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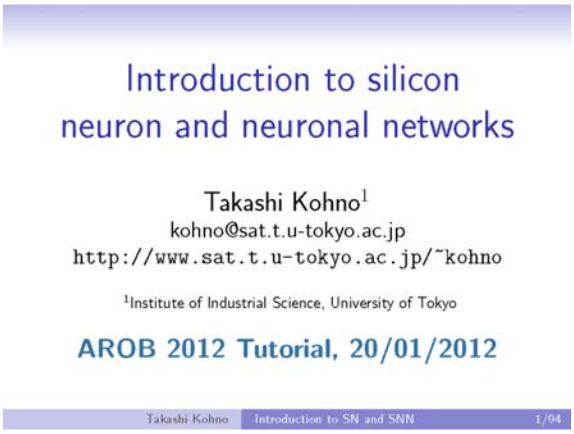
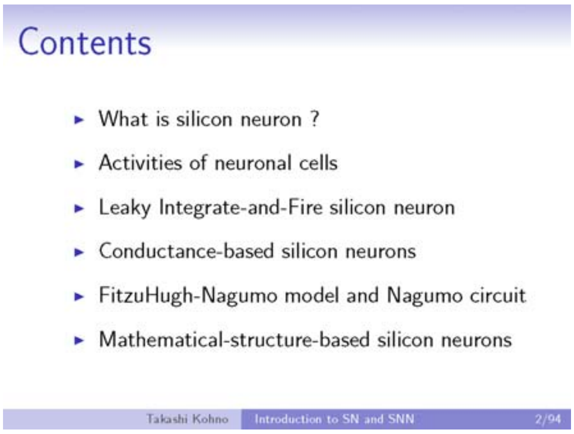
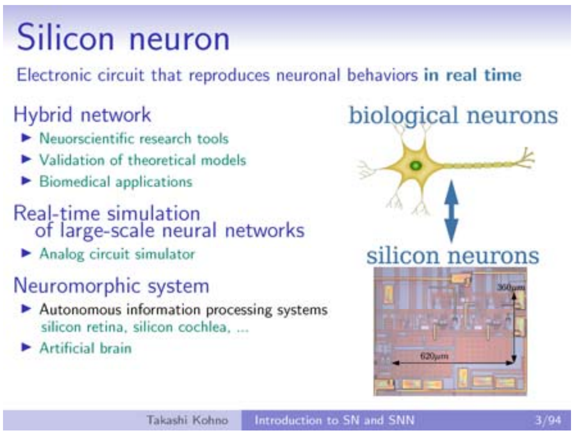
Introduction to silicon neuron and neuronal networks

