 - 19.5887|} = 2.118$$

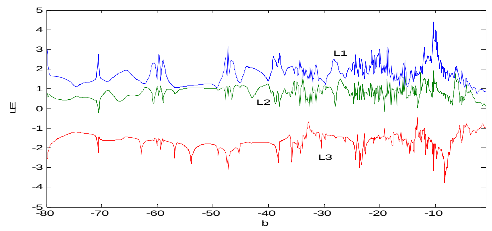
indicating that hyperchaotic system(1) has a fractional dimension with a fractal feather.

4. Lyapunov Exponent

In order to analyze the chaotic characteristics of system(1) as system parameters change, we draw the Lyapunov exponent spectra of system(1) as shown in Fig. 3(As L_4 is always less than -10, so we only give curves of L_1,L_2,L_3) when parameter a,b change in a certain range.From Fig. 3, we can observe that as a,b vary in a larger range, system(1) has two positive Lyapunov exponents, which shows that system(1) presents hyperchatic in a large range as parameters change.



(*a*)q=0.9,b=-14,c=1,d=-1, $a \in [-75,0]$



(*b*) q=0.9,b=-14,c=1,d=-1, $b \in [-80,0]$

Fig.3 Lyapunov exponent spectra of system (1)

5. Power Spectrum

In addition, the power spectrum of system(1) is shown in Fig. 4 when q=0.9,a=-9,b=-14,c=1,d=-1. it can be seen that the power spectrum of system(1) is a continuous decreasing curve with noise background and wide band.

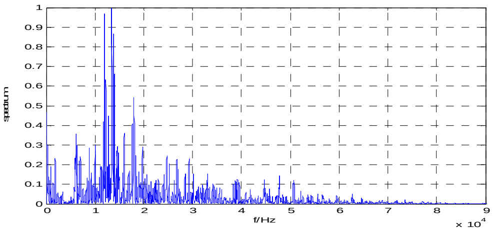


Fig.4 Power spectrum of system(1)

6. Circuit Implementation

we usually adopt the method of the approximation conversion from time domain to frequency domain to design a chaotic circuit. Here we utilize the approximation of $1/s^{0.9}$ with discrepancy 2dB to design the analog circuit of 3.6 order hyperchaotic system(1). According to Ref. [12], we can get

$$\frac{1}{s^{0.9}} \approx \frac{2.2675(s+1.292)(s+215.4)}{(s+0.01292)(s+215.4)(s+359.4)}.$$

In Ref.[13], a circuit unit is proposed to realize the $1/s^{0.9}$ as shown in Fig.5.Where R_1=62.84MΩ,R_2=250KΩ; R_3=2.5KΩ;C_1=1.232μF,C_2=1.84μF,C_3=1.1μF.

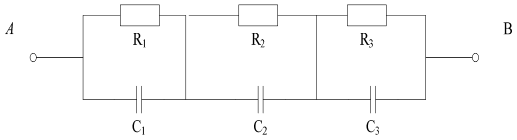


Fig. 5 Circuit unit of $1/s^{0.9}$

Here LF347N is selected as the amplifier and AD633JN (the output gain is 0.1) as the multiplier to design the hyperchaotic circuit. In order to restrict the change of state variables to the operating voltage of the analog circuit, the state variables are decreased by 10 times, namely $(x_1,x_2,x_3,x_4)\rightarrow(20x_1,20x_2,20x_3,20x_4)$. The analog circuit design of 3.6 order hyperchaotic system(1) when *a*=-9,*b*=-14,*c*=1,*d*=-1 is shown in Fig. 6 ,where F09 represents the circuit unit of $1/s^{0.9}$ and R_1=1.825KΩ, R_3=1.111KΩ, R_4=10KΩ,R_6=0.714KΩ, R_7=R_{10}=10KΩ, R_2=R_5=R_8=R_9=50Ω. If we choose R_4 as a reference resistance, we can change the value of *a*,*b*,*c*,*d*, by changing the ratio of R_3,R_6,R_7,R_{10} to R_4 respectively.

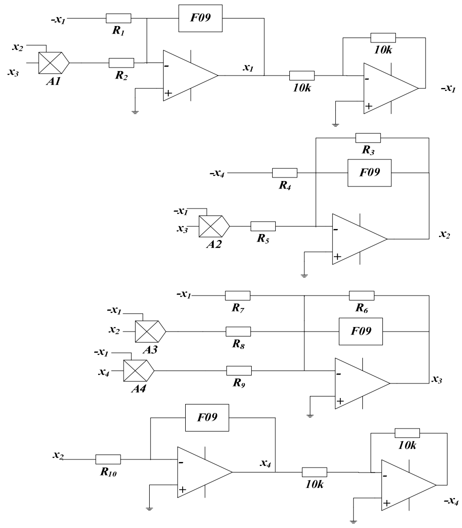
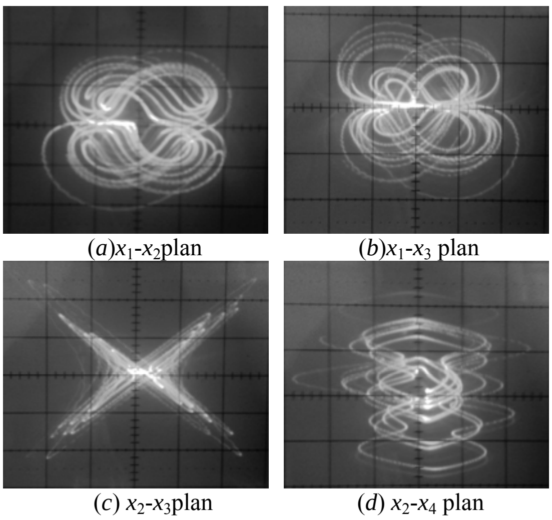


Fig. 6 Circuit diagram of 3.6 order system(1)

Based on the circuit diagram in Fig.6, we conduct the circuit experiment and the result is observed through the YB4365 analog oscilloscope as shown in the Fig.7, which agrees with the numerical simulation result, verifying the existence of hyperchaotic attractor in 3.6 order system(1).



(*a*)x_1-$x_2$plan (*b*)x_1-x_3 plan (*c*) x_2-$x_3$plan (*d*) x_2-x_4 plan

Fig.7 Phase portraits of 3.6 order hyperchaotic system(1) through the oscilloscope YB4365

7. Conclusion

In this paper, the fractional order hyperchaotic generalized augmented Lü system is investigated . By analyzing phase portraits, Poincaré maps, Lyapunov exponent spectra and power spectrum of the fractional order system, it is found that there exists four-wing hyperchaotic attractors in the system and the system is hyperchaos with a fractional dimension in a large range of parameters. Moreover, take 3.6 order system for example, an analog circuit is designed to implement the fractional order hyperchaotic system and circuit experiment results agree with the numerical analysis, providing technical basis for its further application in engineering.

Acknowledgements

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