# Simultaneous Computation of Model Order and Parameter Estimation of a Heating System Based on Particle Swarm Optimization for Autoregressive with Exogenous Model : An Analysis

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**Abstract**

System identification is one of the method for solving a mathematical model of a system by performing on analysis only at its input and output behaviour. In system identification, the procedure of modelling the system is separated into four main parts. The first part is constructing an experiment to collect the input and output data of the system. Then, with some criteria, the model order and structure are selected. The next part is to estimate the parameters of the model. For the final part, the mathematical model is verified. Model order selection and parameter estimation are two important parts of finding the mathematical model for system identification. Previously, a technique called simultaneous model order and parameter estimation (SMOPE), which is based on Particle Swarm Optimisation (PSO) and ARX model, has been introduced to combine these two parts simultaneously. This technique, however, exclude the error term of ARX model. In this study, an analysis is shown to prove that the performance of SMOPE based on PSO and ARX model degraded as the magnitude of error increases.

*Keywords*: ARX, Particle swarm optimization (PSO), System identification.

## 1. Introduction

System identification is a method to find the mathematical model of a dynamic system which uses statistical methods to build mathematical models from measured data. Two types of models are common in the field of system identification: grey-box model and black-box model. Grey-box model refers to the model where some information is partially known from first principle while the rest of the information is obtained from experiment. On the other hand, a black-box model uses no priori physical knowledge of the system. A number of model structures have been proposed in the area of black-box identification system. The most common models are autoregressive model with exogenous inputs (ARX) [1], autoregressive moving average with exogenous inputs (ARMAX) [2], and Box-Jenkins (BJ) model [3].

Autoregressive with exogenous (ARX) model is the simplest model in linear black box identification. In solving the linear black box identification problem of the ARX model, the model order selection and parameter estimation part is solved as separate process. Previously, a technique called simultaneous model order and parameter estimation (SMOPE), which is based on Particle Swarm Optimisation (PSO) and ARX model, has been introduced to combine these two parts simultaneously [4]. This technique, however, exclude the error term of ARX model. In this study, an analysis is conducted to investigate the performance of SMOPE based on PSO and ARX model with error term included.

## 2. Simultaneous Model Order and Parameter Estimation

In ARX model, AR refers to the autoregressive part and X refers to the exogenous input. The single-input single-output (SISO) ARX mathematical model can be defined as:

$$y(t) = \frac{B(q)}{A(q)}u(t) + \frac{1}{A}\varepsilon(t) \quad (1)$$

where $u(t)$ and $y(t)$ are input and output variables, respectively, $\varepsilon(t)$ is a Gaussian white noise process, $A(q)$ and $B(q)$ are polynomials in the backward shift operator. In ARX, a mathematical model of a system can be represented as:

$$G(t) = \frac{Y(t)}{U(t)} = \frac{b_0 + b_1 q^{-1} + \cdots + b_m q^{-m}}{1 + a_1 q^{-1} + \cdots + a_n q^{-n}} \quad (2)$$

In SMOPE, however, the error term $\varepsilon(t)$ is ignored and the maximum order of 9[th] is considered. To decide the parameter '*a*' and '*b*', the constraint $n \geq m$ is taken into account. This is due to the transfer function form which the order value of poles ($n$ value) must be the same or less than the order of zeroes ($m$ value).

The purpose of PSO algorithm [5], as shown in Fig. 1, is to search for the best mathematical model. The combination of model order and parameter for ARX equation are considered in particle representation, which is shown in Table 1. For example, if the model order is 2, then '*n*' value is 2 and all possible mathematical models are subjected to fitness calculation. Specifically, the calculations involve two mathematical models, which are $\frac{b_1 u(t-1)}{a_1 y(t-1) + y(t-2)}$ and $\frac{b_1 u(t-1) + b_2 u(t-2)}{a_1 y(t-1) + y(t-2)}$.

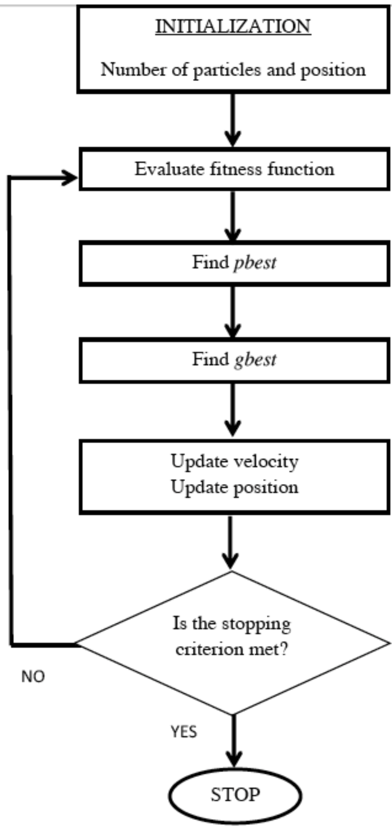



Fig. 1. The flowchart of PSO algorithm.