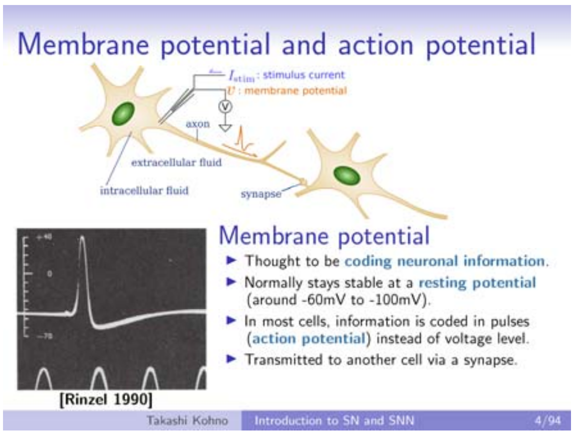
Takashi Kohno

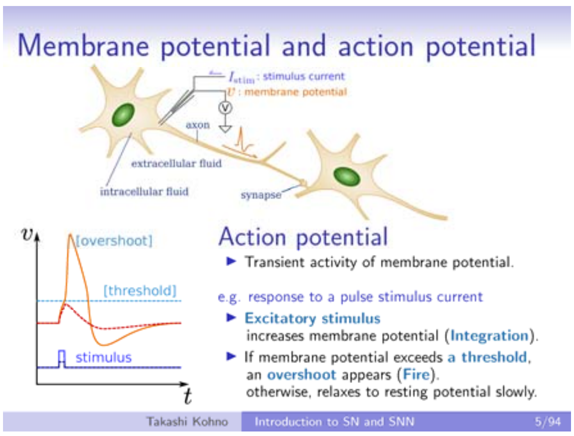
The University of Tokyo

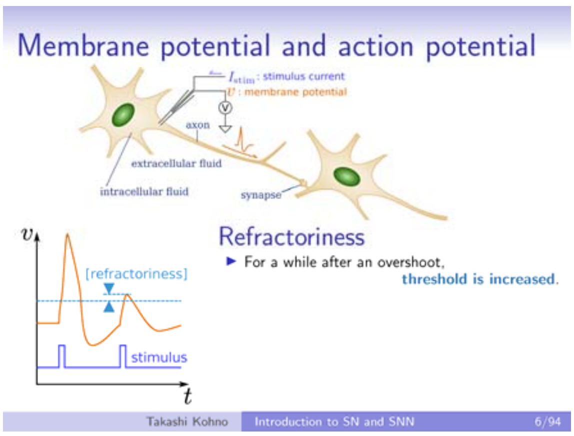


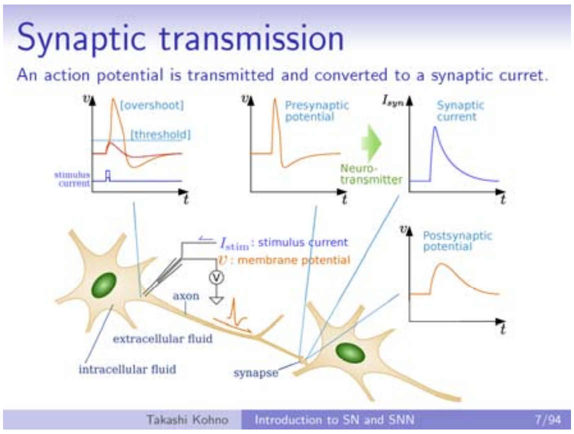











Synaptic transmission
An action potential is transmitted and converted to a synaptic curret.
[overshoot]
[threshold]
stimulus current
Presynaptic potential
Neuro-transmitter
Synaptic current
Postsynaptic potential
$I_{\rm stim}$: stimulus current
V : membrane potential
axon
extracellular fluid
intracellular fluid
synapse
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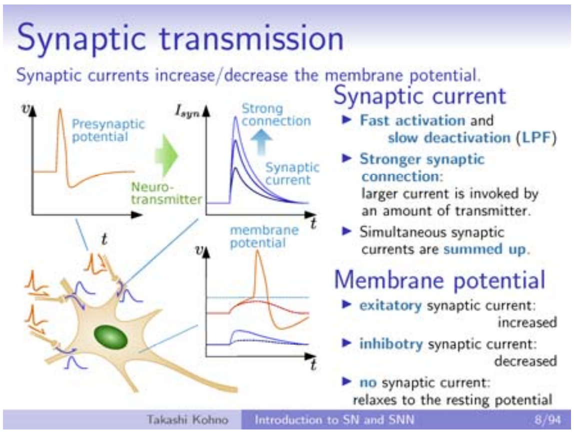
Synaptic transmission
Synaptic currents increase/decrease the membrane potential.
Presynaptic potential
Neuro-transmitter
Strong connection
Synaptic current
membrane potential
Synaptic current
Fast activation and slow deactivation (LPF)
Stronger synaptic connection: larger current is invoked by an amount of transmitter.
Simultaneous synaptic currents are summed up.
Membrane potential
exitatory synaptic current: increased
inhibotry synaptic current: decreased
no synaptic current: relaxes to the resting potential
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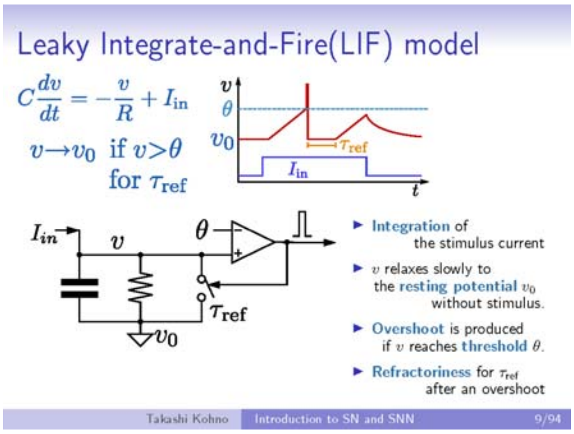
Leaky Integrate-and-Fire(LIF) model
$C\frac{dv}{dt} = -\frac{v}{R} + I_{\rm in}$
$v \to v_0$ if $v > \theta$
for $\tau_{\rm ref}$
$\tau_{\rm ref}$
$I_{\rm in}$
Integration of the stimulus current
v relaxes slowly to the resting potential v_0 without stimulus.
Overshoot is produced if v reaches threshold θ.
Refractoriness for $\tau_{\rm ref}$ after an overshoot
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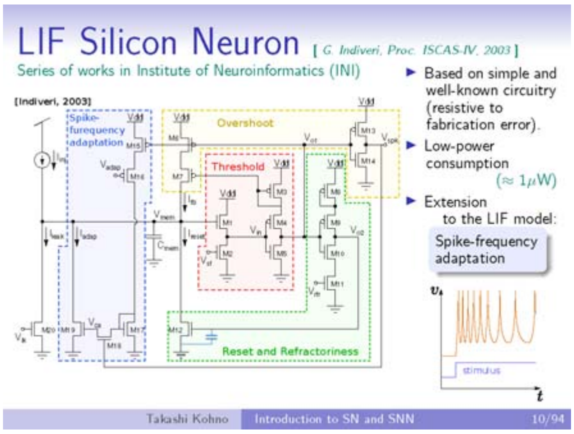
LIF Silicon Neuron [G. Indiveri, Proc. ISCAS-IV, 2003]
Series of works in Institute of Neuroinformatics (INI)
[Indiveri, 2003]
Spike-furequency adaptation
Overshoot
Threshold
Reset and Refractoriness
Based on simple and well-known circuitry (resistive to fabrication error).
Low-power consumption ($\approx 1\mu$W)
Extension to the LIF model:
Spike-frequency adaptation
stimulus
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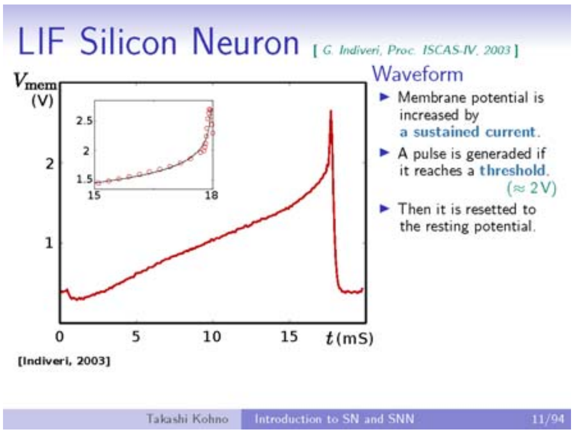
LIF Silicon Neuron [G. Indiveri, Proc. ISCAS-IV, 2003]
$V_{\rm mem}$ (V)
t (mS)
[Indiveri, 2003]
Waveform
Membrane potential is increased by a sustained current.
A pulse is generaded if it reaches a threshold. ($\approx$ 2V)
Then it is resetted to the resting potential.
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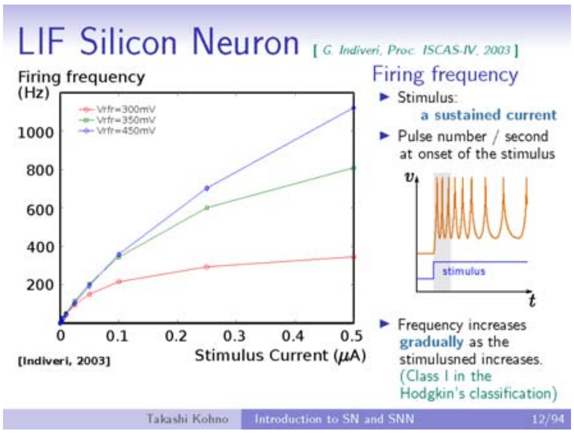
LIF Silicon Neuron [G. Indiveri, Proc. ISCAS-IV, 2003]
Firing frequency (Hz)
Vfr=300mV
Vfr=350mV
Vfr=450mV
Stimulus Current (μA)
[Indiveri, 2003]
Firing frequency
Stimulus: a sustained current
Pulse number / second at onset of the stimulus
stimulus
Frequency increases gradually as the stimulusned increases. (Class I in the Hodgkin's classification)
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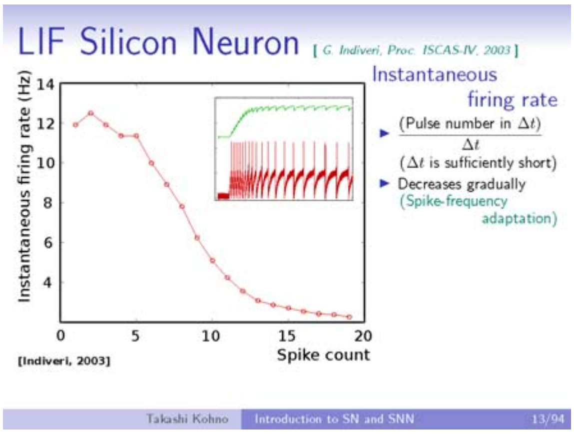
LIF Silicon Neuron [G. Indiveri, Proc. ISCAS-IV, 2003]
Instantaneous firing rate (Hz)
Spike count
[Indiveri, 2003]
Instantaneous firing rate
(Pulse number in Δt) / Δt
(Δt is sufficiently short)
Decreases gradually (Spike-frequency adaptation)
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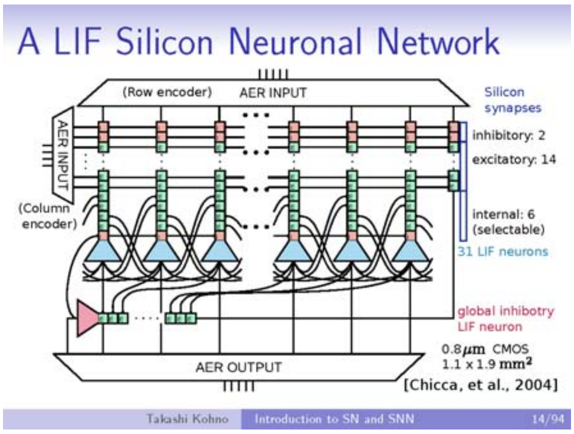
A LIF Silicon Neuronal Network
(Row encoder)
AER INPUT
AER INPUT
(Column encoder)
Silicon synapses
inhibitory: 2
excitatory: 14
internal: 6 (selectable)
31 LIF neurons
global inhibotry LIF neuron
0.8μm CMOS
1.1 x 1.9 mm^2
AER OUTPUT
[Chicca, et al., 2004]
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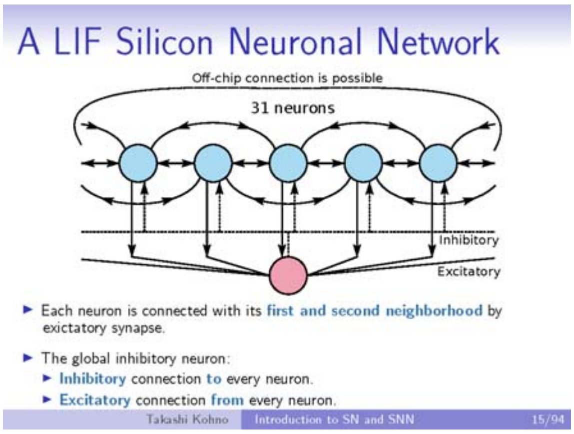
A LIF Silicon Neuronal Network
Off-chip connection is possible
31 neurons
Inhibitory
Excitatory
Each neuron is connected with its first and second neighborhood by exictatory synapse.
The global inhibitory neuron:
Inhibitory connection to every neuron.
Excitatory connection from every neuron.
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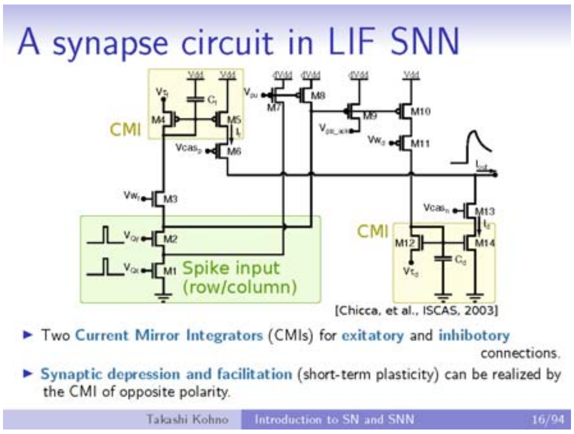
A synapse circuit in LIF SNN
CMI
CMI
Spike input (row/column)
[Chicca, et al., ISCAS, 2003]
Two Current Mirror Integrators (CMIs) for exitatory and inhibotory connections.
Synaptic depression and facilitation (short-term plasticity) can be realized by the CMI of opposite polarity.
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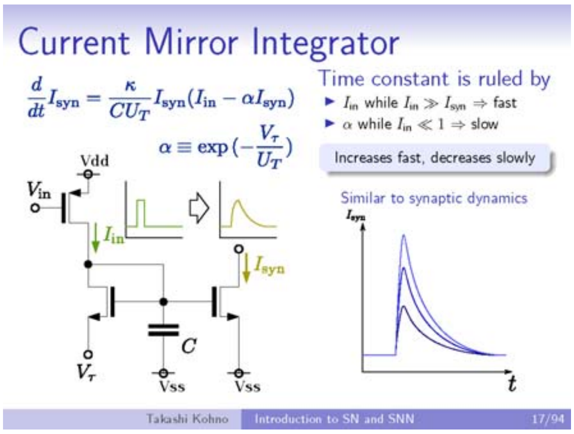
Current Mirror Integrator
$\frac{d}{dt}I_{syn} = \frac{\kappa}{CU_T}I_{syn}(I_{in} - \alpha I_{syn})$
$\alpha \equiv \exp(-\frac{V_\tau}{U_T})$
Time constant is ruled by
I_{in} while $I_{in} \gg I_{syn} \Rightarrow$ fast
α while $I_{in} \ll 1 \Rightarrow$ slow
Increases fast, decreases slowly
Similar to synaptic dynamics
Vdd
V_{in}
I_{in}
I_{syn}
C
V_τ
Vss
Vss
t
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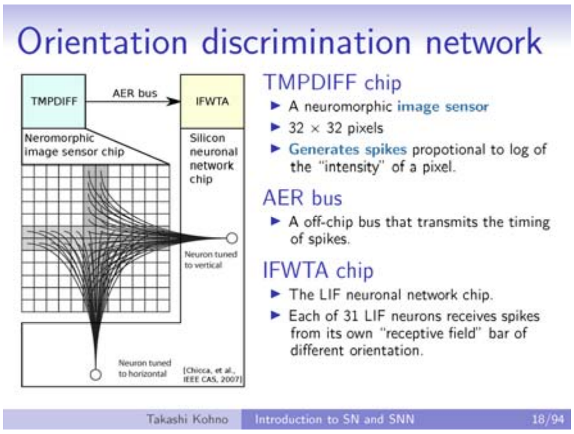
Orientation discrimination network
TMPDIFF
AER bus
IFWTA
Neromorphic image sensor chip
Silicon neuronal network chip
Neuron tuned to vertical
Neuron tuned to horizontal
[Chicca, et al., IEEE CAS, 2007]
TMPDIFF chip
A neuromorphic image sensor
32 × 32 pixels
Generates spikes propotional to log of the "intensity" of a pixel.
AER bus
