Synchronized Response to Grayscale Image Inputs in the Chaotic Cellular Neural Network

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Abstract

In this article, synchronized response in the chaotic cellular neural network for grayscale visual stimulus was studied in the viewpoint of neural coding. Simple gradation patterns were used as visual stimuli and the synchronized response was analyzed by the correlation of spike firing times. As results, synchronized responses were observed for the neurons which have similar input value and they formed chaotic cell assemblies. Each assembly was distinguished from the others in terms of cross-correlation.

Keywords: chaotic synchronization, neural coding, spike response model, visual segmentation

1. Introduction

Recently, several information coding schemes using synchronized firing of neurons have been proposed[1, 2]. As one of such schemes, the correlated firing is also regarded having an important role for information processing in the brain as a binding mechanism of neural information[1]. On the other hand, chaotic system is well-known to produce various complex behaviors by simple equations. When we consider to represent binding information by the correlated firing, the chaotic firing pattern has an advance in its variety compared to the periodic firing pattern. There is some possibility to represent more various information by the chaotic spike sequence than the periodic sequence[3, 4].

Authors have been studied formation of chaotic cell assembly in the chaotic cellular neural network (Chaotic-CNN) that is a two dimensional coupled network of chaotic spike response model (Chaotic-SRM)[5,6]. The Chaotic-SRM is an extended spike response model [7] that exhibits chaotic inter-spike interval by adding the background sinusoidal oscillation [4, 8]. One fundamental goal of this study is to realize visual segmentation using chaotic synchronization.

In our previous study, chaotic cell assemblies is formed in Chaotic-CNN for each localized stimulus when the stimulus is a two-dimensional binary pattern [6]. For the segmentation of real image, it is, however, necessary to analyze the case that the stimulus has analog continuous values. In this study, synchronized chaotic responses of Chaotic-CNN to the grayscale visual stimulus are numerically analyzed.

2. Chaotic-SRM

The spike response model (SRM) was introduced by Gerstner and Kistler[7]. In SRM, the dynamics of neuron is directory described by kernel functions and it is not necessary to solve differential equations for its simulation. We proposed the Chaotic-SRM that is extended SRM to exhibit the chaotic inter-spike intervals[5]. The definitions of Chaotic-SRM is as follows. The membrane potential $u(t)$ of neuron at time t is defined as

$$u(t) = u_{rest} + \eta\left(t - t^{(f)}\right) + \beta, \qquad (1)$$

where u_{rest}, $t^{(f)}$ and β denote the resting potential of neurons, the last firing time of this neuron and the external input, respectively. The kernel function η describes the response of membrane potential after firing. The definition of the kernel function η is

$$\eta\left(t - t^{(f)}\right) = -\eta_{init} exp\left(\frac{t - t^{(f)}}{\tau_{\eta_0}}\right)\theta\left(t - t^{(f)}\right), \qquad (2)$$

where τ_{η_0} is the time constant of spike response and Θ is the step function such that $\Theta(s)$ is 1 for $s \geq 0$ and 0 for the others. In this model, when the membrane potential exceeds the threshold value ϑ, this neuron is firing and the membrane potential is reset by the update of the last firing time $t^{(f)}$. The term $-\eta_{init}$ is an initial value of the kernel function η after firing. In the original SRM, this term is constant. Therefore, the original SRM is periodically firing and its period is determined by the external input value β.

We extended the original SRM to exhibits chaotic response by adding background sinusoidal oscillation in the same way of the bifurcating neuron[8] and the chaotic pulse coupled neural network[4]. In the extended SRM, the background oscillation is added to the term η_{init} and its definition is

