Distributed Terminal Backstepping control for Multi-Agent Euler-Lagrange Systems

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Abstract

This paper presents a distributed terminal (finite-time) backstepping consensus control for multi-agent Euler-Lagrange systems. Terminal virtual error surfaces and virtual controls are proposed to guarantee the finite-time error consensus and formation convergence of a group of one-leader and multi-follower cooperative tracking Euler-Lagrange system. Finite-time stability including infinite-time stability was proved by the finite-time Lyapunov candidate function. Simulation example shows the effectiveness of the proposed finite-time backstepping coordinated tracking controller.

Keywords: Euler-Lagrange multi-agent system, backstepping control, Terminal virtual error surface.

1. Introduction

In recent years, there has been a great interest for researches of multi-agent systems, whose applications include spacecraft, mobile robots, sensor networks, etc. Interesting research directions are containment control, consensus, formation, and flocking control [1]. These problems focus on two cases, namely, the case that there does not exist a leader and the case where there exists a leader. The coordinate tracking problems to track a single leader have been investigate for followers with single-integrator, double-integrator,high-order dynamics, nonlinear or Euler-Lagrange dynamics [2-5]. Linear control theory and variable structure control methods in most researches are used. On the other hand, there are few examples that use the backstepping control technique [6] for nonlinear or Euler-Lagrange multi-agent system. In this method, the problem of unmatched uncertainty and neglecting the efficient nonlinearities is overcome via adopting step-by-step recursive process.

However, although a controller designed using this theorem guarantees the infinite-time stability of a closed-loop system, it has drawbacks such as a slow convergence rate and reduced robustness to uncertainty. On the other hand, systems with finite-time settling-time design possess attractive features such as improved robustness and disturbance rejection properties [7], In this paper, terminal backstepping control based multi-agent consensus control for Euler-Lagrange system with one-leader and multi-followers is developed.

2. Background and Preliminaries

2.1. *Concept of Graph Theory*

In this paper, multi-agent robot Euler-Lagrange systems consisting of one leader and n followers are considered. Graph theory is introduced to solve the coordination problem and model information exchange between agents. The communication topology is a directed graph, $\mathcal{G}=\{\mathcal{V},\mathcal{E}\}$, where $\mathcal{V}=\{0,1,2, \ldots,n\}$ is the set of nodes, node i represents the ith agent, $\mathcal{E}$ is the set of edges, and an edge in $\mathcal{G}$ is denoted by an ordered pair (i, j). $(i, j) \in \mathcal{E}$ if and only if the ith agent can send information to jth agent directly, but not necessarily vice versa. A directed tree is a directed graph, where every node has exactly one parent except for the root, and the root has directed paths to every other node. A directed graph , $\mathcal{G}=\{\mathcal{V},\mathcal{E}\}$, has a directed spanning tree if and only if $\{\mathcal{V},\mathcal{E}\}$ has at least one node with a directed path to all other nodes. $A=[a_{i,j}]\in R^{(n+1)\times(n+1)}$ is called the weighted adjacency matrix of $\mathcal{G}$, where $a_{ii}=0$ and $a_{ij}\ge 0$ with $a_{ij}>0$ if there is an edge between the ith agent and jth . The Laplacian of the weighted graph can be defined as $L=D-A\in R^{(n+1)\times(n+1)}$, where $D=diag(d_0,d_1,...,d_n)$ $\in R^{(n+1)\times(n+1)}$ is the degree matrix and $d_i=\sum_{j=0}^{n}a_{ij}$ for $i=0,1,...,n$. For simplicity, it is assumed that $a_{ij}=1$ if $(i, j)\in\mathcal{E}$ and 0 otherwise. The connection weight between agent i and the leader is denoted by b_i such that $b_i=1$ if agent i connected to the leader and 0 otherwise.