A off-chip bus that transmits the timing of spikes.
IFWTA chip
The LIF neuronal network chip.
Each of 31 LIF neurons receives spikes from its own "receptive field" bar of different orientation.
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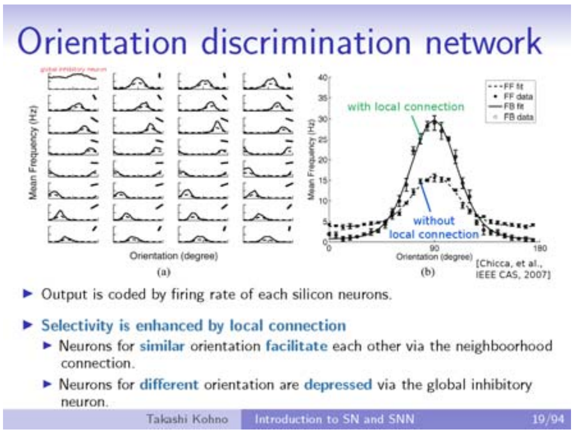
Orientation discrimination network
Mean Frequency (Hz)
Orientation (degree)
(a)
with local connection
without local connection
Orientation (degree)
(b)
[Chicca, et al., IEEE CAS, 2007]
Output is coded by firing rate of each silicon neurons.
Selectivity is enhanced by local connection
Neurons for similar orientation facilitate each other via the neighboorhood connection.
Neurons for different orientation are depressed via the global inhibitory neuron.
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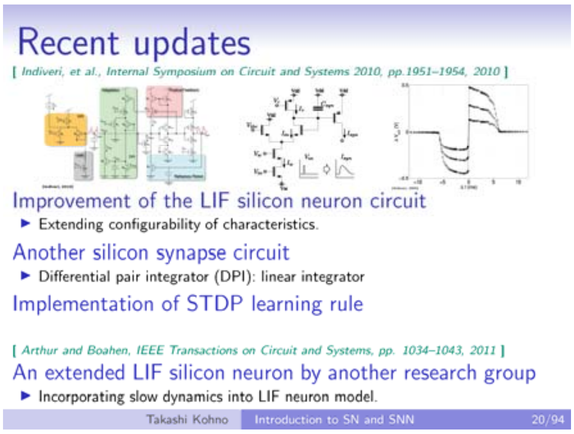
Recent updates
[Indiveri, et al., Internal Symposium on Circuit and Systems 2010, pp.1951–1954, 2010]
Improvement of the LIF silicon neuron circuit
Extending configurability of characteristics.
Another silicon synapse circuit
Differential pair integrator (DPI): linear integrator
Implementation of STDP learning rule
[Arthur and Boahen, IEEE Transactions on Circuit and Systems, pp. 1034–1043, 2011]
An extended LIF silicon neuron by another research group
Incorporating slow dynamics into LIF neuron model.
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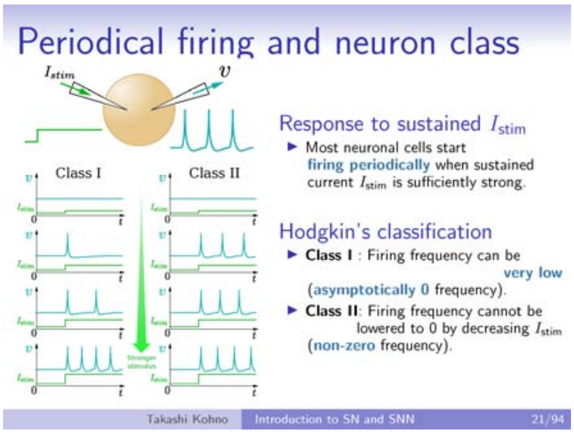
Periodical firing and neuron class
I_{stim}
v
Class I
Class II
Response to sustained I_{stim}
Most neuronal cells start firing periodically when sustained current I_{stim} is sufficiently strong.
Hodgkin's classification
Class I : Firing frequency can be very low (asymptotically 0 frequency).
Class II: Firing frequency cannot be lowered to 0 by decreasing I_{stim} (non-zero frequency).
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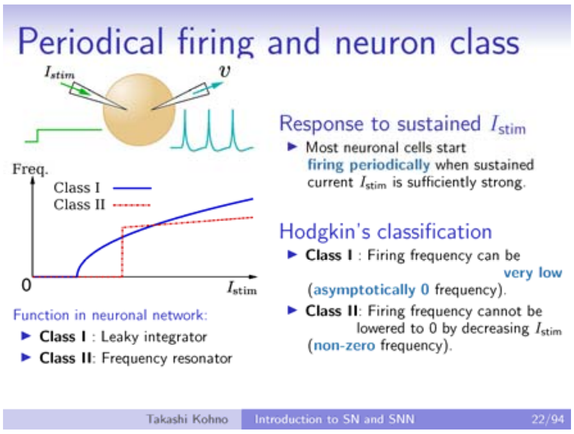
Periodical firing and neuron class
I_{stim}
v
Freq.
Class I
Class II
0
I_{stim}
Function in neuronal network:
Class I : Leaky integrator
Class II: Frequency resonator
Response to sustained I_{stim}
Most neuronal cells start firing periodically when sustained current I_{stim} is sufficiently strong.
Hodgkin's classification
Class I : Firing frequency can be very low (asymptotically 0 frequency).
Class II: Firing frequency cannot be lowered to 0 by decreasing I_{stim} (non-zero frequency).
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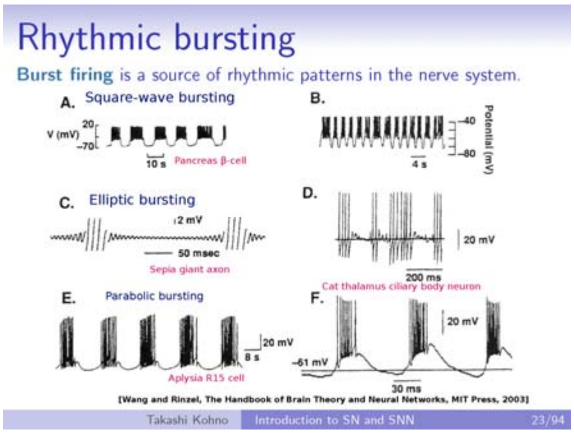
Rhythmic bursting
Burst firing is a source of rhythmic patterns in the nerve system.
A. Square-wave bursting
Pancreas β-cell
B.
C. Elliptic bursting
Sepia giant axon
D.
Cat thalamus ciliary body neuron
E. Parabolic bursting
Aplysia R15 cell
F.
[Wang and Rinzel, The Handbook of Brain Theory and Neural Networks, MIT Press, 2003]
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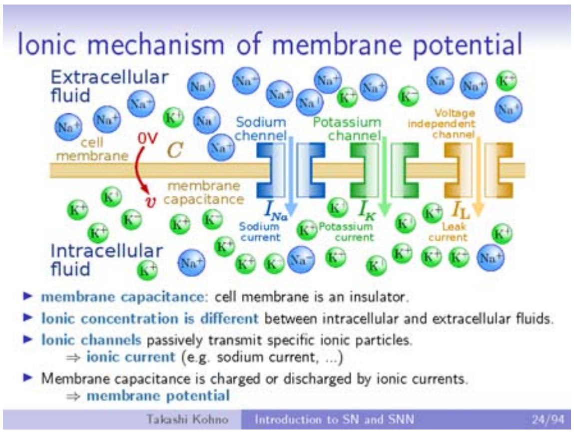
Ionic mechanism of membrane potential
Extracellular fluid
Sodium chennel
Potassium channel
Voltage independent channel
cell membrane
membrane capacitance
Sodium current
Potassium current
Leak current
Intracellular fluid
membrane capacitance: cell membrane is an insulator.
Ionic concentration is different between intracellular and extracellular fluids.
Ionic channels passively transmit specific ionic particles.
⇒ ionic current (e.g. sodium current, ...)
Membrane capacitance is charged or discharged by ionic currents.
⇒ membrane potential
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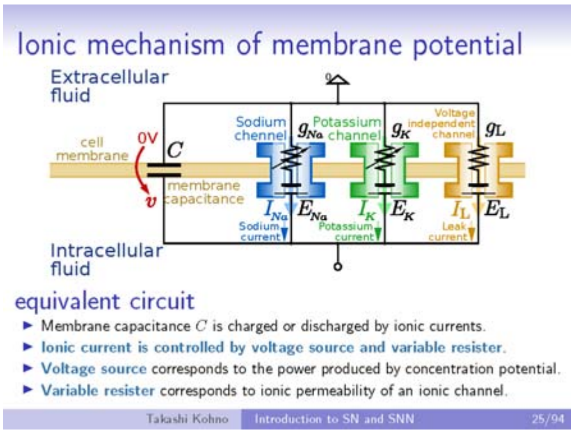
Ionic mechanism of membrane potential
Extracellular fluid
Intracellular fluid
equivalent circuit
Membrane capacitance C is charged or discharged by ionic currents.
Ionic current is controlled by voltage source and variable resister.
Voltage source corresponds to the power produced by concentration potential.
Variable resister corresponds to ionic permeability of an ionic channel.
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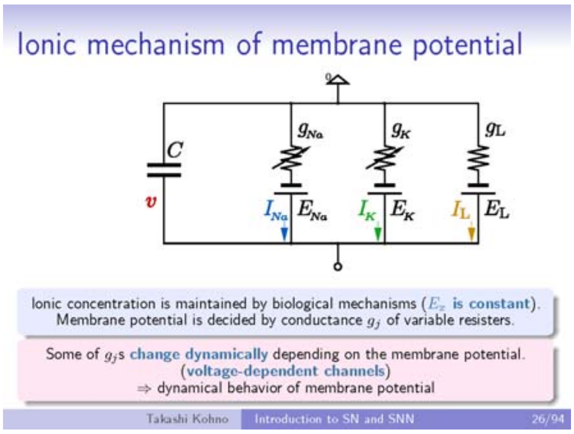
Ionic mechanism of membrane potential
Ionic concentration is maintained by biological mechanisms (E_x is constant).
Membrane potential is decided by conductance g_j of variable resisters.
Some of g_js change dynamically depending on the membrane potential.
(voltage-dependent channels)
⇒ dynamical behavior of membrane potential
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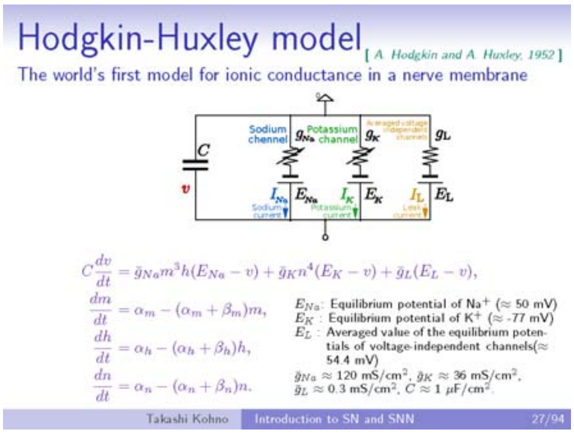
Hodgkin-Huxley model [A. Hodgkin and A. Huxley, 1952]
The world's first model for ionic conductance in a nerve membrane
$C\frac{dv}{dt} = \bar{g}_{Na}m^3h(E_{Na}-v) + \bar{g}_K n^4(E_K - v) + \bar{g}_L(E_L - v),$
$\frac{dm}{dt} = \alpha_m - (\alpha_m + \beta_m)m,$
$\frac{dh}{dt} = \alpha_h - (\alpha_h + \beta_h)h,$
$\frac{dn}{dt} = \alpha_n - (\alpha_n + \beta_n)n.$
E_{Na}: Equilibrium potential of Na^+ (≈ 50 mV)
E_K : Equilibrium potential of K^+ (≈ -77 mV)
E_L : Averaged value of the equilibrium potentials of voltage-independent channels(≈ 54.4 mV)
$\bar{g}_{Na} \approx 120$ mS/cm², $\bar{g}_K \approx 36$ mS/cm², $\bar{g}_L \approx 0.3$ mS/cm², $C \approx 1$ μF/cm².
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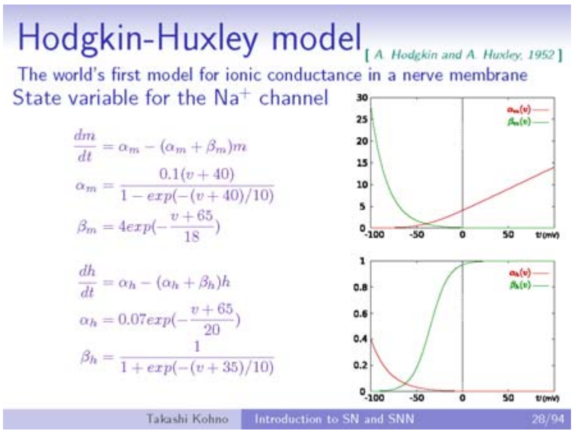