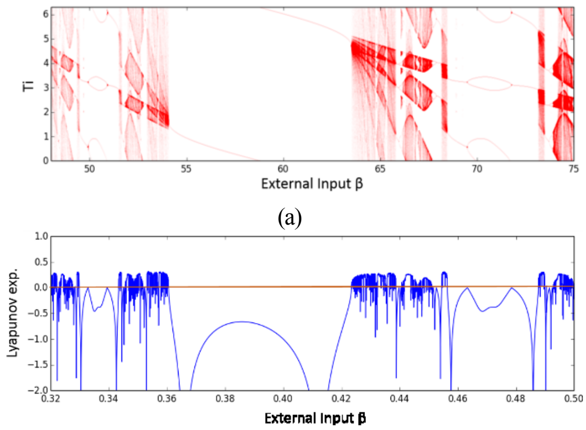
$$\eta_{init} = \eta_0 - A_{\eta_0} sin\left(2\pi\omega_{\eta_0} t^{(f)}\right), \qquad (3)$$

where η_0 denotes the constant η_{init} in the original model and, A_{η_0} and ω_{η_0} denote the amplitude and the frequency of background oscillation. In this article, we call this extended model the Chaotic-SRM. The Chaotic-SRM exhibits various chaotic behavior depending on the parameter values of A_{η_0} and the external input β.

The bifurcation diagram and the Lyapunov exponent for the external input β are obtained by numerical simulation[6] (Fig.1), where the parameter values of Chaotic-SRM are set as follows: $u_{rest} = -70\text{mv}$, $\theta = -35\text{mv}$, $\eta_0 = 55$, $\tau_\eta = 10\text{msec}$, $\omega_{\eta_0} = 0.75/2\pi$ and $A_{\eta_0} = 10.9$. In the bifurcation diagram (Fig.1(a)), the period doubling bifurcation that is typical behavior of chaotic system were observed. In Fig. 1(b), several regions where the Lyapunov exponents is positive, are existing and these regions correspond to the chaotic region in the bifurcation diagram.

3. Chaotic-CNN

The Chaotic-CNN is defined as a two dimensional coupled system of Chaotic-SRM[6]. In the Chaotic-CNN, each neuron is put on the $N \times M$ lattice and connected to



(a)

(b)

Fig.1. (a) The bifurcating diagram and (b) Lyapunov Exponent [6].

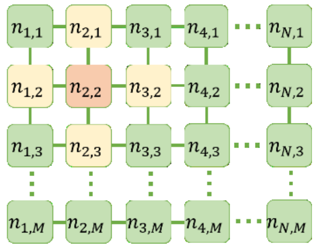


Fig. 2. Chaotic-CNN [6].

the nearest neighbors in four directions (up, down, left and right) as shown in Fig. 2. Let $n_{x,y}$ be a neuron that allocated at the location (x, y). The membrane potential of $n_{x,y}$ is denoted by $u_{x,y}$ and it is defined as

$$\begin{aligned} u_{x,y}(t) &= u_{rest} + \eta\left(t - t_{x,y}^{(f)}\right) + \beta_{x,y} \\ &+\xi \times \left[o_{x-1,y}(t) + o_{x+1,y}(t) + o_{x,y-1}(t) \right. \\ &\left. + o_{x,y+1}(t)\right], \end{aligned} \qquad (4)$$

where $t_{x,y}^{(f)}$, $\beta_{x,y}$ and ξ denote the last firing time, the external input of $n_{x,y}$ and the coupling weight, respectively. The boundary condition is fixed as follows: $o_{x^*,y}(t) = o_{x,y^*}(t) = 0 \quad (x^* \in \{0, N+1\}, y^* \in \{0, M+1\})$.

The function $o_{x',y'}$ is the output from the connected neuron $n_{x',y'}$ and it is also defined as

$$o_{x',y'}(t) = \sum_{t_{x,y}^{(f)} < t_{x',y'}^{(k)} < t} \varepsilon(t - t_{x',y'}^{(k)}), \qquad (5)$$

where $t_{x',y'}^{(k)}$ denotes the k-th firing time of the neuron $n_{x',y'}$. The kernel function ε describes the response of synaptic connection. The definition of the kernel function ε is

$$\varepsilon(s) = \frac{s}{\tau_\varepsilon} exp\left(-\frac{s}{\tau_\varepsilon}\right)\Theta(s), \qquad (6)$$

where τ_ε is the time constant of the synaptic connection.

