Associative Memory with Class I and II Izhikevich Model

Yoshika Osawa
Department of Electrical Engineering and Information Systems, The University of Tokyo
7-3-1 Hongo, Bunkyo-ku, Tokyo 113-8656, Japan

Takashi Kohno
Institute of Industrial Science, University of Tokyo
4-6-1, Komaba, Meguro-ku, Tokyo, 153-8505, Japan
E-mail: osawa@sat.t.u-tokyo.ac.jp, kohno@sat.t.u-tokyo.ac.jp
www.sat.t.u-tokyo.ac.jp/~kohno/

Abstract

Spiking neural network is a system that qualitatively reproduce the nervous system. It was shown in previous researches that the performance of associative memory task in all-to-all connected networks is higher when they are composed of Class II neurons than Class I neurons. The Izhikevich model in its Class II mode, however, does not have this performance boost. In this study, we focus on phase resetting curve as an index that reflects neuronal properties related to neuron classes in detail.

Keywords: Spiking neural network, associative memory, Izhikevich model, Phase resetting curve, Hodgkin's classification

1. Introduction

Nervous system has robust, autonomous, and power-efficient information processing capacity. Neurons generate overshoot of their membrane potential called spike, when sufficiently large stimulus is applied, which travel through their axon and transmit signals to post-synaptic neurons via synapses by transmitter.

There are several approaches in modeling neuronal activities. Conductance-based models that focus on the current through the cell membrane such as the Hodgkin-Huxley model[1] can reproduce those activities precisely. Since this type of models are composed of complex differential equations, large-scale simulation is impractical. The Leaky Integrate-and-Fire (LIF) model[2] represents the spike by resetting of membrane potential. This method yields simple and low-dimensional system equations. It is very easy to simulate using this model but it reproduces limited aspects of neuronal behaviors. In the meanwhile simplified expression while maintaining the mechanism of the conductance-based models develops the qualitative modeling. One of them, Digital Spiking Silicon Neuron (DSSN) model[3] is composed of two differential equations that expresses the spike generation process without reset of system variables. The Izhikevich (IZH) model[4] captures only the spike decision process qualitatively, and represents the spike by reset of its system variables. It is possible to reproduce a variety of spike patterns.

Neurons are classified by characteristics of the periodic firing in the Hodgkin's classification[5]. Associative memory simulation using the DSSN model built by Li et al.[6] showed that an all-to-all connected network of Class II neurons has higher recall ability than that of

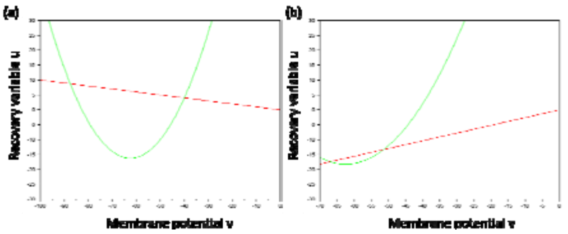

Fig. 1. The phase plane of Izhikevich model in (a) Class I and (b) Class II modes.

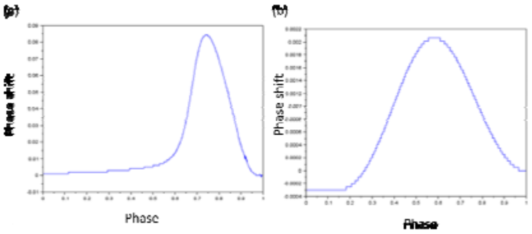

Fig. 2. Phase resetting curve of Izhikevich model in (a) Class I and (b) Class II modes.

Class I neurons. Higher recall ability here means that an original stored pattern is recalled from an associated input pattern with higher error. We configured similar associative memory using the IZH model and evaluated the difference of its performance by focusing on the shape of the phase resetting curve (PRC)[7].