Hodgkin-Huxley model [A. Hodgkin and A. Huxley, 1952]
The world's first model for ionic conductance in a nerve membrane
State variable for the Na^+ channel
$\frac{dm}{dt} = \alpha_m - (\alpha_m + \beta_m)m$
$\alpha_m = \frac{0.1(v+40)}{1-exp(-(v+40)/10)}$
$\beta_m = 4exp(-\frac{v+65}{18})$
$\frac{dh}{dt} = \alpha_h - (\alpha_h + \beta_h)h$
$\alpha_h = 0.07exp(-\frac{v+65}{20})$
$\beta_h = \frac{1}{1+exp(-(v+35)/10)}$
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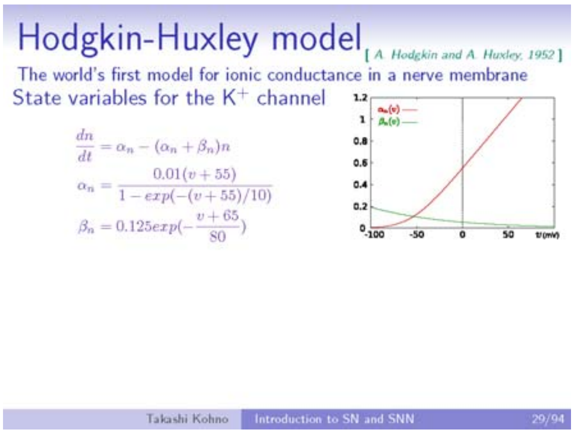
Hodgkin-Huxley model [A. Hodgkin and A. Huxley, 1952]
The world's first model for ionic conductance in a nerve membrane
State variables for the K^+ channel
$\frac{dn}{dt} = \alpha_n - (\alpha_n + \beta_n)n$
$\alpha_n = \frac{0.01(v+55)}{1-exp(-(v+55)/10)}$
$\beta_n = 0.125exp(-\frac{v+65}{80})$
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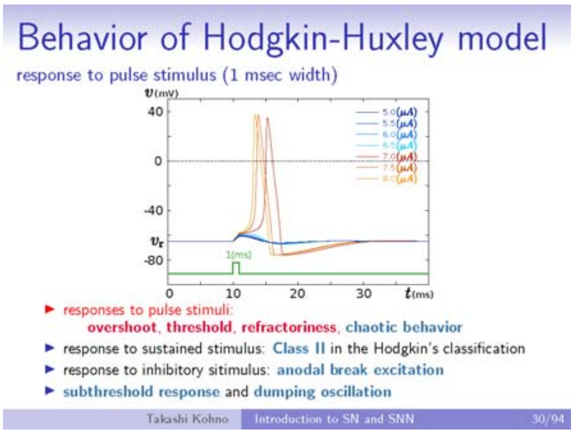
Behavior of Hodgkin-Huxley model
response to pulse stimulus (1 msec width)
responses to pulse stimuli:
overshoot, threshold, refractoriness, chaotic behavior
response to sustained stimulus: Class II in the Hodgkin's classification
response to inhibitory stimulus: anodal break excitation
subthreshold response and dumping oscillation
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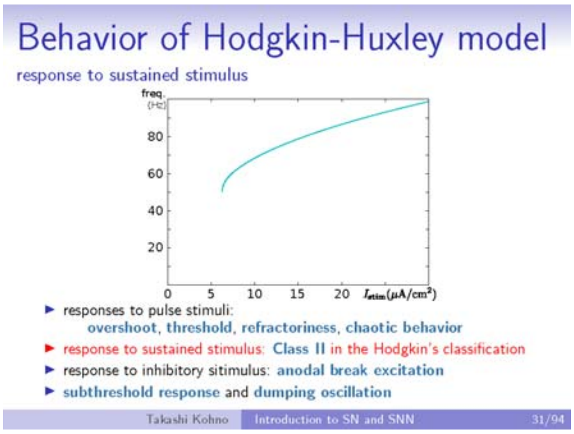
Behavior of Hodgkin-Huxley model
response to sustained stimulus
freq. (Hz)
80
60
40
20
0
5
10
15
20
$I_{stim}(\mu A/cm^2)$
responses to pulse stimuli:
overshoot, threshold, refractoriness, chaotic behavior
response to sustained stimulus: Class II in the Hodgkin's classification
response to inhibitory sitimulus: anodal break excitation
subthreshold response and dumping oscillation
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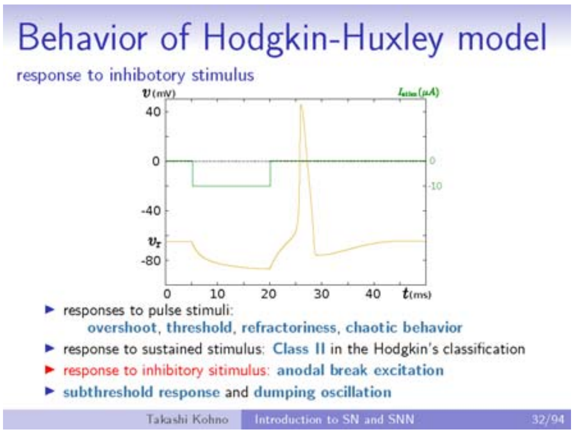
Behavior of Hodgkin-Huxley model
response to inhibotory stimulus
v(mV)
$I_{stim}(\mu A)$
40
0
-40
v_r
-80
-10
0
10
20
30
40
t(ms)
responses to pulse stimuli:
overshoot, threshold, refractoriness, chaotic behavior
response to sustained stimulus: Class II in the Hodgkin's classification
response to inhibitory sitimulus: anodal break excitation
subthreshold response and dumping oscillation
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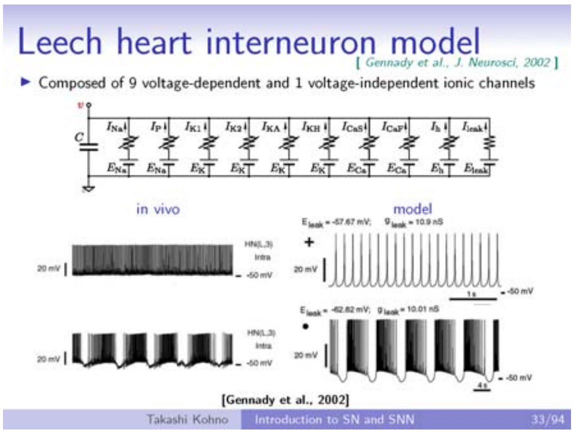
Leech heart interneuron model
[Gennady et al., J. Neurosci, 2002]
Composed of 9 voltage-dependent and 1 voltage-independent ionic channels
in vivo
model
20 mV
-50 mV
[Gennady et al., 2002]
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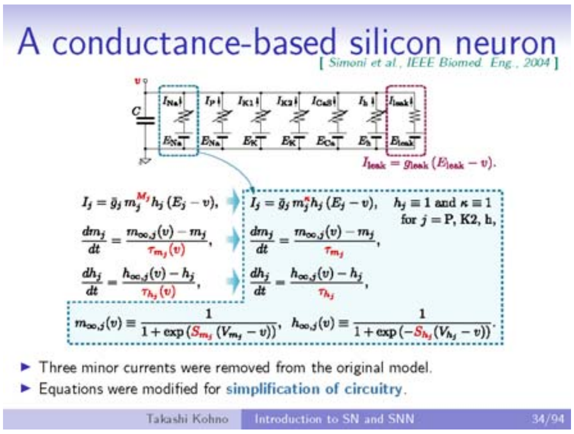
A conductance-based silicon neuron
[Simoni et al., IEEE Biomed. Eng., 2004]
$I_{leak} = g_{leak}(E_{leak} - v)$
$I_j = \bar{g}_j m_j^{M_j} h_j (E_j - v)$, $\frac{dm_j}{dt} = \frac{m_{\infty,j}(v) - m_j}{\tau_{m_j}(v)}$, $\frac{dh_j}{dt} = \frac{h_{\infty,j}(v) - h_j}{\tau_{h_j}(v)}$
$I_j = \bar{g}_j m_j^{\kappa} h_j (E_j - v)$, $h_j \equiv 1$ and $\kappa \equiv 1$ for j = P, K2, h,
$\frac{dm_j}{dt} = \frac{m_{\infty,j}(v) - m_j}{\tau_{m_j}}$, $\frac{dh_j}{dt} = \frac{h_{\infty,j}(v) - h_j}{\tau_{h_j}}$
$m_{\infty,j}(v) \equiv \frac{1}{1+\exp(S_{m_j}(V_{m_j} - v))}$, $h_{\infty,j}(v) \equiv \frac{1}{1+\exp(-S_{h_j}(V_{h_j} - v))}$
Three minor currents were removed from the original model.
Equations were modified for simplification of circuitry.
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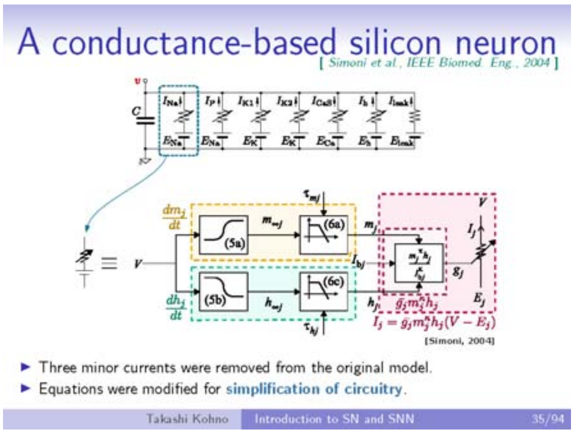
A conductance-based silicon neuron
[Simoni et al., IEEE Biomed. Eng., 2004]
$I_j = \bar{g}_j m_j^{\kappa} h_j (V - E_j)$
[Simoni, 2004]
Three minor currents were removed from the original model.
Equations were modified for simplification of circuitry.
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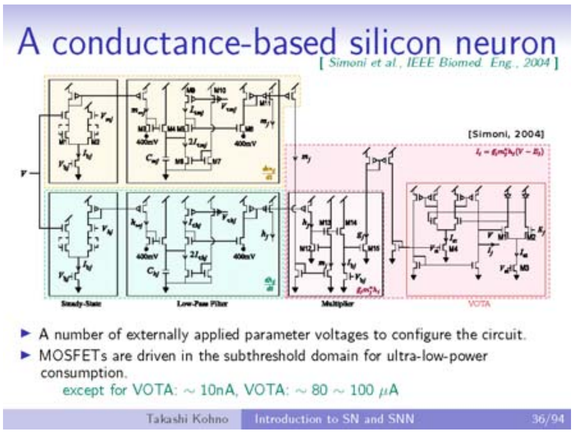
A conductance-based silicon neuron
[Simoni et al., IEEE Biomed. Eng., 2004]
[Simoni, 2004]
Steady-State
Low-Pass Filter
Multiplier
VOTA
A number of externally applied parameter voltages to configure the circuit.
MOSFETs are driven in the subthreshold domain for ultra-low-power consumption.
except for VOTA: ~ 10nA, VOTA: ~ 80 ~ 100 μA
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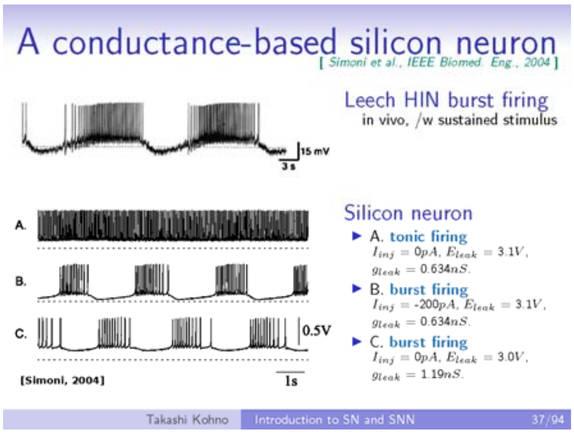
A conductance-based silicon neuron
[Simoni et al., IEEE Biomed. Eng., 2004]
Leech HIN burst firing
in vivo, /w sustained stimulus
15 mV
3 s
Silicon neuron
A. tonic firing
I_{inj} = 0pA, E_{leak} = 3.1V, g_{leak} = 0.634nS.
B. burst firing
I_{inj} = -200pA, E_{leak} = 3.1V, g_{leak} = 0.634nS.
C. burst firing
I_{inj} = 0pA, E_{leak} = 3.0V, g_{leak} = 1.19nS.
0.5V
1s
[Simoni, 2004]
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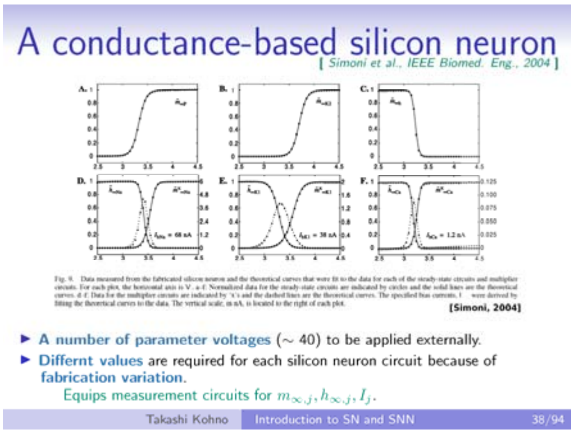
A conductance-based silicon neuron
[Simoni et al., IEEE Biomed. Eng., 2004]
[Simoni, 2004]
A number of parameter voltages (~ 40) to be applied externally.
Differnt values are required for each silicon neuron circuit because of fabrication variation.
Equips measurement circuits for $m_{\infty,j}, h_{\infty,j}, I_j$.
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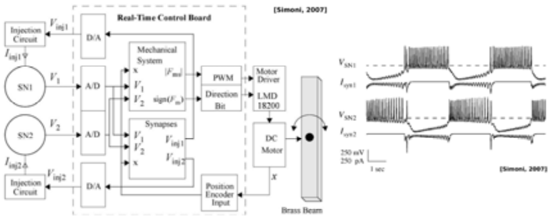
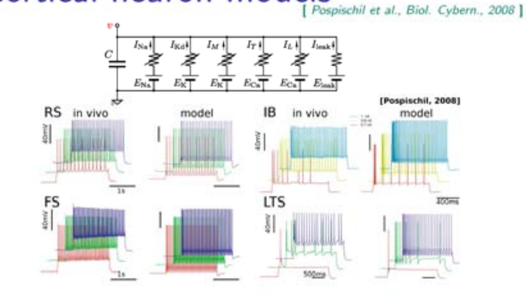
A conductance-based silicon neuronal network
[Simoni et al., IEEE Biomed. Eng., 2007]