In the earlier stage of PSO algorithm, some parameters are initialized. The PSO parameter values used in this study is shown in Table 2. The PSO parameter includes the number of particles, the inertia weight, cognitive component, $c_1$, social component, $c_2$, and the maximum number of iterations, $k$. The initial position and velocity of particle is randomly located in a search space. After the initialization stage is done, the fitness function is calculated as follows:

$$best\ fit = 100\left[1 - \frac{norm(y_{estimation} - y)}{norm(y - \bar{y})}\right]\% \quad (3)$$

The personal best or *pbest* is the best solution found by each particle and *gbest* is defined as the best *pbest*. Both *pbest* and *gbest* are updated at every iteration. The velocity of a particle is updated using Eq. (4):

$$v_i^{k+1} = \omega v_i^k + c_1 rand(p_{best} - s_i^k) + c_2 rand(g_{best} - s_i^k) \quad (4)$$

Table 1. Particle representation.

| Dimension | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Variable in ARX | Order, $n$ | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ | $a_8$ | $a_9$ | $b_1$ | $b_2$ | $b_3$ | $b_4$ | $b_5$ | $b_6$ | $b_7$ | $b_8$ | $b_9$ |

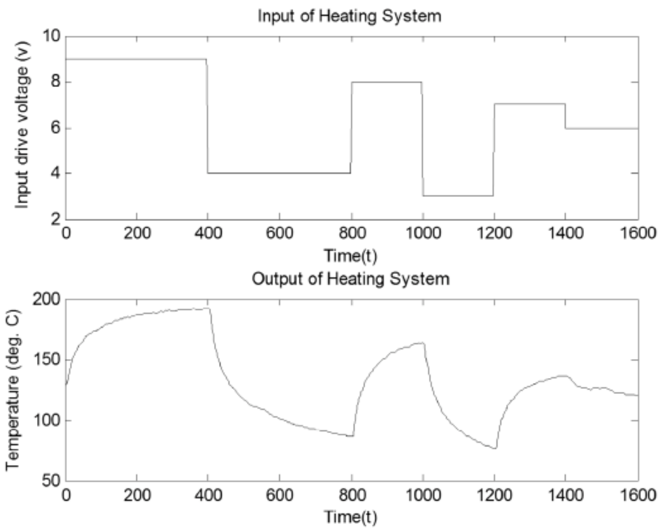



Fig. 2. Input and output behavior of heating system.

Table 2. PSO parameters.

| Parameters | Value |
|---|---|
| Maximum number of particles, $i$ | 10 |
| Decrease inertia weight, $\omega$ | 0.9~0.4 |
| Cognitive Component, $c_1$ | 2 |
| Social Component, $c_2$ | 2 |
| Random value, *rand* | [0,1] |
| Maximum iterations, $k$ | 150 |
| Number of run | 100 |

where $v_i^k$ is the velocity particle $j$ at iteration $k$, *rand* is random numbers [0,1], $\omega$ is inertia weight, and $c_1$ and $c_2$ denote the cognitive and social coefficients, respectively. The particle's new velocity is then used to update the particle's position using Eq. (5).

$$s_i^{k+1} = s_i^k + v_i^{k+1} \qquad (5)$$

where $s_i^k$ is the position of particle $i$ at iteration $k$ . In this study, the linear dynamic inertia weight is used and calculated according to Eq. (6) as follows:

$$\omega = \omega_{max} - \frac{\omega_{max} - \omega_{min}}{k_{max}} \times k \qquad (6)$$

where $\omega_{max}$ and $\omega_{min}$ denote the maximum and minimum values of inertia weight, respectively, and $k_{max}$ is the maximum iteration.

Note that the PSO will assign floating values to every dimension of a particle even though the value of model order is discrete. Hence, for the first dimension, the floating value is converted to discrete data by rounding its value. If the value is not in range, the previous value is used instead. Then, the particle refers to other parameters for the calculation of fitness.

