Probability of mixing up a nearest neighbor robot under target enclosure by robot swarm

Masao KUBO
Dep. of Computer Science National Defense Academy of Japan
Yokosuka,Kanagawa,239-8686,Japan Email: masaok@nda.ac.jp
Hiroshi Sato
Dep. of Computer Science National Defense Academy of Japan
Yokosuka,Kanagawa,239-8686,Japan Email: hsato@nda.ac.jp
Akihiro Yamaguchi
Department of Information and Systems Engineering
Fukuoka Institute of Technology Fukuoka, 811-0295, Japan Email: aki@fit.ac.jp

***Abstract*—This paper presents a robot swarm model to enclose a target. The robots use information of the target and their each neighbors. Every robots have a own but same torch like signal emitter and they can observe the sum of the intensity of their torches. In this paper, the robot uses a direction with strongest intensity of the emitter as direction of its nearest neighbor. We expect that this new approach makes the robot swarm more simple and scalable. We confirm this model by computer simulations.**

I. INTRODUCTION

In this paper, a new implementation of a robot swarm for enclosing a target is examined. The swarm employs a same limited transmission range signal emitter. We suppose that this communication system makes a large swarm be built more easier.

Target enclosure task, which is useful for monitoring disaster sites and unknown vehicles, has recently become an important goal for multiple robots. Robots can operate in dangerous circumstances, replacing human presence. These sites are usually far from where its operator is. It happens that the group of robots notices the fact of the exact number of sites to be observed and their location. Therefore, it is desirable that the larger number of robots than the necessary size is employed. At least, it will accept the large number of targets than their expectation.

We focused on the study of Takayama et al.[6]. In this model, each robot needs information of directions to one neighbor and to its target. As in other studies, this model also requires the Hamiltonian cycle constraint[6][3]. Recently, research[1] uses hybrid system theory[7][4] and shows robots controlled by Takayama's work referencing nearest neighbors can also enclose a target. The robot swarm with this new reference model can enclose targets while the size of group is changing[2].

We focus on a system with limited range homogeneous signal emitter. We call it torch system. For example, Kilobot[5] has a same small LED emitter as a communication device and a flock of 1000 kilobots can form a large shape. However, the transmission range of their LED light is about 6 robot length. Swarmbot[8] proposes that the robot having a homogeneous light can collect the sufficient number of robots to solve a task. Each robot has same color lights around its body. The intensity of light becomes strong as robots increase. If the intensity of light is too strong a robot will not join the group.

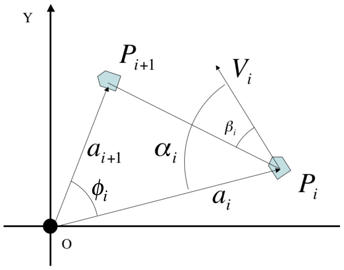

Fig. 1. Model of Takyama's enclosing a target algorithm: α, β.

Advantage of this torch system is its high procurability. The very limited transmission range provides less interference of communication. Therefore, emitters which use same signal band can be installed into many robots. Also, its small energy consumption is also an advantage. The more smaller range communication only needs smaller energy consumption.

In this paper, we examine an robotic swarm with torch system for the target enclosing task. We adopt Takayama's work for the control scheme of robot but the referencing robot is different to this work. We propose that a most powerful signal direction is used as the direction of the referencing robot. We call this direction "MOPS" direction. In this paper, we show the proposed robotic swarm can enclose a target successfully by computer simulation. We discuss the probability of mixing up a nearest neighbor robot under the target enclosure task by robot swarm.

II. TAKAYAMA'S TARGET ENCLOSING MODEL

Firstly, Takayama's target enclosing model is explained.