- A silicon half-center oscillator
  - Half-center oscillator: the simplest biological motion pattern generator
  - Two neurons are connected each other via inhibitory synapses.
- Synapses are emulated by ADC, PC, and DAC (dynamic clamp)

Cortical neuron models
[Pospischil et al., Biol. Cybern., 2008]



- Composed of 5 voltage-dependent and 1 voltage-independent ionic channels

A conductance-based silicon cortical neuron
[Grassia et al., Frontiers Neurocsi., 2011]

$I_{leak} = g_{leak}(E_{leak} - v)$.

$I_j = \bar{g}_j m_j^{p_j} h_j^{q_j} (E_j - v)$,

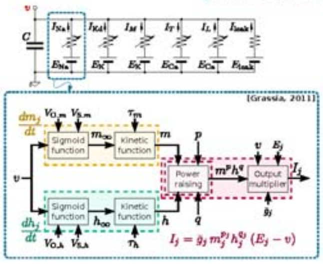
$$\frac{dm_j}{dt} = \frac{m_{\infty,j}(v) - m_j}{\tau_{m_j}(v)}, \quad \frac{dm_j}{dt} = \frac{m_{\infty,j}(v) - m_j}{\tau_{m_j}},$$

$$\frac{dh_j}{dt} = \frac{h_{\infty,j}(v) - h_j}{\tau_{h_j}(v)}, \quad \frac{dh_j}{dt} = \frac{h_{\infty,j}(v) - h_j}{\tau_{h_j}},$$

$$m_{\infty,j}(v) \equiv \frac{1}{1+\exp((V_{m_j} - v)/V_{s,m_j})}, h_{\infty,j}(v) \equiv \frac{1}{1+\exp((V_{h_j} - v)/V_{s,h_j})}$$

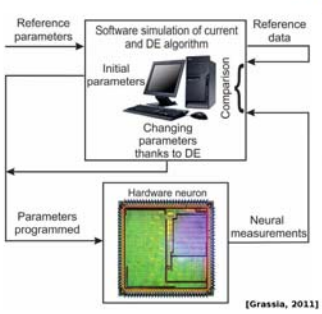
- Equations were modified for simplification of circuitry.

A conductance-based silicon cortical neuron
[Grassia et al., Frontiers Neurocsi., 2011]



- Several nFs of capacitors are required: off-chip capacitor.
- Parameter voltages are stored in on-chip dynamic analog memory.
- 0.35μm BiCMOS SiGe process, total power consumption 550mW (5 neurons)

A conductance-based silicon cortical neuron
[Grassia et al., Frontiers Neurocsi., 2011]



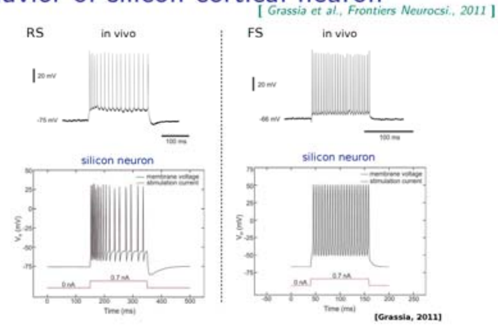
Configuration

- Numbers of parameter voltages to be tuned (maximum 28).
- Fabrication variation is inevitable:
  ⇒ Each instance of silicon neurons requires different parameter voltages.

Automated configuration

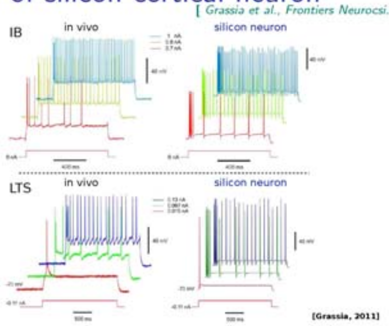
- Parameter voltages are tuned up by differential evolution algorithm.
- Minimize behavioral difference to the target cortical neuron models.
- Requires about a day.

Behavior of silicon cortical neuron
[Grassia et al., Frontiers Neurocsi., 2011]



- Successfull automated tuning system.

Behavior of silicon cortical neuron
[Grassia et al., Frontiers Neurocsi., 2011]



- Successfull automated tuning system.

Qualitative neuron models

Ionic conductance models are too complex to explain the mechanism of the neuronal behaviors.

Simple models with similar dynamics were developed by techniques in nonlinear mathematics.