2.2. *Multi-Agent Euler-Lagrange Systems*

The nonlinear dynamics of a group of $n+1$ fully actuated Euler-Lagrange systems are described as follows:

$$M_i(q_i)\ddot{q}_i+C_i(q_i,\dot{q}_i)\dot{q}_i+G_i(q_i)+\tau_{di}=\tau_i\ ,\ i=1,...,n+1\ , \qquad (1)$$

where $M_i(q_i)$ is a symmetric and positive definite inertia matrix; $C_i(q_i,\dot{q}_i)$ is a velocity-dependent centripetal and Coriolis forces matrix; $G_i(q_i)$ is a gravitational vector; τ_{di} is a bounded unknown disturbance including unmodelled dynamics and exogenous disturbance; and τ_i is an input torque. The simple dynamic equation can be expressed as the following state space model:

$$\begin{aligned}&\dot{x}_{i,1}=x_{i,2}\ ,\\&\dot{x}_{i,2}=f_i(\overline{x}_2)+g_i(\overline{x}_2)u_i\ ,\\&y_i=x_{i,1}\ ,\quad i=1,...,n+1\ ,\end{aligned} \qquad (2)$$

where $x_{i,1}=q_i$, $x_{i,2}=\dot{q}_i$, $\overline{x}_2=[x_{i,1},\ x_{i,2}]^T$, $f_i(\overline{x}_2)=$ $-M_i^{-1}C_i(\overline{x}_{i,2})x_{i,2}\ -M_i^{-1}G_i(x_{i,1})-M_i^{-1}\tau_{di}$, $g_i=M_i^{-1}$, and $u_i=\tau_i$.

Assumption 1. $\|M_i^{-1}\tau_{di}\|\le\delta_{di}$, $\|K_i^C-M_i^{-1}C_i(\overline{x}_{i,2})\|\le\delta_{ci}$, $\|K_i^G x_{i,1}-M_i^{-1}G_i(x_{i,1})\|\le\delta_{gi}$, and $\delta_{ci}+\delta_{gi}+\delta_{di}\le\delta_{hi}$, where K_i^C and K_i^G are positive definite diagonal matrices and vectors, respectively, and $\delta_{hi}>0$ are upper bounds.

3. Distributed Terminal backstepping Controller Design and Stability Analysis

3.1. *Controller Design*

The tracking errors and virtual error surfaces are defined as follows:

$$z_{i,1}=\sum_{j=1}^{n}a_{ij}(y_i-y_j)+b_i(y_i-x_0)\ , \qquad (3)$$

$$z_{i,2}=x_{i,2}+c_{i,1}sig(z_{i,1})^{\gamma_{i,1}}-\alpha_{i,1}\ ,\ i=1,...,n\ , \qquad (4)$$

where x_0 is the position of the leader, $\alpha_{i,1}$ are the virtual controls, $sig(z_{i,1})=\|z_{i,1}\|^{\gamma_{i,1}}\operatorname{sgn}(z_{i,1})$, $c_{i,1}>0$ are constants, and $\gamma_{i,1}=\xi_{i,1}/\zeta_{i,1}$, $\xi_{i,1}$ and $\zeta_{i,1}$ are positive odd numbers, $\xi_{i,1}<\zeta_{i,1}<2\xi_{i,1}$, $\operatorname{sgn}(z_{i,1})$ is a sign function,. (3) can be changed for the formation control case as follows:

$$z_{i,1}=\sum_{j=1}^{n}a_{ij}(y_i+\Delta_i-y_j-\Delta_j)+b_i(y_i+\Delta_i-x_0-\Delta_0) \qquad (5)$$

The time derivative of the first error surfaces $z_{i,1}$ along (2) is

$$\begin{aligned}\dot{z}_{i,1}&=(d_i+b_i)(z_{i,2}-c_{i,1}sig(z_{i,1})^{\gamma_{i,1}}+\alpha_{i,1})\\&\quad-\sum_{j=1}^{n}a_{ij}x_{i,2}-b_i\dot{x}_0\ .\end{aligned} \qquad (6)$$

The Lyapunov function candidate $V_{i,1}=z_{i,1}^T z_{i,1}/2$ to design the distributed virtual controller. Differentiating $V_{i,1}$ yields

$$\begin{aligned}\dot{V}_{i,1}&=z_{i,1}^T[(d_i+b_i)(z_{i,2}-c_{i,1}sig(z_{i,1})^{\gamma_{i,1}}+\alpha_{i,1})\\&\quad-\sum_{j=1}^{n}a_{ij}x_{i,2}-b_i\dot{x}_0]\ .\end{aligned} \qquad (7)$$

Choosing the distributed virtual control as

$$\alpha_{i,1}=\frac{1}{d_i+b_i}\left(-k_{i,1}z_{i,1}+\sum_{i=1}^{n}a_{ij}x_{i,2}+b_i\dot{x}_0\right), \qquad (8)$$