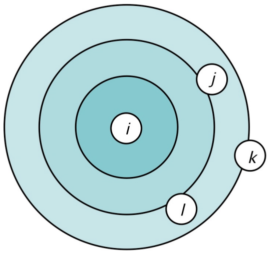

Fig. 2. Diffusion of light of torch

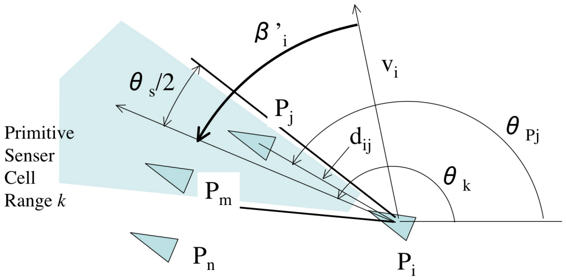

Fig. 3. The most powerful signal(MOPS) direction

We assume that all robots choose the same target. We assume that on a two-dimensional (2D) plane, there is only one target O at the origin and n robots. Robots are numbered counterclockwise as $P_1, \ldots, P_n$, and r_i is the position vector of the robot P_i.

To achieve this task, Takayama et al.[6] proposed the following model. Each robot determines its control input, speed v_i, and angular velocity ω_i using two aspects of angular information: relative angles with respect to the target and an anterior neighboring robot,denoted as α_i and β_i, respectively. As a result, rotational movement occurs with a central focus on the target.

$$v_i = f\beta_i \tag{1}$$

$$\omega_i = v_i/\bar{r} - k\cos\alpha_i \tag{2}$$

where the parameters $\bar{r}, k$, and $f > 0$ specified beforehand. P_{i+1} is the robot to which P_i refers, and $\bar{r}$ is the expected distance to the target. In Takayama et al.'s model, the i-th robot refers to the $i+1$-th robot, and the n-th robot refers to the first robot P_1.

A. Nearest neighboring robot as the reference

[2] examined a new reference robot scheme in which each robot considers its nearest neighboring robot as its reference robot. Each robot controls itself as described in equations 1 and 2, but it chooses its nearest neighbor as its reference robot. This system has higher scalability because individual robots need not be identified to observe the nearest robot.

III. THE PROPOSED REFERENCE MODEL: THE MOST POWERFUL SIGNAL DIRECTION REFERENCE MODEL,MOPS MODEL

We propose a new robotic swarm for the taget enclosure task. Each robot has the own but same specification torch. Additionally we propose a most powerful direction of their signal is used as the direction of the referencing robot. First, we explain the torch which the robot has.

A. Torch

Every agent has a same torch and it can turn on/off its torch. The maximum intensity of all the torch is same and known beforehand. We suppose that the speed of propagation of the signal of the torch is so fast that an agent can observe the signal immediately.As shown in Fig.2, during the flight of the signal, the power of the signal is attenuated. The signal propagation function $p(d)$ is described as follows.

$$p(d) = \begin{cases} \frac{\gamma^{(d/L)}}{(d/L)^2} & d > L \\ 1 & (otherwise) \end{cases} \tag{3}$$

where d is the distance from the emitter, L is the length of an agent, γ is the attenuation coefficient per L. The equation 3 represents the spherical diffusion of a light. An agent closer to an emitter receives more stronger signal.

B. target enclosing model with MOPS direction reference model

We explain procedure of receiving signal of the proposed method.

Agent P_i uses the most powerful signal direction instead of the angle from P_i to P_j(=β_i). Also, we call the angle β'_i.

Fig.3 shows this angle. Now, we suppose agent P_i has several signal receivers. We call it *sensor primitive*. The direction of k-th sensor primitive sp_k is θ_k. If $|\theta_k - \theta_{ij}| \leq \theta_s/2$, the sensor primitive sp_k can sense the signal of P_j. We call θ_s a visible range of sensor primitive.

The power of sensor primitive sp_k, s_k is

$$s_k = \sum_{P_m \in visible} p(d_{i,m}) \tag{4}$$

where $d_{i,m}$ is the distance between P_m and P_i and p is the signal propagation function in eq.3. β'_i is the substitution of β_i.

$$\beta'_i = \angle v_i O \theta_{\underset{k}{arg-max}\, s_k} \tag{5}$$

IV. COMPUTER SIMULATION

The setting of experiment is explained. There is 1 target at the origin. There are $|N|$ agent which are deployed at random far from the origin. We suppose that the agents use strategy B and every agent can receive signal of torch of all agents.