FitzHugh-Nagumo model
Similar dynamics to the Hodgkin-Huxley model

$$\frac{du}{dt} = c(-v + u - \frac{u^3}{3} + I(t)),$$
$$\frac{dv}{dt} = u - bv + a.$$

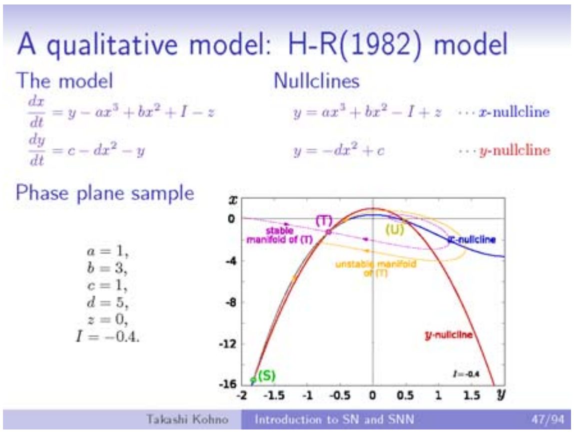
Hindmarsh-Rose model
A simplest burst firing model

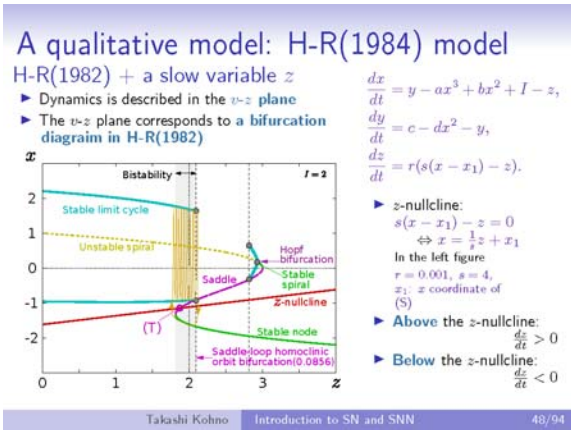
$$\frac{dx}{dt} = y - x^3 + 3x^2 - z,$$
$$\frac{dy}{dt} = 1 - 5x^2 - y,$$
$$\frac{dz}{dt} = r(s(x - x_1) - z).$$

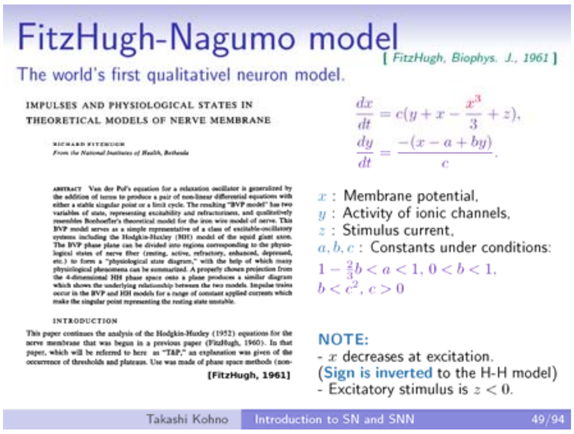
Reduction of the Hodgkin-Huxleymodel by Rinzel

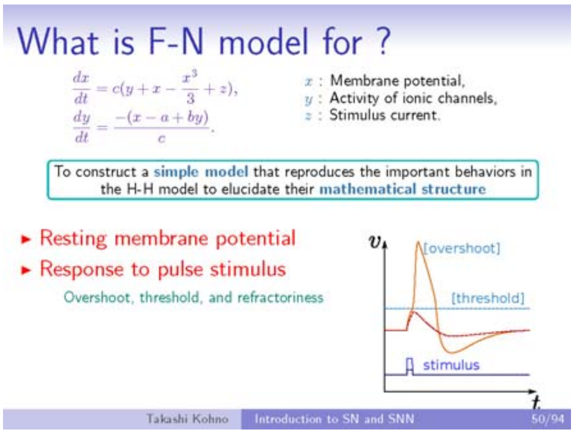
$$h = b - an.$$

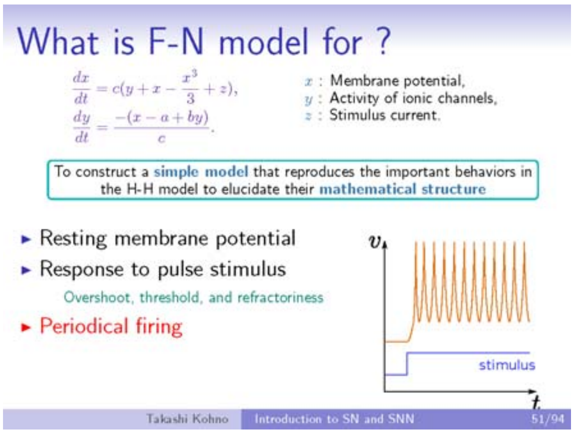
The Seventeenth International Symposium on Artificial Life and Robotics 2012 (AROB 17th '12),
B-Con Plaza, Beppu, Oita, Japan, January 19-21, 2012