(7) becomes

$$\dot{V}_{i,1}=-k_{i,1}z_{i,1}^T z_{i,1}-(d_i+b_i)c_{i,1}\|z_{i,1}\|^{\gamma_{i,1}+1}+(d_i+b_i)z_{i,1}^T z_{i,2}\ , \qquad (9)$$

where $k_{i,1}>0$ are constants. Differentiating the Lyapunov function, $V_{i,2}=V_{i,1}+z_{i,2}^T z_{i,2}/2+\tilde{\delta}_{hi}^2/2\eta_i$, along (2) and (4),

$$\begin{aligned}\dot{V}_{i,2}&=-k_{i,1}z_{i,1}^T z_{i,1}-(d_i+b_i)c_{i,1}\|z_{i,1}\|^{\gamma_{i,1}+1}+(d_i+b_i)z_{i,1}^T z_{i,2}\\&\quad+z_{i,2}^T[f_i(\overline{x}_2)+g_i(\overline{x}_2)u_i+c_{i,1}\gamma_{i,1}\|z_{i,1}\|^{\gamma_{i,1}-1}\dot{z}_{i,1}-\dot{\alpha}_{i,1}]\\&\quad-\tilde{\delta}_{hi}\dot{\hat{\delta}}_{hi}/\eta_i\ .\end{aligned} \qquad (10)$$

Choosing the control inputs and adaptive laws as

$$\begin{aligned}u_i&=g_i^{-1}[-k_{i,2}z_{i,2}-(d_i+b_i)z_{i,1}^T+K_i^C x_{i,2}+K_i^G x_{i,1}\\&\quad-c_{i,1}\gamma_{i,1}\|z_{i,1}\|^{\gamma_{i,1}-1}\dot{z}_{i,1}-c_{i,2}sig(z_{i,2})^{\gamma_{i,2}}\\&\quad+\frac{\hat{\delta}_{hi}z_{i,2}}{\|z_{i,2}\|+\kappa_{i,2}}+\dot{\alpha}_{i,1}]\ ,\end{aligned} \qquad (11)$$

$$\dot{\hat{\delta}}_{hi}=\eta_i\left(\|z_{i,2}\|^2/(\|z_{i,2}\|+\kappa_{i,2})-\eta_i'\hat{\delta}_{hi}\right), \qquad (12)$$

where $k_{i,2} > 0$, $\eta_i > 0$, $\eta_i' > 0$, $c_{i,2} > 0$, and $0.5 < \gamma_{i,2} < 1$ are constants, $\tilde{\delta}_{hi} = \delta_{hi} - \hat{\delta}_{hi}$, $\hat{\delta}_{hi}$ are estimates of δ_{hi}, we obtain the following expression:

$$\dot{V}_{i,2} \le -\sum_{k=1}^{2} k_{i,1} z_{i,k}^T z_{i,k} - (d_i + b_i) c_{i,1} \left\| z_{i,1} \right\|^{\gamma_{i,1}+1} - c_{i,2} \left\| z_{i,2} \right\|^{\gamma_{i,2}+1}$$
$$+ \tilde{\delta}_{hi} (z_{i,2}^T - \dot{\hat{\delta}}_{hi} / \eta_i)$$
$$\le -\sum_{k=1}^{2} k_{i,1} z_{i,k}^T z_{i,k} - \frac{\eta_i' \tilde{\delta}_i^2}{2} - \sum_{k=1}^{2} \beta_{i,k} \left\| z_{i,k} \right\|^{\gamma_{i,k}+1} + \eta_i' \delta_i^2 / 2$$
$$\le -\left(\sum_{k=1}^{2} k_{i,k} z_{i,k}^T z_{i,k} + \frac{\eta_i' \tilde{\delta}_i^2}{2} \right) - \left(\sum_{k=1}^{2} \beta_{i,k} z_{i,k}^T z_{i,k} + \frac{\eta_i' \tilde{\delta}_i^2}{2} \right)^{\frac{\gamma_{i,k}+1}{2}}$$
$$+ \mu_i$$
$$\le -a V_{i,2} - b^{\frac{\gamma_{i,k}+1}{2}} V_{i,2}^{\frac{\gamma_{i,k}+1}{2}} + \mu_i \,, \qquad (13)$$

where $\beta_{i,k} = \min[(d_i + b_i) c_{i,1}, c_{i,2}]$, $a = \min[2k_{i,1}, 2k_{i,2}, \eta_i']$,