Example of process of enclosing the target are shown by Fig.4. In this case, we set that $|N| = 60$ and the attenuation coefficient $\gamma = 0.9999$. The circle indicates the orbit on which agents enclose the target. A triangle indicates an agent. As you see In Fig.4, the agents can enclose the target.

[dvipdfmx]graphicx amsmath fancyhdr

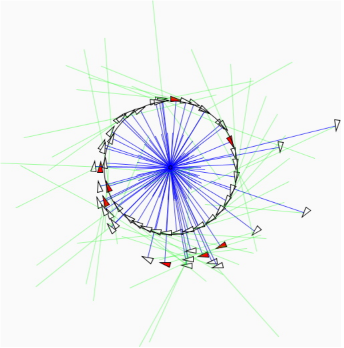

Fig. 4. An example of near completion of taget enclosure with 60 agents

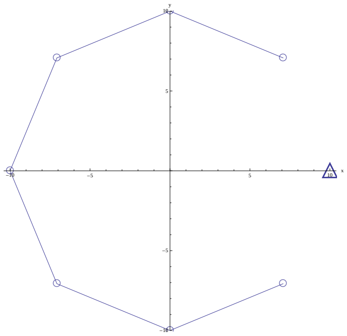

Fig. 5. The location of agents when they enclose the target completely ($|N| = 8$). The target is at the origin O and △ indicates the reference agent and ◯ means its neighbor agents.

V. ANALYSIS OF THE PROPOSED DIRECTION REFERENCE MODEL

In this section, we describe the result of analysis of the proposed MOPS reference model. [6] proved that an agent for any reference model converges to the circle having a radius of $\bar{r}$. [2] proved that a group of 3 and 4 agents with the nearest neighbor agent reference model can achieve a uniform deployment on the circle. In this section, we clarify that it is possible for a set of adequate sensor primitives to detect the nearest neighbor agent direction if the variance of location of robot is small.

A. Definition of this target enclosing task

The following 2 conditions for successful enclosing a target are required. For any agent P_i, the first condition is that the distance between P_i and the target at O is equal to $\bar{r}$. The second condition is that for any agent P_i angle $\angle P_iOP_j = 2\pi/|N|$ where P_j is the nearest neighbor agent of P_i.

Now, we suppose that an agent, $i = 0$ is at $(r_i, \theta_i) = (\bar{r}, 0)$. We call the agent with $i = 0$ the reference agent. Also other agents, $i = i = 1, \ldots, |N| - 1$ are called neighbor agent. When they enclose the target successfully, the location of the neighbor agent are $(r_i, \theta_i) = (\bar{r}, iC_\theta), C_\theta = 2\pi/|N|, i = i = 1, \ldots, |N| - 1$. We call this location regular position. Even if the reference agent moves while they keep enclosing a target successfully, the relative position to the other agents is same. The distance to a neighbor agent $i, i = 1, \ldots, |N| - 1$ is

$$d_i = \sqrt{2\bar{r}^2(1 - \cos(iC_\theta))}. \tag{6}$$

Therefore,

$$d_{i=1} = d_{i=|N|-1} < d_{i\neq 0,1,|N|-1} \tag{7}$$

There are 2 nearest neighbor agents for the reference agent when they enclose the target.