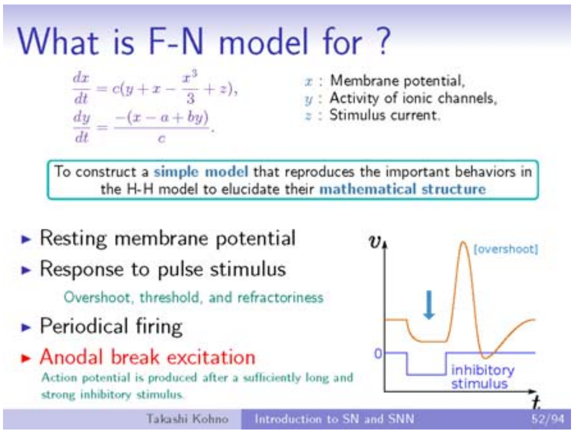


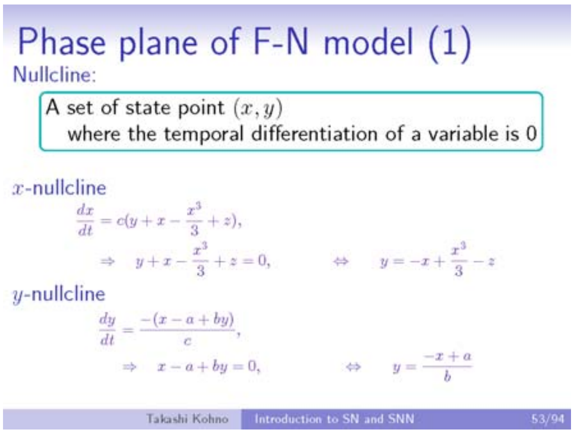


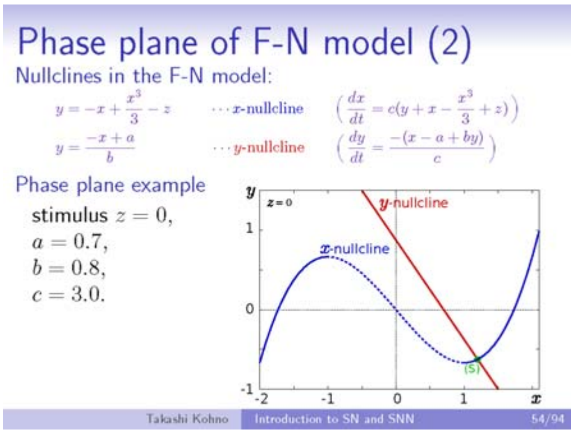


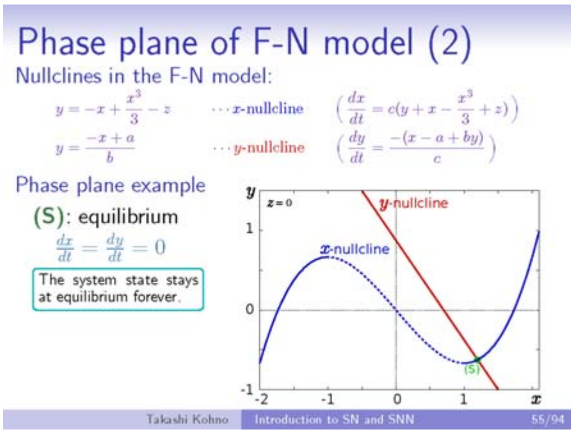











Phase plane of F-N model (2)
Nullclines in the F-N model:
$y = -x + \frac{x^3}{3} - z$ ··· x-nullcline $\left(\frac{dx}{dt} = c(y + x - \frac{x^3}{3} + z)\right)$
$y = \frac{-x + a}{b}$ ··· y-nullcline $\left(\frac{dy}{dt} = \frac{-(x - a + by)}{c}\right)$
Phase plane example
(S): equilibrium
$\frac{dx}{dt} = \frac{dy}{dt} = 0$
The system state stays at equilibrium forever.
z = 0
y-nullcline
x-nullcline
(S)
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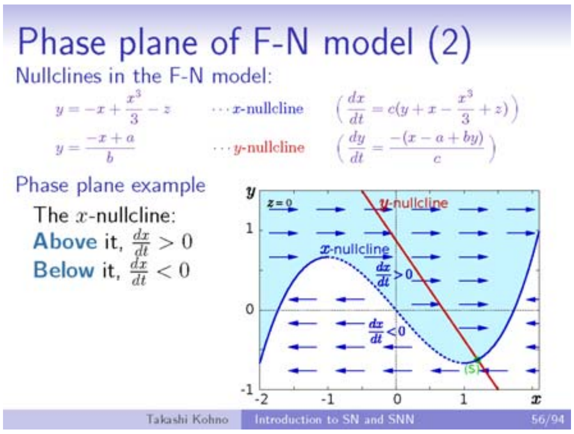
Phase plane of F-N model (2)
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$y = \frac{-x + a}{b}$ ··· y-nullcline $\left(\frac{dy}{dt} = \frac{-(x - a + by)}{c}\right)$
Phase plane example
The x-nullcline:
Above it, $\frac{dx}{dt} > 0$
Below it, $\frac{dx}{dt} < 0$
z = 0
y-nullcline
x-nullcline
$\frac{dx}{dt} > 0$
$\frac{dx}{dt} < 0$
(S)
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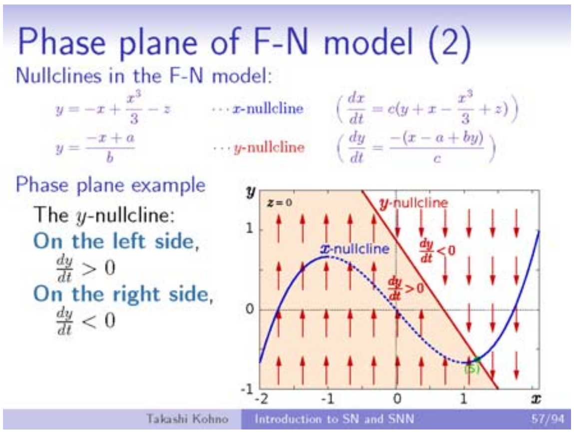
Phase plane of F-N model (2)
Nullclines in the F-N model:
$y = -x + \frac{x^3}{3} - z$ ··· x-nullcline $\left(\frac{dx}{dt} = c(y + x - \frac{x^3}{3} + z)\right)$
$y = \frac{-x + a}{b}$ ··· y-nullcline $\left(\frac{dy}{dt} = \frac{-(x - a + by)}{c}\right)$
Phase plane example
The y-nullcline:
On the left side, $\frac{dy}{dt} > 0$
On the right side, $\frac{dy}{dt} < 0$
z = 0
y-nullcline
x-nullcline
$\frac{dy}{dt} < 0$
$\frac{dy}{dt} > 0$
(S)
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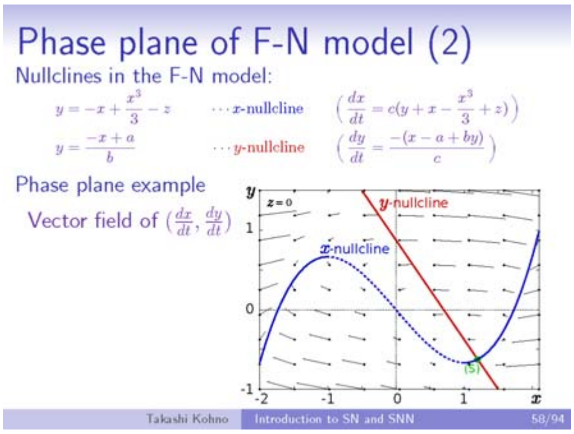
Phase plane of F-N model (2)
Nullclines in the F-N model:
$y = -x + \frac{x^3}{3} - z$ ··· x-nullcline $\left(\frac{dx}{dt} = c(y + x - \frac{x^3}{3} + z)\right)$
$y = \frac{-x + a}{b}$ ··· y-nullcline $\left(\frac{dy}{dt} = \frac{-(x - a + by)}{c}\right)$
Phase plane example
Vector field of $(\frac{dx}{dt}, \frac{dy}{dt})$
z = 0
y-nullcline
x-nullcline
(S)
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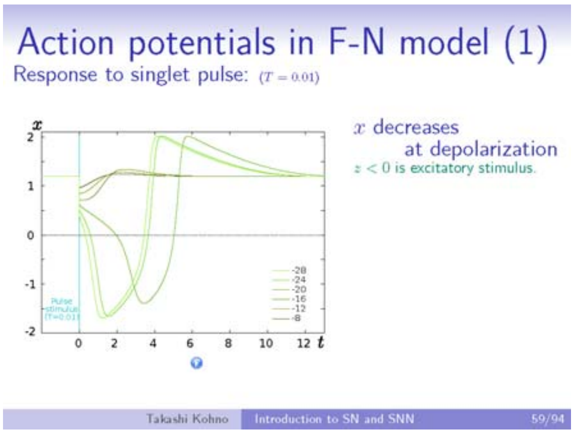
Action potentials in F-N model (1)
Response to singlet pulse: $(T = 0.01)$
x decreases at depolarization
$z < 0$ is excitatory stimulus.
Pulse stimulus (T=0.01)
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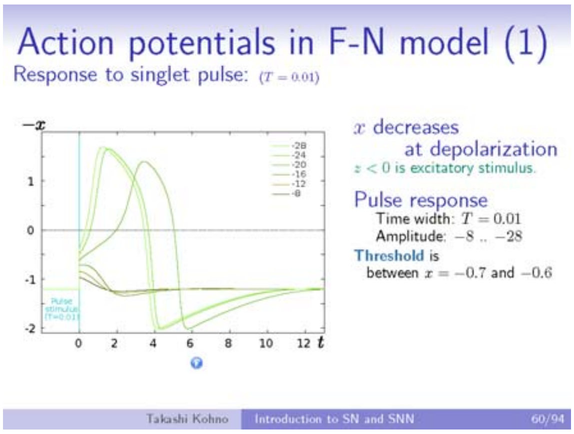
Action potentials in F-N model (1)
Response to singlet pulse: $(T = 0.01)$
x decreases at depolarization
$z < 0$ is excitatory stimulus.
Pulse response
Time width: $T = 0.01$
Amplitude: -8 .. -28
Threshold is between $x = -0.7$ and -0.6
Pulse stimulus (T=0.01)
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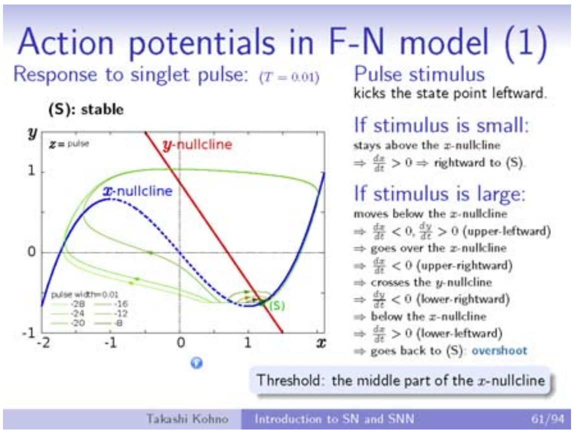
Action potentials in F-N model (1)
Response to singlet pulse: $(T = 0.01)$
(S): stable
Pulse stimulus
kicks the state point leftward.
If stimulus is small:
stays above the x-nullcline
$\Rightarrow \frac{dx}{dt} > 0 \Rightarrow$ rightward to (S).
If stimulus is large:
moves below the x-nullcline
$\Rightarrow \frac{dx}{dt} < 0, \frac{dy}{dt} > 0$ (upper-leftward)
$\Rightarrow$ goes over the x-nullcline
$\Rightarrow \frac{dx}{dt} < 0$ (upper-rightward)
$\Rightarrow$ crosses the y-nullcline
$\Rightarrow \frac{dy}{dt} < 0$ (lower-rightward)
$\Rightarrow$ below the x-nullcline
$\Rightarrow \frac{dx}{dt} > 0$ (lower-leftward)
$\Rightarrow$ goes back to (S): overshoot
Threshold: the middle part of the x-nullcline
z = pulse
y-nullcline
x-nullcline
pulse width=0.01
(S)
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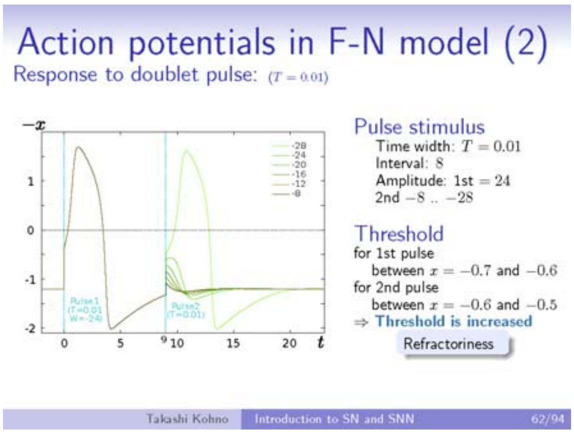
Action potentials in F-N model (2)
Response to doublet pulse: $(T = 0.01)$
Pulse stimulus
Time width: $T = 0.01$
Interval: 8
Amplitude: 1st = 24
2nd -8 .. -28
Threshold
for 1st pulse
between $x = -0.7$ and -0.6
for 2nd pulse
between $x = -0.6$ and -0.5
$\Rightarrow$ Threshold is increased
Refractoriness
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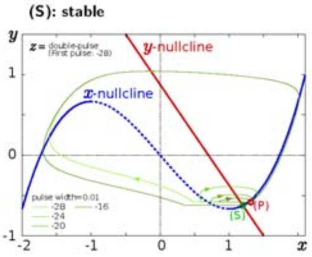
Action potentials in F-N model (2)

Response to doublet pulse: $(T = 0.01)$



The 2nd pulse is given when the state point is at (P)

(8 timeunit after the 1st pulse) ⇒ jumps leftward from (P)

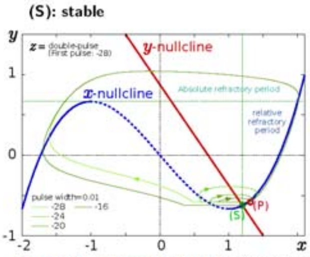
Threshold: the middle part of the x-nullcline

[The x coordinate of threshold for (P)] > [The x coordinate of threshold for (S)]

Refractoriness

Action potentials in F-N model (2)

Response to doublet pulse: $(T = 0.01)$



The 2nd pulse is given when the state point is at (P)

(8 timeunit after the 1st pulse) ⇒ jumps leftward from (P)

Threshold: the middle part of the x-nullcline

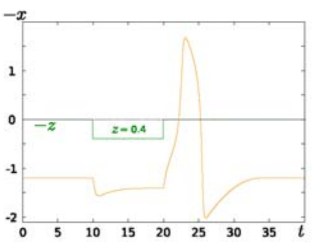
[The x coordinate of threshold for (P)] > [The x coordinate of threshold for (S)]

Refractoriness

- **Absolute refractory period**: No threshold exist
- **Relative refractory period**: Threshold is high

Anodal break excitation in F-N model

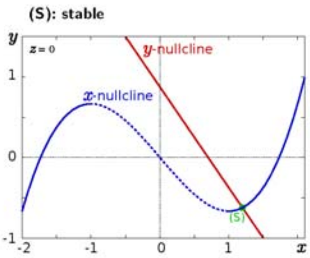
Response to inhibitory stimulus: $(z = 0.4, T = 10)$



Sufficiently long inhibitory stimulus invokes an action potential at its end.

Anodal break excitation in F-N model

Response to inhibitory stimulus: $(z = 0.4, T = 10)$

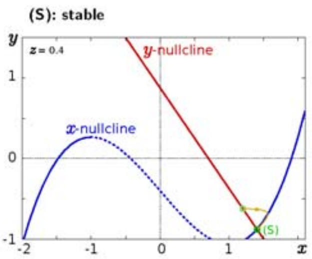


Sufficiently long inhibitory stimulus invokes an action potential at its end.

- Inhibitory stimulus $z = 0.4$
  ⇒ x-nullcline is displaced **downward**
  (∵ x-nullcline: $y = -x + \frac{x^3}{3} - z$)
  ⇒ (S) moves **lower-rightward**

Anodal break excitation in F-N model

Response to inhibitory stimulus: $(z = 0.4, T = 10)$



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Anodal break excitation in F-N model

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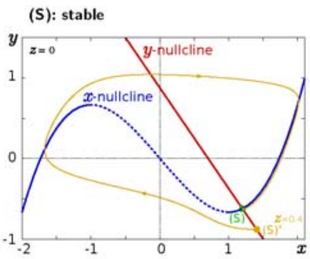
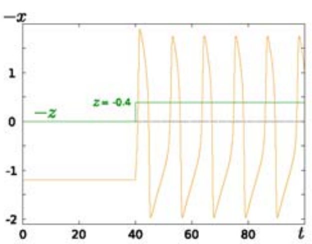


Sufficiently long inhibitory stimulus invokes an action potential at its end.

- Inhibitory stimulus $z = 0.4$
  ⇒ x-nullcline is displaced **downward**
  (∵ x-nullcline: $y = -x + \frac{x^3}{3} - z$)
  ⇒ (S) moves **lower-rightward**
- Stimulus ends $z = 0$
  ⇒ The system state is **below the x-nullcline**, where $\frac{dx}{dt} < 0$
  ⇒ it moves leftward and action potential starts.

Periodic firing in F-N model (1)

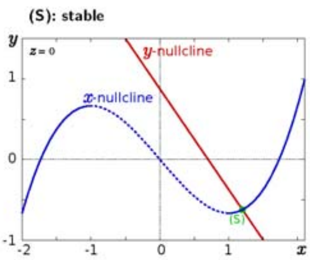
Response to sustained stimulus: $(z = 0 \to -0.4)$



Excitatory sustained stimulus invokes periodic firing.

Periodic firing in F-N model (1)

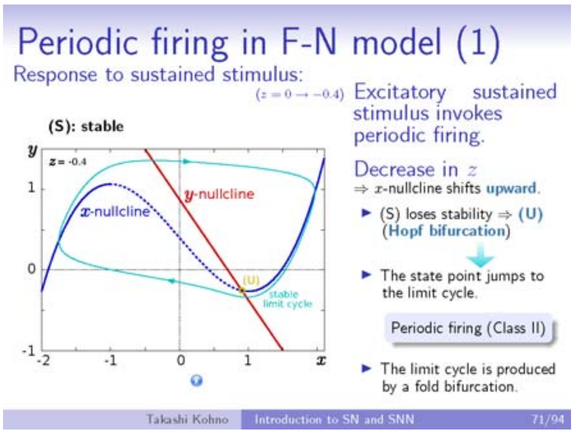
Response to sustained stimulus: $(z = 0 \to -0.4)$



Excitatory sustained stimulus invokes periodic firing.

Decrease in z
⇒ x-nullcline shifts **upward**.