4. Correlation Analysis

In this article, a synchronized response is analyzed by the correlation. As an index of the synchronized response, the cross-correlation between two neurons are analyzed. Let $S_{x,y}$ be a set of the firing times of the neuron $n_{x,y}$. The cross-correlation between $S_{x,y}$ and $S_{x',y'}$ with time shift ϕ is defined as

$$CC(S_{x,y}, S_{x',y'}; \phi) = \frac{\#(\{t_{x,y}^{(k)} | \exists t_{x',y'}^{(l)} \in S_{x',y'}, |t_{x,y}^{(k)} - t_{x',y'}^{(l)} - \phi| \le \Delta s\})}{\#(S_{x,y})} \quad (7)$$

where $\#(X)$ denotes the number of elements of X and Δs is a time resolution of coincident firing. In this article, Δs is set to 0.5 msec. The auto-correlation is also defined as $AC(S_{x,y}; \phi) = CC(S_{x,y}, S_{x,y}; \phi)$. The maximal value of the cross-correlation is defined as

$$CC^*(S_{x,y}, S_{x',y'}) = \max_{\phi} CC(S_{x,y}, S_{x',y'}; \phi). \quad (8)$$

In the case that $CC^* = CC(S_{x,y}, S_{x',y'}; \phi) \simeq 1$, $S_{x,y}$ and $S_{x',y'}$ are synchronized with the time shift ϕ. In the case that $AC^* = AC(S_{x,y}; \phi) \simeq 1$, $S_{x,y}$ is periodic with the period ϕ. When the spike sequence $S_{x,y}$ is chaotic, $AC(S_{x,y}; \phi)$ exponentially decays for the time shift ϕ.

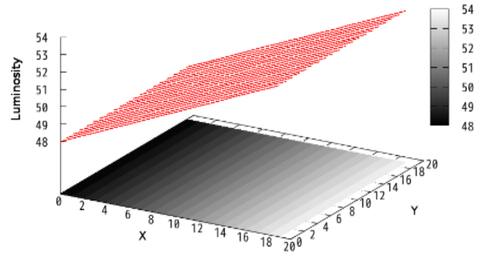
5. Numerical Experiments

As numerical experiments, we simulated the Chaotic-CNN for 20×20 grayscale image patterns shown in Fig. 4(a) and Fig. 5(a), where the parameters of the single neuron are set as the same values mentioned in the section 2 and the parameters of coupling are set as follows: $\tau_\varepsilon = 0.5$msec and $\xi = 4$. The grayscale pixel value $g_{x,y} \in \{0,1,\cdots,255\}$ is mapped to the external input $\beta_{x,y} \in [\beta_0, \beta_1]$ such that

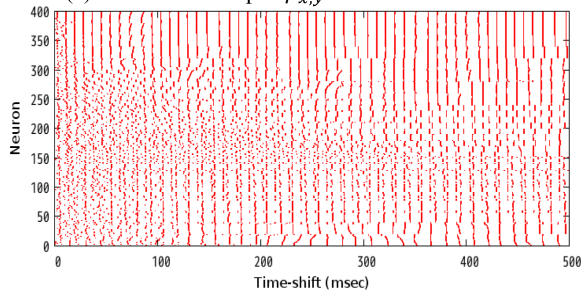
$$\beta_{x,y} = (\beta_1 - \beta_0) \times \frac{g_{x,y}}{255} + \beta_0. \quad (9)$$

In this simulation, we chose the interval [48,54] as $[\beta_0, \beta_1]$ that includes the chaotic region in the bifurcation diagram (Fig.1).

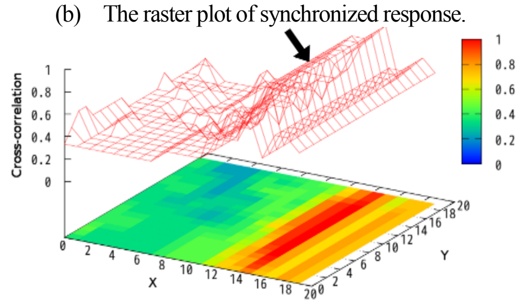
The input pattern shown in Fig. 4(a) is a simple gradation pattern from black to white in the direction of x-axis. The neurons aligned in the direction of y-axis have the same input value. A response of the Chaotic-CNN is shown in Fig. 4(b) as a raster plot of firing times, where the abscissa is time and the ordinate is an id of neuron such that $id = 20x + y$ for the neuron $n_{x,y}$. Spike responses are roughly synchronized for each input values. The maximal cross-correlations between the neuron indicated by the arrow and the others were calculated (Fig.4(c)). As shown in Fig.4(c), high correlation is observed for the neurons in the direction of y-axis and relatively lower correlation is observed for the others. The auto-correlation profile was also calculated for the neuron indicated in Fig. 4(c). As a result, exponential decay of auto-correlation is observed as shown in Fig. 4(d). These results support the formation of the chaotic cell assemblies.