B. Fluctuation of power of signal caused by displacement of neighbor agents

The power of signal of neighbor agent i at the reference agent can be approximated by Taylor expansion. It is described as follows.

$$S_i = A_i + K_i\Delta_i, A_i = \frac{\gamma^{\frac{d_i}{L}} L^2}{d_i^2}, K_i = [k_{i,r} k_{i,\theta}] \tag{8}$$

$$k_{i,r} = -\frac{L\gamma^{\frac{d_i}{L}}\bar{r}(1 - \cos(iC_\theta))\left(2\bar{r}^2\log(\gamma)\cos(iC_\theta) - 2\bar{r}^2\log(\gamma) + 2Ld_i\right)}{d_i^5} \tag{9}$$

$$k_{i,\theta} = -\frac{L\gamma^{\frac{d_i}{L}}\bar{r}^2\sin(iC_\theta)\left(2\bar{r}^2\log(\gamma)\cos(iC_\theta) - 2\bar{r}^2\log(\gamma) + 2Ld_i\right)}{d_i^5} \tag{10}$$

where $\Delta_i = [\Delta r_i \Delta\theta_i]^T$. Δr_i is a displacement along the target direction from its regular position, $\Delta\theta_i$ is a circumferential displacement from its regular position. We assume that the displacement $\Delta r_i, \Delta\theta_i$ follows the normal distributions having average 0 and variance $\sigma_{r_i}^2, \sigma_{\theta_i}^2$ respectively. Also, we assume that the fluctuation of power of signal at the reference agent follows a normal distribution having σ_i. In this case, the average of power of the signal at the reference agent is A_i, and its distribution σ_i^2 is

$$\sigma_i^2 = K_i\Sigma_i K_i^T = k_{i,r}^2\sigma_{r_i}^2 + k_{i,\theta}^2\sigma_{\theta_i}^2 \tag{11}$$

where $\Sigma_i = diag(\sigma_{r_i}^2, \sigma_{\theta_i}^2)$.

The deviation of the signal at the reference agent can be calculated if $|N| and \sigma_{r_i}, \sigma_{\theta_i}$ are known. We suppose that I_s means the set of agents which are in visible range of sensor primitive s. The average $\bar{p_s}$ and the standard deviation M_s of signal at the reference agent are described as follows.

$$\bar{p_s} = \sum_{j\in I_s} A_j, M_s = \sqrt{\sum_{j\in I_s}\sigma_j^2} \tag{12}$$

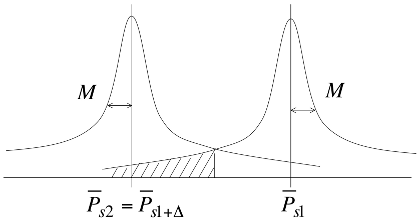

Fig. 6. The probability with which an farther neighbor is recognized as a closer neighbor.

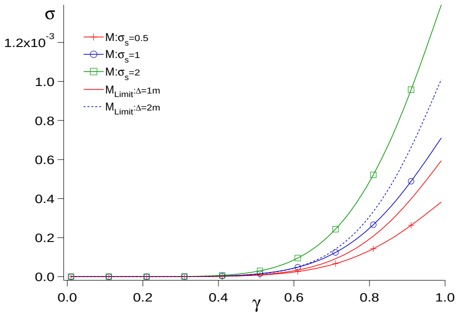

Fig. 7. Limit of variance of signal under which an agent can detect the nearest neighbor agents

C. Limit of variance of signal under which an agent can detect the nearest neighbor agents

We assume that there are 2 sensor primitives $s1$,$s2$ of the reference agent. Each sensor primitive observes one of the nearest neighbor agents in eq.7. Also visible ranges of $s1$ and $s2$ are so narrow that it receives signal from only one of the nearest neighbor agents. Namely, $|I_{s1}| = |I_{s2}| = 1$.

Now, we suppose that a small displacement at the nearest neighbor agents occurs while all of the agents move. We assume that the difference of distance between the 2 nearest neighbor agents is Δ. In this case, it can be deduced by using the 3-σ rule the condition for the reference agent to choose a sensor primitive correctly which observes the closer agent.

$$\bar{p}_{s1} - kM > \frac{\bar{p}_{s1} + \bar{p}_{s2}}{2} \tag{13}$$

where $\bar{p}_{s1} \geq \bar{p}_{s2}$ and k is a positive constant. Here k=3 in the rest of this paper.