x-nullcline: $y = -x + \frac{x^3}{3} - z$


Periodic firing in F-N model (1)
Response to sustained stimulus:
Excitatory sustained stimulus invokes periodic firing.
(S): stable
z = -0.4
y-nullcline
x-nullcline
(U)
stable limit cycle
Decrease in z
⇒ x-nullcline shifts upward.
(S) loses stability ⇒ (U) (Hopf bifurcation)
The state point jumps to the limit cycle.
Periodic firing (Class II)
The limit cycle is produced by a fold bifurcation.
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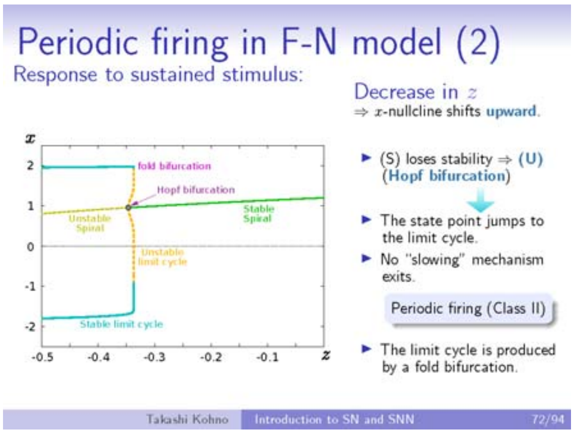
Periodic firing in F-N model (2)
Response to sustained stimulus:
Decrease in z
⇒ x-nullcline shifts upward.
fold bifurcation
Hopf bifurcation
Unstable Spiral
Stable Spiral
Unstable limit cycle
Stable limit cycle
(S) loses stability ⇒ (U) (Hopf bifurcation)
The state point jumps to the limit cycle.
No "slowing" mechanism exits.
Periodic firing (Class II)
The limit cycle is produced by a fold bifurcation.
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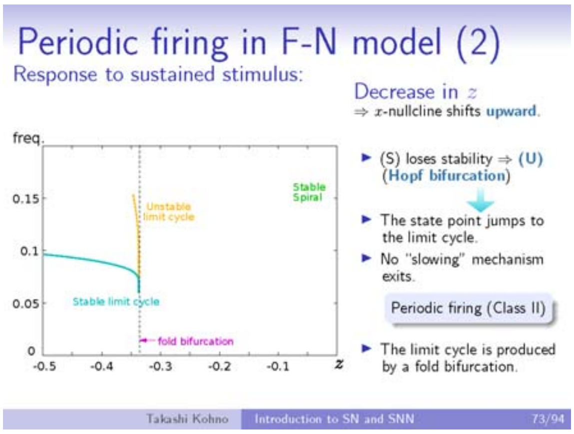
Periodic firing in F-N model (2)
Response to sustained stimulus:
Decrease in z
⇒ x-nullcline shifts upward.
freq.
Unstable limit cycle
Stable Spiral
Stable limit cycle
fold bifurcation
(S) loses stability ⇒ (U) (Hopf bifurcation)
The state point jumps to the limit cycle.
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Periodic firing (Class II)
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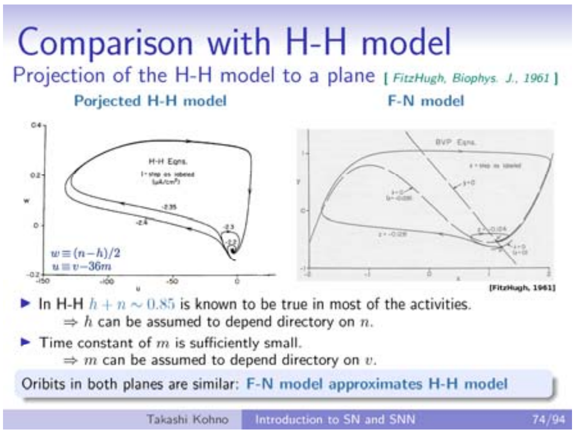
Comparison with H-H model
Projection of the H-H model to a plane [FitzHugh, Biophys. J., 1961]
Porjected H-H model
F-N model
H-H Eqns.
BVP Eqns.
[FitzHugh, 1961]
In H-H h + n ~ 0.85 is known to be true in most of the activities.
⇒ h can be assumed to depend directory on n.
Time constant of m is sufficiently small.
⇒ m can be assumed to depend directory on v.
Oribits in both planes are similar: F-N model approximates H-H model
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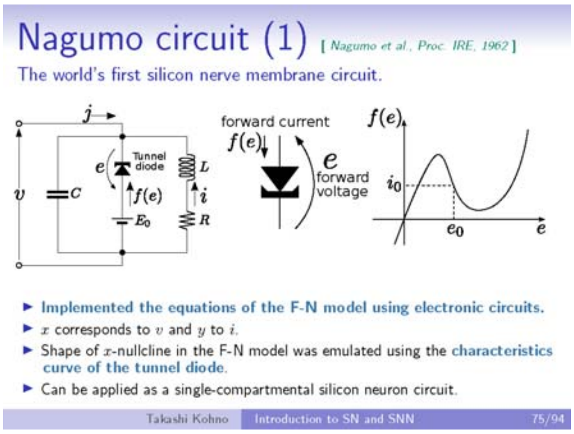
Nagumo circuit (1) [Nagumo et al., Proc. IRE, 1962]
The world's first silicon nerve membrane circuit.
Tunnel diode
forward current
forward voltage
Implemented the equations of the F-N model using electronic circuits.
x corresponds to v and y to i.
Shape of x-nullcline in the F-N model was emulated using the characteristics curve of the tunnel diode.
Can be applied as a single-compartmental silicon neuron circuit.
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Introduction to SN and SNN
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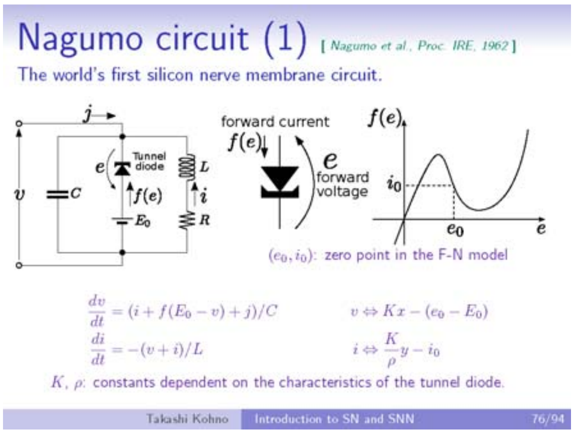
Nagumo circuit (1) [Nagumo et al., Proc. IRE, 1962]
The world's first silicon nerve membrane circuit.
Tunnel diode
forward current
forward voltage
(e0, i0): zero point in the F-N model
dv/dt = (i + f(E0 − v) + j)/C
di/dt = −(v + i)/L
v ⇔ Kx − (e0 − E0)
i ⇔ (K/ρ)y − i0
K, ρ: constants dependent on the characteristics of the tunnel diode.
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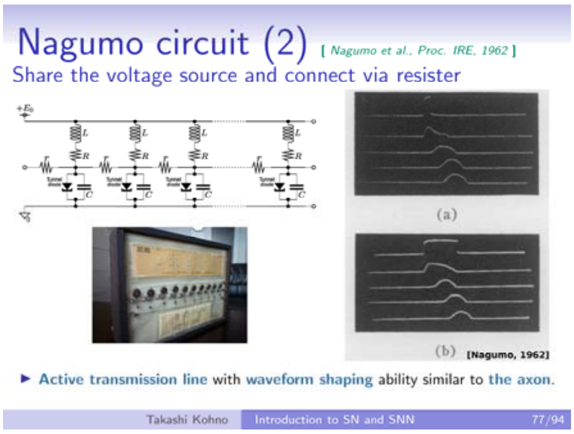
Nagumo circuit (2) [Nagumo et al., Proc. IRE, 1962]
Share the voltage source and connect via resister
(a)
(b)
[Nagumo, 1962]
Active transmission line with waveform shaping ability similar to the axon.
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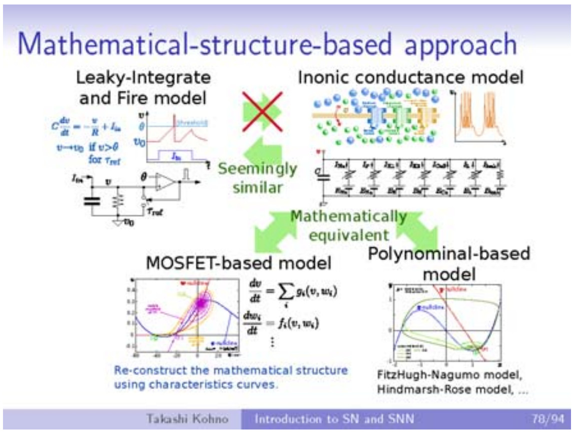
Mathematical-structure-based approach
Leaky-Integrate and Fire model
Inonic conductance model
Seemingly similar
Mathematically equivalent
MOSFET-based model
Polynominal-based model
Re-construct the mathematical structure using characteristics curves.
FitzHugh-Nagumo model, Hindmarsh-Rose model, ...
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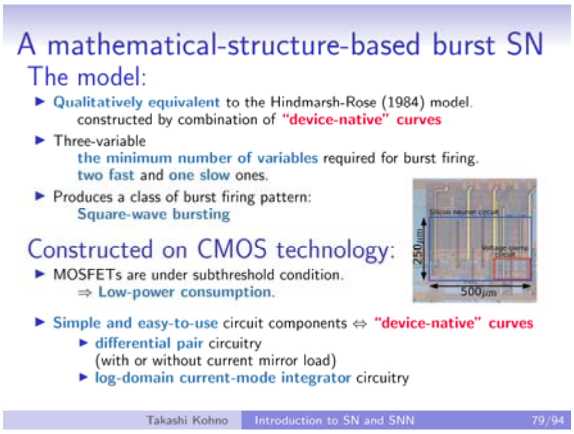
A mathematical-structure-based burst SN
The model:
Qualitatively equivalent to the Hindmarsh-Rose (1984) model.
constructed by combination of "device-native" curves
Three-variable
the minimum number of variables required for burst firing.
two fast and one slow ones.
Produces a class of burst firing pattern:
Square-wave bursting
Constructed on CMOS technology:
MOSFETs are under subthreshold condition.
⇒ Low-power consumption.
Simple and easy-to-use circuit components ⇔ "device-native" curves
differential pair circuitry
(with or without current mirror load)
log-domain current-mode integrator circuitry
250μm
500μm
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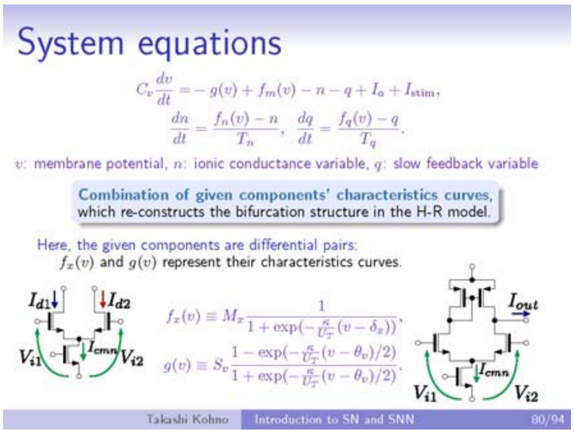
System equations
$C_v \frac{dv}{dt} = -g(v) + f_m(v) - n - q + I_a + I_{stim},$
$\frac{dn}{dt} = \frac{f_n(v) - n}{T_n}, \quad \frac{dq}{dt} = \frac{f_q(v) - q}{T_q}.$
v: membrane potential, n: ionic conductance variable, q: slow feedback variable
Combination of given components' characteristics curves,
which re-constructs the bifurcation structure in the H-R model.
Here, the given components are differential pairs:
$f_x(v)$ and $g(v)$ represent their characteristics curves.
$f_x(v) \equiv M_x \frac{1}{1+\exp(-\frac{\kappa}{U_T}(v-\delta_x))},$
$g(v) \equiv S_v \frac{1-\exp(-\frac{\kappa}{U_T}(v-\theta_v)/2)}{1+\exp(-\frac{\kappa}{U_T}(v-\theta_v)/2)}.$
I_{d1}
I_{d2}
I_{cmn}
V_{i1}
V_{i2}
I_{out}
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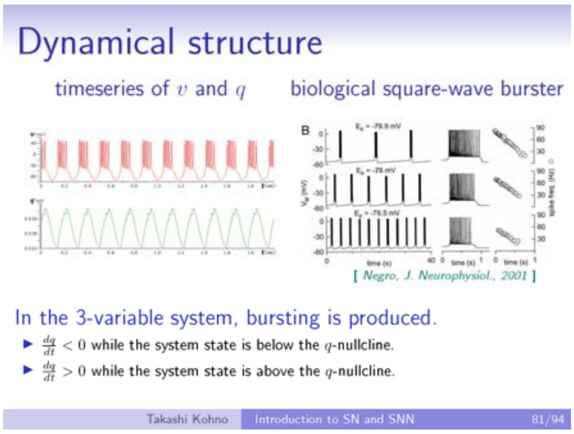
Dynamical structure
timeseries of v and q
biological square-wave burster
[Negro, J. Neurophysiol., 2001]
In the 3-variable system, bursting is produced.
$\frac{dq}{dt} < 0$ while the system state is below the q-nullcline.
$\frac{dq}{dt} > 0$ while the system state is above the q-nullcline.
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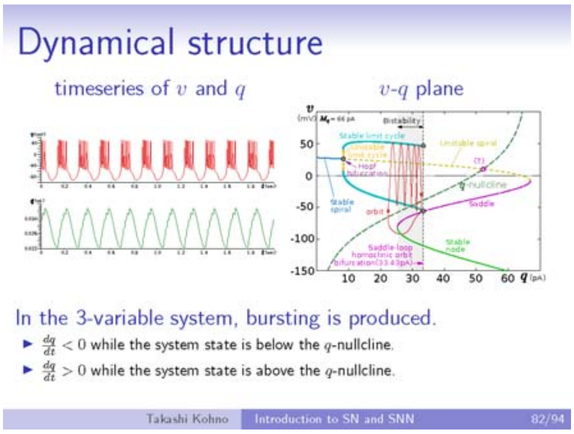
Dynamical structure
timeseries of v and q
v-q plane
Bistability
Stable limit cycle
Unstable spiral
Stable spiral
Saddle
Stable node
Saddle-loop homoclinic orbit bifurcation
q-nullcline
In the 3-variable system, bursting is produced.
$\frac{dq}{dt} < 0$ while the system state is below the q-nullcline.
$\frac{dq}{dt} > 0$ while the system state is above the q-nullcline.
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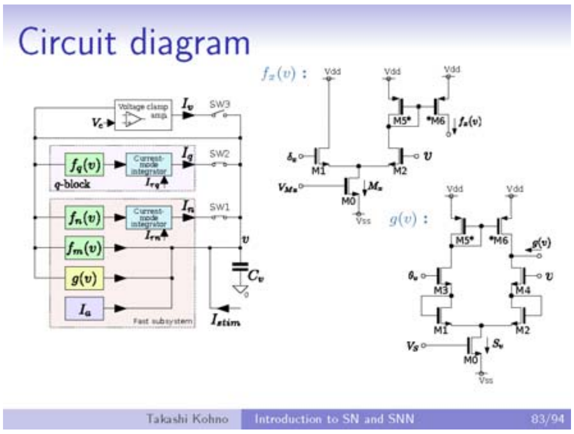
Circuit diagram
$f_x(v)$:
$g(v)$:
Voltage clamp amp.
$f_q(v)$
q-block
$f_n(v)$
$f_m(v)$
$g(v)$
I_a
Current-mode integrator
Fast subsystem
C_v
I_{stim}
Takashi Kohno
Introduction to SN and SNN
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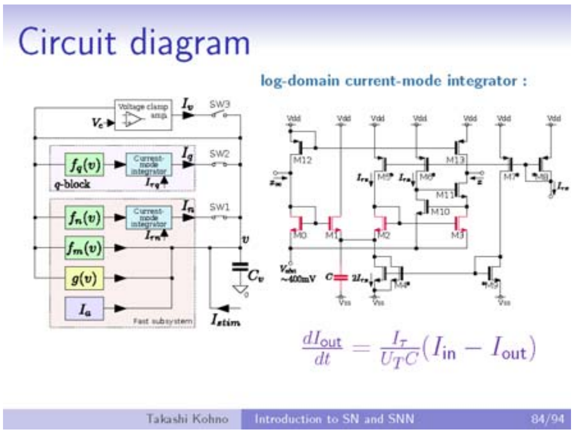
Circuit diagram
log-domain current-mode integrator :
$\frac{dI_{out}}{dt} = \frac{I_T}{U_T C}(I_{in} - I_{out})$
Takashi Kohno
Introduction to SN and SNN
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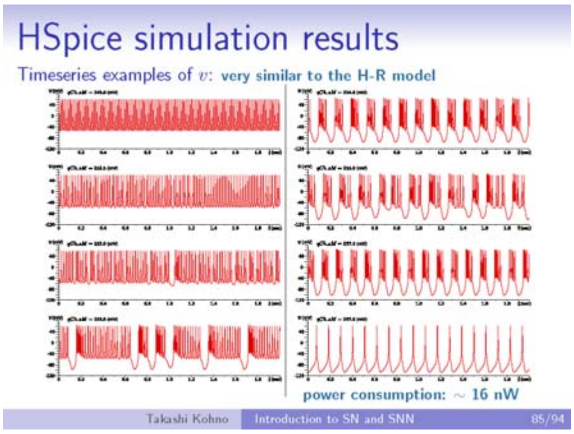
HSpice simulation results
Timeseries examples of v: very similar to the H-R model
power consumption: ~ 16 nW
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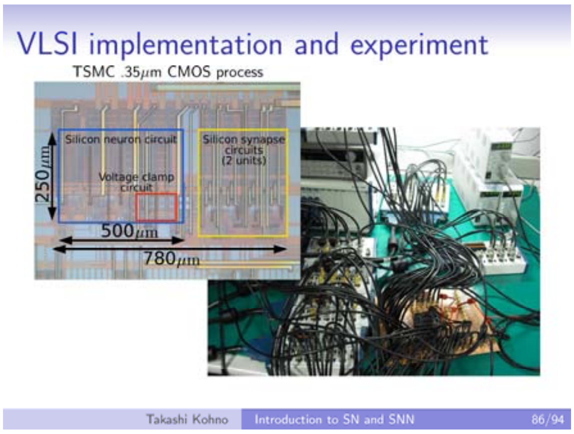
VLSI implementation and experiment
TSMC .35μm CMOS process
Silicon neuron circuit
Silicon synapse circuits (2 units)
Voltage clamp circuit
250μm
500μm
780μm
Takashi Kohno
Introduction to SN and SNN
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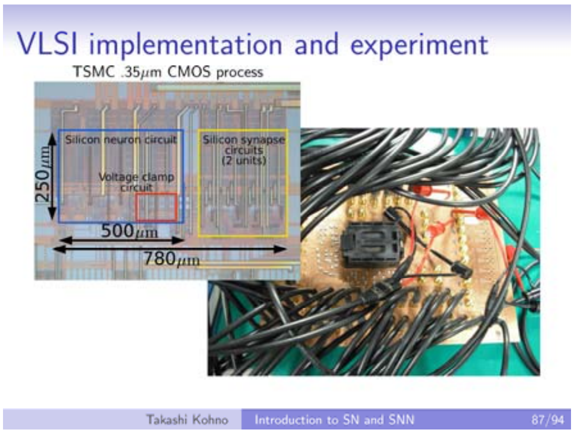
VLSI implementation and experiment
TSMC .35μm CMOS process
Silicon neuron circuit
Silicon synapse circuits (2 units)
Voltage clamp circuit
250μm
500μm
780μm
Takashi Kohno
Introduction to SN and SNN
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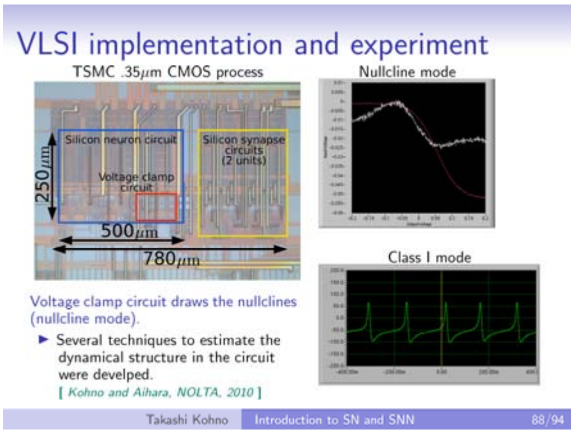
VLSI implementation and experiment
TSMC .35μm CMOS process
Silicon neuron circuit
Silicon synapse circuits (2 units)
Voltage clamp circuit
250μm
500μm
780μm
Nullcline mode
Class I mode
Voltage clamp circuit draws the nullclines (nullcline mode).
Several techniques to estimate the dynamical structure in the circuit were develped.
[Kohno and Aihara, NOLTA, 2010]
Takashi Kohno
Introduction to SN and SNN
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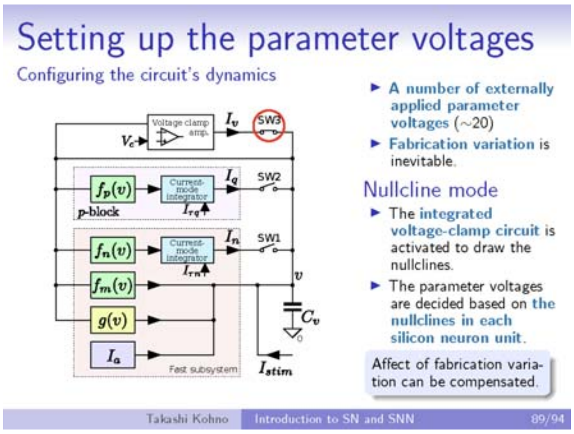
Setting up the parameter voltages
Configuring the circuit's dynamics
Voltage clamp amp.
SW3
SW2
SW1
p-block
Current-mode integrator
Fast subsystem
A number of externally applied parameter voltages (~20)
Fabrication variation is inevitable.
Nullcline mode
The integrated voltage-clamp circuit is activated to draw the nullclines.
The parameter voltages are decided based on the nullclines in each silicon neuron unit.
Affect of fabrication variation can be compensated.
Takashi Kohno
Introduction to SN and SNN
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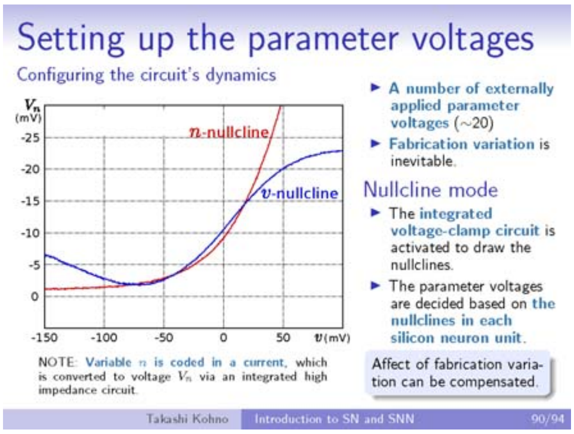
Setting up the parameter voltages
Configuring the circuit's dynamics
n-nullcline
v-nullcline
A number of externally applied parameter voltages (~20)
Fabrication variation is inevitable.
Nullcline mode
The integrated voltage-clamp circuit is activated to draw the nullclines.
The parameter voltages are decided based on the nullclines in each silicon neuron unit.
NOTE: Variable n is coded in a current, which is converted to voltage Vn via an integrated high impedance circuit.
Affect of fabrication variation can be compensated.
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Introduction to SN and SNN
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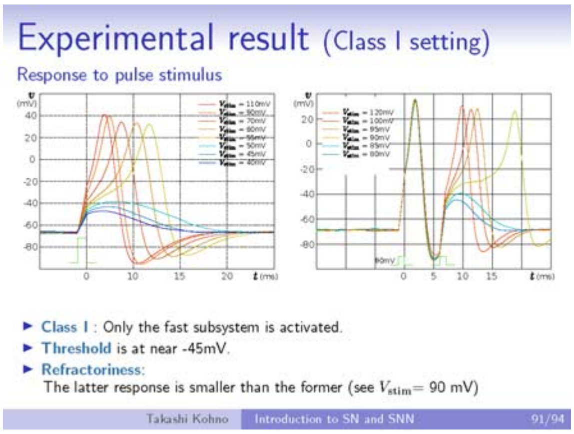
Experimental result (Class I setting)
Response to pulse stimulus
Class I : Only the fast subsystem is activated.
Threshold is at near -45mV.
Refractoriness:
The latter response is smaller than the former (see Vstim= 90 mV)
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Introduction to SN and SNN
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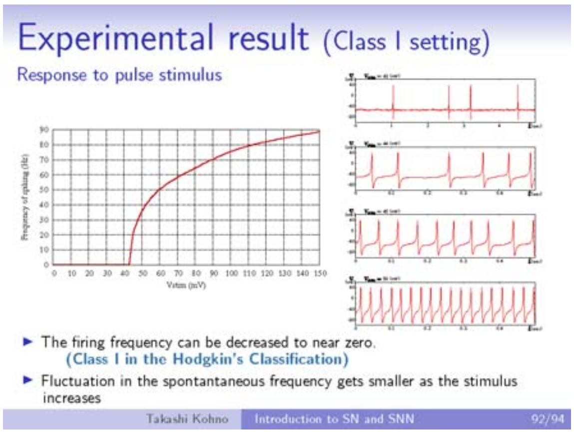
Experimental result (Class I setting)
Response to pulse stimulus
The firing frequency can be decreased to near zero.
(Class I in the Hodgkin's Classification)
Fluctuation in the spontaneous frequency gets smaller as the stimulus increases
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Introduction to SN and SNN
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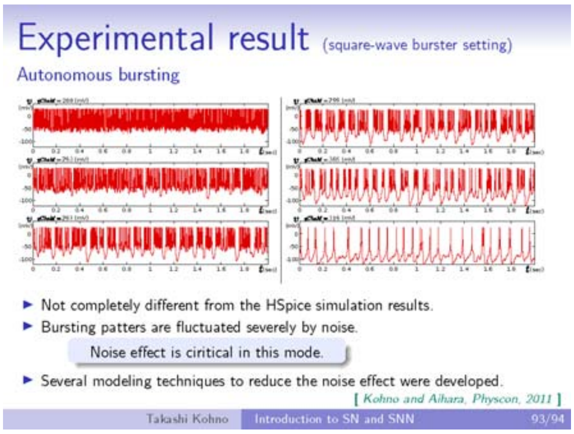
Experimental result (square-wave burster setting)
Autonomous bursting
Not completely different from the HSpice simulation results.
Bursting patters are fluctuated severely by noise.
Noise effect is ciritical in this mode.
Several modeling techniques to reduce the noise effect were developed.
[Kohno and Aihara, Physcon, 2011]
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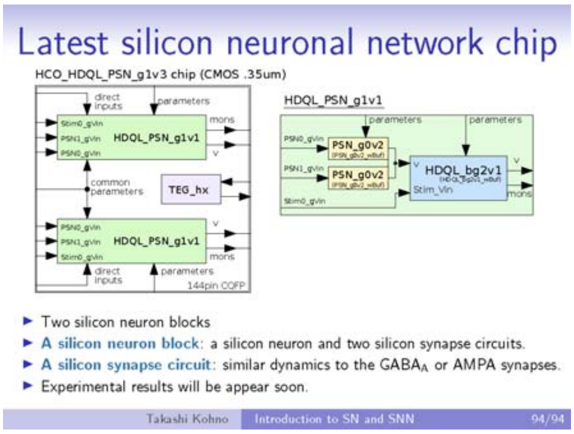
Latest silicon neuronal network chip
HCO_HDQL_PSN_g1v3 chip (CMOS .35um)
HDQL_PSN_g1v1
TEG_hx
PSN_g0v2
HDQL_bg2v1
144pin CQFP
Two silicon neuron blocks
A silicon neuron block: a silicon neuron and two silicon synapse circuits.
A silicon synapse circuit: similar dynamics to the GABAA or AMPA synapses.
Experimental results will be appear soon.
Takashi Kohno
Introduction to SN and SNN
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