2. Neuron Model and Synapse Model

2.1. Izhikevich Model

The IZH model captures the mechanism of spike decision process based on its bifurcation structure. Saddle-node and Hopf bifurcations produce Class I and II properties, respectively. In the IZH model, this mechanism is described by two differential equation (1), and membrane potential *v* is reset when it exceeds 30.

$$
\begin{aligned}
&\frac{dv}{dt} = 0.04v^2 + 5v + 140 - u + I \\
&\frac{du}{dt} = a(bv - u) \\
&\text{if } v > 30 \text{ then} \\
&v \leftarrow c \\
&u \leftarrow u + d
\end{aligned}
\tag{1}
$$

Here, *u* is a variable that represents membrane recovery the parameters *a*, *b*, *c*, and *d* are constants that specify the model's firing property, and *I* represents a stimulus input. The phase portraits of Class I and II neuron modes of this model are described in Fig. 1. Class II

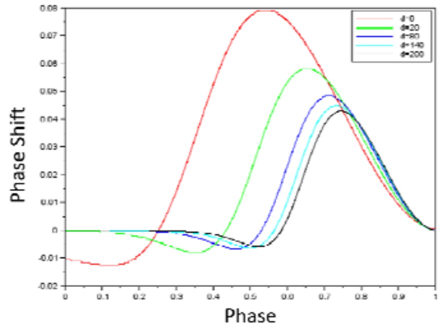

Fig. 3. Change of the shape of the phase resetting curve depending on the resetting parameter *d*.

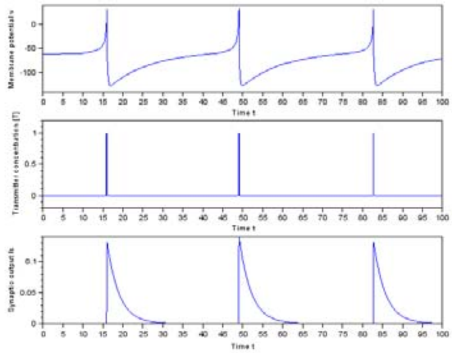

Fig. 4. Time course example of membrane potential, transmitter concentration, and synaptic output.

neuron arise periodic firing before he intersection of the two lines disappear although no occur in Class I. The PRC is a plot of spiking phase shift in a neuron at periodically spiking state induced by sufficiently small pulse stimulus. Its horizontal axis is the phase at which the stimulus is applied. Fig. 2 shows the PRC of the IZH model. There are two types of the shape. The Type 1 is always positive, and Type 2 is biphasic. The PRC of the IZH model is Type 1 and 2 in Class I and II models, respectively. The PRC of Class II has a jump point induced by reset. Its jump is reduced by reset parameter *d* (Fig. 3).

2.2. Synapse Model

Synapses transmit signals to post-synaptic neurons. Chemical synapses release transmitter when its pre-synaptic neuron fires. In our model, the neurotransmitter is in the same manner as Li et al., approximated to vary in a pulse form while the spike exceeds a threshold. The expression of synaptic output I_s is shown in Eq. (2)[8].

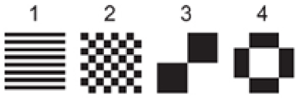

Fig. 5. Stored patterns in our associative memory simulation.

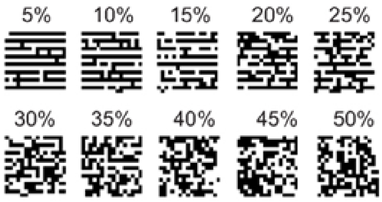

Fig. 6. Examples of input patterns, generated by applying errors to the stored pattern 1.

$$\frac{dI_s}{dt} = \begin{cases} \alpha(1 - I_s) & ([T] = 1) \\ -\beta I_s & ([T] = 0) \end{cases} \tag{2}$$

Here, α and β are constants that represent the time constant of rising and phase of synaptic outputs, respectively. The amount of transmitter [T] is set to 1 when $v > 0$, and 0 when $v < 0$. Figure 4 represents example of time variation of the membrane potential, neurotransmitter concentration, and the synaptic current.