$b = \min[2\beta_{i,1}, 2\beta_{i,2}, \eta_i']$, $\mu_i = \eta_i' \left\| z_{i,k} \tilde{\delta}_i \right\|^2 + \left(\frac{\eta_i' \tilde{\delta}_i^2}{2} \right)^2 + \frac{\eta_i' \delta_i^2}{2}$

3.2. *Finite-Time Stability Analysis*

((13) can be rewritten the following two forms:

$$\dot{V}_{i,2} \le -a V_{i,2} - \left(b^{\frac{\gamma_{i,k}+1}{2}} - \frac{\mu_i}{V_{i,2}^{\frac{\gamma_{i,k}+1}{2}}} \right) V_{i,2}^{\frac{\gamma_{i,k}+1}{2}}$$
$$= -a' V_{i,2} - b' V_{i,2}^{\gamma'} \,, \qquad (14)$$

where $a' = a - \mu_i / V_{i,2}$, $b' = b^{\frac{\gamma_{i,k}+1}{2}} - \mu_i / V_{i,2}^{\frac{\gamma_{i,k}+1}{2}}$, and $\gamma' = \frac{\gamma_{i,k}+1}{2}$. From (14), if a and b is selected such that $a > \mu_i / V_{i,2}$ and $b > \mu^{\frac{2}{\gamma_{i,k}+1}} / V_{i,2}$, respectively. Then, from the definition of finite-time stability [7], the equilibrium point $x = 0$ is globally finite-time stable and the settling time t_s can be given by

$$t_s \le \frac{1}{a(1-\gamma')} \ln \frac{a V^{1-\gamma'}(x_0) + b'}{b'} \,. \qquad (15)$$

4. Simulation Example

To validate the proposed control scheme, the following group of one leader indexed by 0 and four followers indexed by 1, 2, 3, and 4, respectively as shown in Fig. 1. The strict feedback state equations of each agent are expressed as

$$x_{i,1} = x_{i,2} \,,$$
$$x_{i,2} = f_{i,2}(x_{i,1}) + g_{i,2} u_i \,, \qquad (16)$$

where $f_{i,2} = -[G_i(q_i) + \tau_{di}] / J_i$, $g_i = 1 / J_i$, $u_i = \tau_i$ $J_i = m_i L_i^2 / 3$, $G_i(q_i) = m_i L_i \cos q_i$, the mass of the link $m_i = 1kg$, and the length of link $L_i = 0.25m$. Let the initial condition of four followers be $x_{1,1} = 1, x_{1,2} = 0$, $x_{2,1} = 1.2, x_{2,2} = 0$, $x_{3,1} = 2, x_{3,2} = 0$, $x_{4,1} = -1.2$, $x_{4,2} = 0$. The Laplacian can be written as

$$L = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & -1 & -1 & -1 \\ 0 & 0 & 1 & -1 & 0 \\ -1 & 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 \end{bmatrix}, \; b_3 = 1, \; b_4 = 1 \,.$$

Simulation results are obtained with the time-varying control input to the leader being designed as $u_0 = -\sin(x_{0,1}) / (1 + e^{-t})$, $x_{0,1} = \pi / 2$, and $x_{0,2} = 0$.

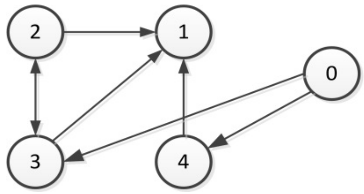
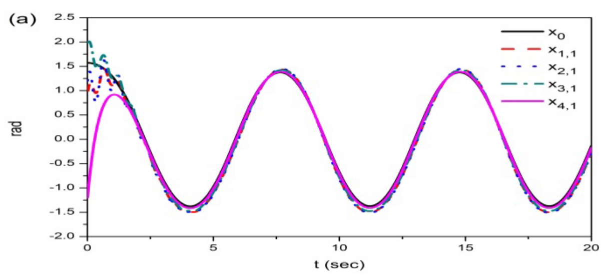


Fig. 1. Directed graph of the manipulator group

The error functions for the illustration of the formation) control are changed into (5), where $\Delta_1 = -1$, $\Delta_2 = -2$, $\Delta_3 = -3$, and $\Delta_4 = -4$. Simulation results are presented in Fig. 2 (consensus control) and Fig. 3 (formation control), where the settling time of the proposed TBSC system is 31% faster than that of the BSC system. In addition, the steady state errors of the TBSC system are smaller compared to the BSC system.