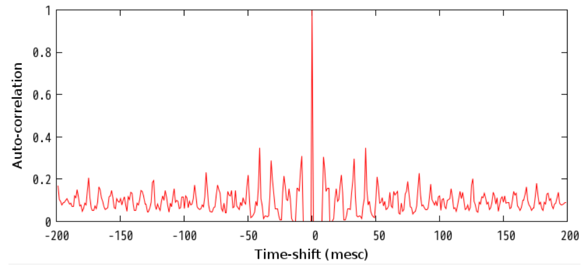
(a) The external input $\beta_{x,y}$ as a visual stimulus.



(b) The raster plot of synchronized response.

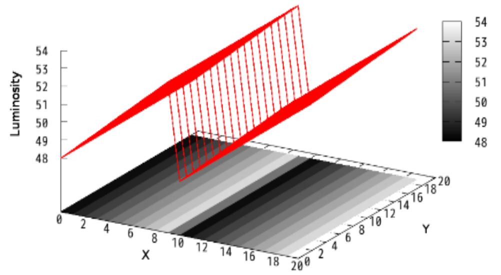


(c) The maximal cross-correlation between the neuron indicated by arrow and the others.

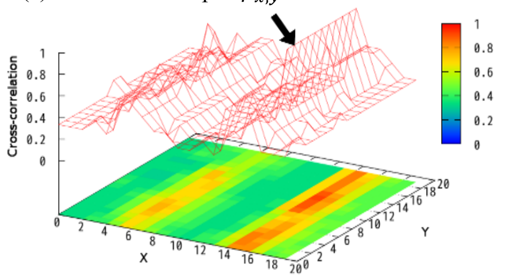


(d) The auto-correlation profile of the neuron indicated in (c).

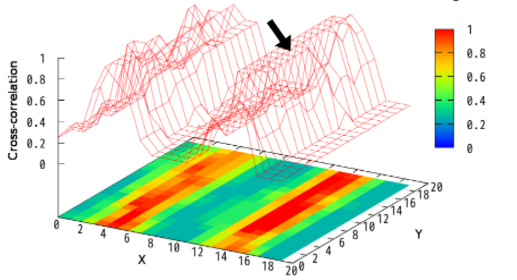
Fig.4. Response to the simple gradation pattern.



(a) The external input $\beta_{x,y}$ as a visual stimulus.



(b) The maximal cross-correlation for the chaotic response.



(c) The maximal cross-correlation for the periodic response.

Fig.5. Response to the repeated gradation pattern.

In order to demonstrate a difference between the chaotic cell assembly and the periodic one, responses to a twice repeated gradation pattern shown in Fig. 5(a) was analyzed. The maximal cross-correlation for the case of the chaotic spike response is shown in Fig. 5(b) and for the case of the periodic one is shown in Fig. 5(c). For the latters, the amplitude of background oscillation A_{η_0} was set to 0 to generate periodic spike responses. For this case, the neuron indicated by arrow has high correlation with neurons in the two regions where neurons have similar input value (Fig. 5(c)). In terms of correlation, these two regions are not distinguished from each other. On the other hand, for the case of the chaotic spike response, the neuron indicated by arrow has high correlation with neurons in one region included itself and has relatively low correlation with the other neurons. In this case, chaotic cell assemblies invoked by the grayscale input is distinguished from the others even if they have similar input value.

6. Conclusions

For grayscale visual stimulus, synchronized response of the Chaotic-CNN was analyzed by the correlation of spike firing times. As results, synchronized responses to the similar input value and a formation of chaotic cell assembly were observed. These results indicate a possibility of visual segmentation using synchronized chaotic response. Analysis for the real image input is one of our future works.

Acknowledgements

This work was partially supported by JSPS KAKENHI Grant Numbers 24650120.

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