3. Simulation

We examined the performance of the associative memory composed of Class II Izhikevich model varying resseting parameter d. The associative memory is composed of all-to-all connected networks and outputs a stored pattern similar to the input data. In this work, p patterns of N dots which are orthogonal to each other are stored ($p = 4$, $N = 256$). The patterns are listed in Fig. 5. The weight W_{ij} of the synapses from i-th to j-th neuron is calculated by Eq. (3).

$$W_{ij} = \begin{cases} \sum_{\mu=1}^{p} x_i^{\mu} x_j^{\mu} & (i \neq j) \\ 0 & (i = j) \end{cases} \tag{3}$$

Input patterns are generated by applying random errors to a stored pattern (Fig. 6). We made 100 input patterns for every error rate. The error level vary from 5 to 50% with basically 5% steps and 1% in a region where the recall rate starts to decrease rapidly.

Firstly, we apply a pulse stimulus only to the neurons corresponding black dot. Then, stimulus current to all neurons was set to the value that induce periodic firing.

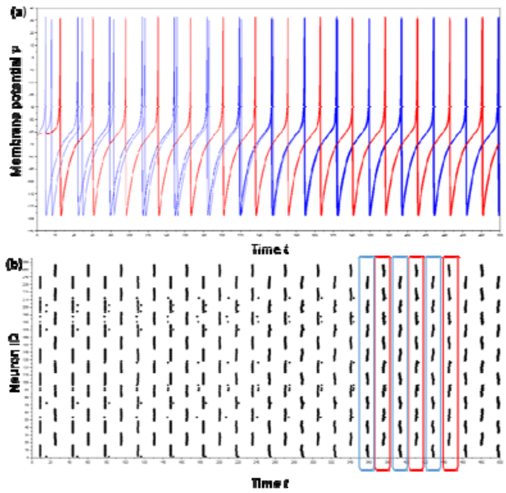

Fig. 7. (a) Waveform example of membrane potential and (b) its raster plot. Input pattern is the upper left in Fig. 6. Blue and red waveforms and squares correspond to the black and white dots of the stored pattern 1 in Fig. 5.

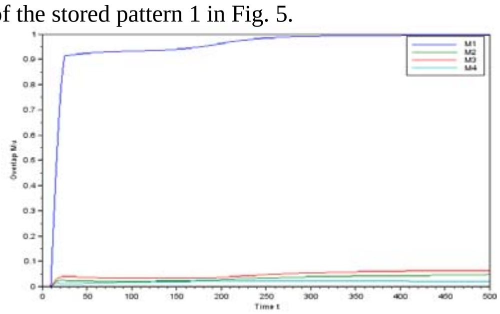

Fig. 8. Time course of the overlap $M_u(t)$ that corresponds to Fig. 7(a).

Figure 7(a) is an example of time series data of 256 neuron's membrane potential when the upper left pattern was input. Figure 7(b) shows the raster plot of this example. In this figure, we can see that at early phase of the recall process the neurons fire relatively in an asynchronous manner. The synchronicity increases as the neurons continue firing. And finally the neurons that correspond to black (white) dots fire synchronously. These two synchronously firing neuron groups are antiphasic. To assess this quantitatively, we used overlap $M_u(t)$ that calculates the matching degree of u-th stored pattern from the phase in the period of each neuron at a time t (Eq. (4)).

$$\phi_j(t) = 2\pi k + 2\pi \frac{t - t_j^k}{t_j^{k+1} - t_j^k}$$
$$M_u(t) = \frac{1}{N} \left| \sum_{j=1}^{N} x_j^u \exp\left(i\phi_j(t)\right) \right| \tag{4}$$

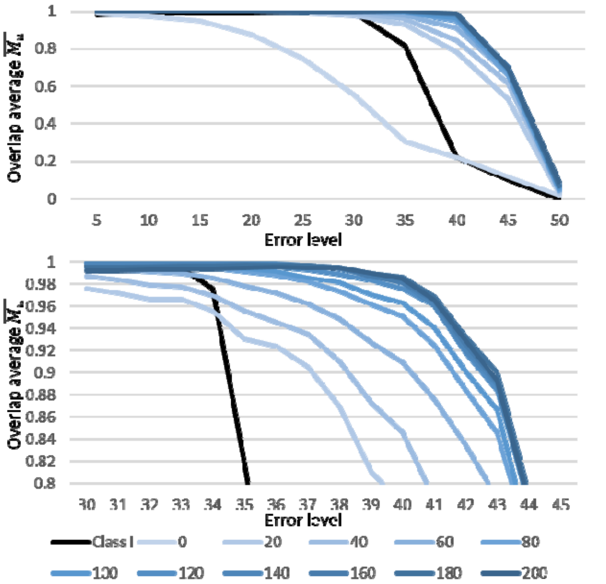

Fig. 9. Overlap average $\overline{M_u}$ of each error level. (a) From 5% to 50% by 5%. (b) From 30% to 45% by 1%.