## 3. The Heating System

In heating system, the input drives a 300 Watt Halogen lamp, suspended several inches above a thin steel plate. The output is a thermocouple measurement taken from the back of the plate. The input and output of the experiment is shown in Fig. 2, which is taken from http://www.esat.kuleuven.ac.be/sista/daisy. The data collect from the system is halfly separated for estimation and validation. This estimation data is used to calculate the model order and parameter of the ARX while the validation data is used to validate the model.

## 4. Experiment, Result, and Discussion

Using MATLAB for simulation and based on the PSO parameter shown in Table 2, an example of a convergence curve is shown in Fig. 3. The minimum and maximum value of best fit obtained in estimation is 99.0018% and 99.3211%, respectively. Average value of best fit is 99.2242%. In validation, the minimum, maximum, and average best fit are 92.6557%, 98.7011%, and 98.0763%, respectively. This experiment gives model order value as 3 and mathematical model as follows:

$$G_{HD}(z) = \frac{0.0024z^{-1} + 0.0020z^{-2} + 0.0720z^{-3}}{1 - 1.6169z^{-1} + 0.7306z^{-2} - 0.0833z^{-3} - 0.0321z^{-4}} \qquad (7)$$

where the model order, $n$ = 4, parameter value $b_1$ = 0.0024, $b_2$ = 0.0020, $b_3$ = 0.0720, $a_1$ = −1.6169, $a_2$ = 0.7306, $a_3$ = 0.0833, and $a_4$ = −0.0321.

An analysis has been conducted to investigate the performance of SMOPE based on PSO and ARX model with error term, $\varepsilon(t)$, included. The complete ARX model shown in Eq. (1) and the same PSO parameters have been used. The error term is a white noise, which can be generated in MATLAB using `rand` command.

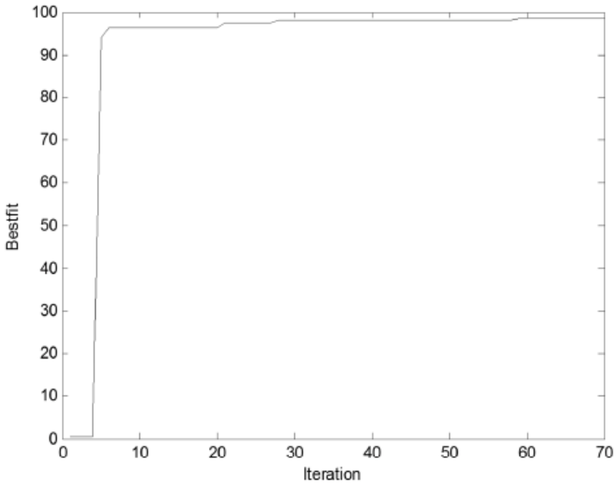



Fig. 3. An example of convergence curve.

The average best fit obtained is 98%, which is very much similar to the result of the previous experiment. Additional experiments with different magnitude of noise have been done and the result is shown in Fig. 4. Note that only 10 runs are considered. This analysis shows that the best fit decreases as the magnitude of error increases.

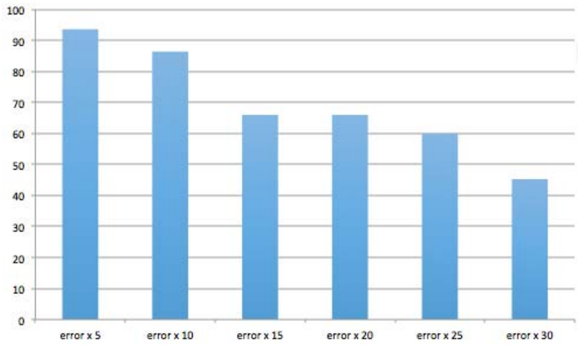
## 5. Conclusion

In the previous study, a system identification results from a heating system case study has been presented to investigate the performance of simultaneous model order and parameter estimation technique based on PSO and ARX model. However, the error term is excluded from the previous experiment. In this paper, with different magnitude of white noise are considered and included in the experiment. Based on the 10 runs and different magnitude of white noise up to 30, this study proved that the performance of SMOPE based on PSO and ARX model degraded as the magnitude of error increased.


## Acknowledgements

This work is financially supported by the Exploratory Research Grant Scheme (RDU130605) awarded by the Ministry of Higher Education (MOHE) to Universiti Malaysia Pahang (UMP).


Fig. 4. Best fit decreases as the magnitude of error increases.

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