Therefore, if the standard deviation of the sensor primitive M is smaller than M_{Limit}=$\frac{p(d_1)-p(d_1+\Delta)}{2k}$, the reference agent can almost certainly select the correct sensor primitive. M_{Limit} is described as follows.

$$M \quad \leq \quad \frac{p(d_1) - p(d_1 + \Delta)}{2k} \quad = \quad M_{Limit} \tag{14}$$

M_{Limit} is changed by signal's attenuation coefficient γ. Fig.7 illustrates M and M_{Limit} while γ changes. M with $\sigma_{\theta_i} = 0.5, 1, 2$ degree are illustrated by line with marker +,◦, and respectively . $|N| = 8, \bar{r} = 10m$. $\sigma_{r_i} = 0.1$m.

The variance of power of signal at reference agent becomes smaller as the σ_{θ_i} becomes small.

The solid and dashed line represent M_{Limit} with Δ=1m and 2m respectively.

In order to detect the difference in distance of Δm between the nearest agents almost certainly, the variation of the received signal M must be smaller than M_{Limit} lines. For example, this figure shows that even if the nearest neighbor agent's location is distributed $\sigma_{\theta_i} = 0.5$ degree, the reference agent can detect 1m difference between the 2 nearest for any γ. On the other hand, the reference agent cannot detect it if the nearest neighbor agent's location is distributed $\sigma_{\theta_i} = 1$ and 2 degree.

By this experiment, we expect that if the σ_{θ_i} is sufficient small, the reference agent can select a correct sensor primitive which observes the nearest neighbor agent that is Δ closer than 2nd nearest neighbor agent.

VI. CONCLUSION

In this paper, a new implementation of a robot swarm for enclosing a target is examined. Every robots have a own but same specification torch and they can observe the sum of the intensity of light of their torches. The robot uses the direction with strongest intensity of the light which is an alternative to direction to its nearest neighbor agent.

We analyzed the relation between the most powerful signal direction and the direction of the nearest neighbor. We can expect that if the fluctuation of agent is sufficient small and a gap of distance Δ between the nearest neighbor agent and the second nearest neighbor agent is given, the agent can select a correct sensor primitive which observes the nearest neighbor agent.

REFERENCES

[1] Masao Kubo, Tatsurou Yoshimura, Akihiro Yamaguchi, Hiroshi Sato, Takashi Matsubara, Analysis of Angular Information based Target Enclosure Model with Nearest Neighbor Topology,Journal of Japan Society for Fuzzy Theory and Intelligent Informatics 24(3), 778-790, 2012

[2] Masao Kubo, Hiroshi Sato, Tatsuro Yoshimura, Akihiro Yamaguchi, Takahiro Tanaka, Multiple targets enclosure by robotic swarm, Robotics and Autonomous Systems 62,pp1294-1304,2014

[3] Joshua A Marshall,Mireille E.Brouvke and Bruce A Francis:Formations of Vehicles in Cyclic Pursuit;IEEE Transactions on Automatic Contorol, vol49,no11 (2004)

[4] Masubuchi I., Zhai G.,Control of Hybrid Systems-V : Analysis and Control of Switched Systems,Systems, Control and Information,52(1), 25-31(2008)

[5] Michael Rubenstein Christian Ahler, Nick Hoff, Adrian Cabrera, Radhika Nagpal, Kilobot: A low cost robot with scalable operations designed for collective behaviors,AAMAS 2013.

[6] Takayama Y., Yamamoto S., Takimoto T., Distributed Formation Control of a Nonholonomic Multi-agent System for Target-enclosing Operations, Proceedings of 9th SICE System Integration Division Annual Conference (2008)

[7] Xuping Xu; Guisheng Zhai,Some Results on Practical Asymptotic Stabilizability of Switched Systems,44th IEEE Conference on Decision and Control, 2005 and 2005 European Control Conference. CDC-ECC '05,3998 - 4003(2005)

[8] F. Ducatelle, A. Förster, G. D. Caro, and L. Gambardella, New task allocation methods for robotic swarms, in 9th IEEE/RAS Conference on Autonomous Robot Systems and Competitions, May 2009.