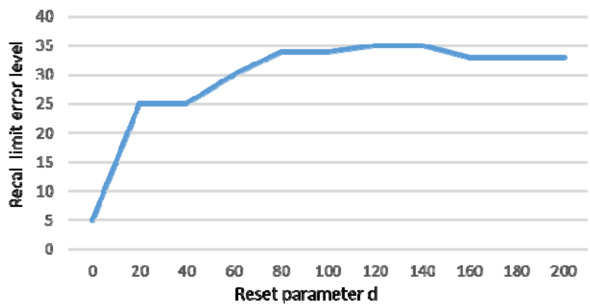

Fig. 10. Maximum input error level with which the recall is correct (M_u > 0.95 for all trial).

Here, t_i^k is the time that *k*-th spike is generated. M_u = 1 indicates that the *u*-th pattern is recalled successfully. The time course of $M_u(t)$ calculated by applying Eq. (4) to the data in Fig.7 is shown in Fig. 8. This result indicates that the stored pattern 1 is recalled correctly.

We performed simulation for 100 input patterns for each error level. The reset parameter *d* of each neuron was varied from 0 to 200 by 20 in this simulation. Figure 9 shows the overlap average $\overline{M_u}$ for each error level. Since that the change of $\overline{M_u}$ is relatively large when input error level is 30-45%, we varied the error level by 1% step. We define that the recall is correct when M_u > 0.95 for all trials. Figure 10 plots the maximum input error rate from which our network recalled the correct pattern. As shown in Fig. 10, it got almost saturated in the vicinity of *d* = 120. Compared with the shape of PRC shown in Fig. 3, the balance between the regions of positive and negative value of the phase shift may be important for the performance of associative memory.

4. Conclusion

We have configured the associative memory consisting of 256 neurons of the IZH model. The performance of Class II network was worse than Class I, but the performance of the associative memory is improved by increasing the reset parameter *d*, by which the shape of the PRC changes. In the future, the validation by a phase model representing the shape of the quantitative PRC will be performed to quantitatively evaluate how the shape of the PRC contributes to the performance of the associative memory.

Acknowledgements

This work was supported by JSPS Grant-in-Aid for scientific Exploratory Research Number 25240045

References

1. Hodgkin, A. L, and Huxley, A. F, A quantitative description of membrane current and its application to conduction and excitation in nerve. (J. Physiol. 117, 500–544, 1952).
2. Stein R.B, Some models of neuronal variability (Biophysical Journal, 7: 37–68, 1967).
3. Kohno, T. and Aihara, K, Digital spiking silicon neuron: concept and behaviors in GJ-coupled network (inProceedings of International Symposium on Artificial Life and Robotics, Beppu, OS3–OS6, 2007).
4. Izhikevich, E. M, Dynamical Systems in Neuroscience: The Geometry of Excitability and Bursting (Cambridge: MIT Press, 2006).
5. Hodgkin, A. L, The local electric changes associated with repetitive action in a nonmedullated axon (J. Physiol. 107, 165-181, 1948).
6. J. Li, Katori, Y, and Kohno, T, An FPGA-Based Silicon Neuronal Network with Selectable Excitability Silicon Neurons (Frontiers in neuroscience 6 183, 2012).
7. Ermentrout B, Type I membranes, phase resetting curves and synchrony (Neural Computation 8:979-1001, 1996).
8. Destexhe, A., Mainen, Z. F., and Sejnowski, T. J, Kinetic models of synaptic transmissionm, (in Methods in Neuronal Modeling, eds C. Koch and I. Segev Cambridge, FL: MIT Press, 1–25, 1998).