5. Conclusion

A terminal backstepping control scheme to guarantee the fast error convergence and small tracking error performance for a multi-agent Euler-Lagrange system is developed in this paper. A virtual finite-time error surface is defined to design a virtual control. The finite-time convergence is proved by the finite-time stability analysis of Lyapunov function. Simulation for one-link manipulator agents confirms the theoretical proposal.



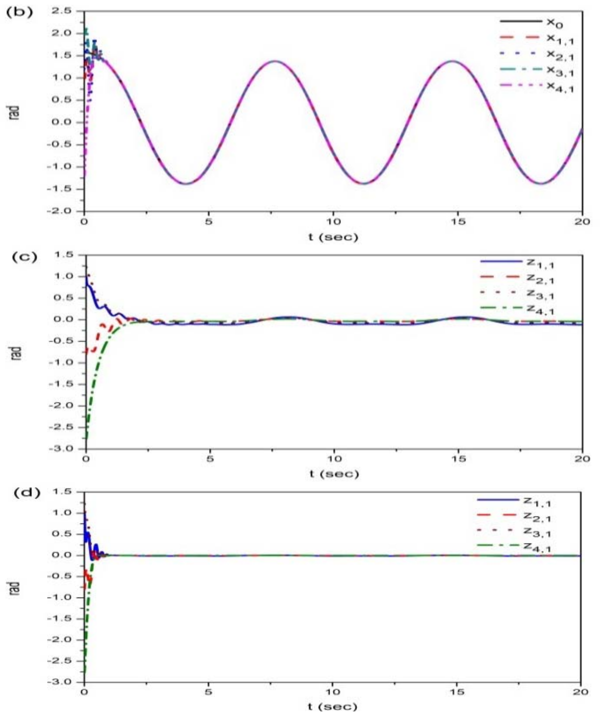


Fig. 2. Consensus control simulation results. (a) Tracking outputs of BSC system. (b) Tracking outputs of TBSC system. (c) $z_{1,1}$ of BSC. (d) $z_{1,1}$ of TBSC.

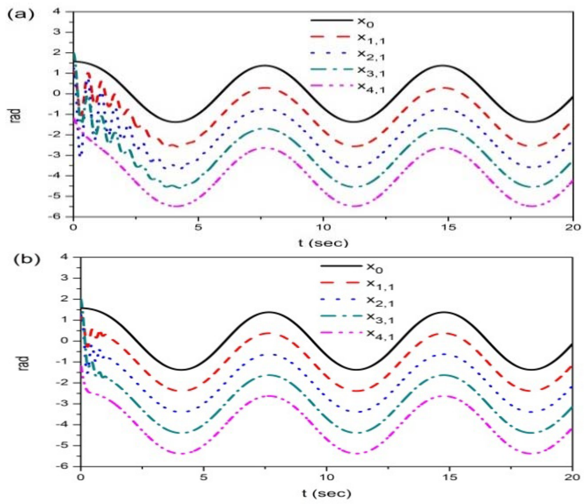


Table 1. Settling time (sec) of BSC and TBSC systems

	Consensus		Formation	
	BSC	TBSC	BSC	TBSC
$z_{1,1} \le 0.01$	2.03 s	0.77 s	2.23 s	0.83 s
$z_{2,1} \le 0.05$	2.20 s	0.67 s	2.63 s	0.91 s
$z_{3,1} \le 0.01$	2.07 s	0.76 s	2.83 s	0.86 s
$z_{4,1} \le 0.05$	2.15 s	0.42 s	1.46 s	0.29 s
Mean (%)	100%	32%	100%	31%

Acknowledgements

This research was supported by the MOTIE (Ministry of Trade, Industry & Energy), Korea, under the Industry Convergence Liaison Robotics Creative Graduates Education Program supervised by the KIAT (N0001126).

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