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Individual Recognition-Free Target Enclosure Model

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Abstract: Target enclosure by autonomous robots is useful for many practical applications, for example, surveillance of disaster sites. Scalability is important for autonomous robots because a larger group is more robust against breakdown, accidents, and failure. However, it is more difficult to operate a larger group of robots because their individual capacity for recognizing teammates should be higher. In this paper, to achieve a highly scalable target enclosure model, we demonstrate a new condition for Takayama's enclosure model. The original model requires a static relationship between agents. However, robots can form an enclosure even under a dynamic topology on the basis of a nearest neighbor graph; hence, they do not require recognition capability. We confirm this by an analytical discussion of switched systems and a series of computer simulations.

Keywords: Collective Intelligence, Distributed robotics, Formation, Cyclic pursuit, robot swarm

1 INTRODUCTION

In this paper, we propose a new condition for Takayama's target enclosure model [10] that can allocate robots to an unspecified number of targets.

Target enclosure, which is useful for monitoring disaster sites and unknown vehicles, has recently become an important goal for multiple robots. Robots can operate in dangerous circumstances, replacing human presence.

Disaster sites are usually far from an operator. In this case, a group of robots examines the exact number of sites to be observed and their locations. Therefore, it is desirable that more robots than necessary are employed, which enables them to accept a larger number of targets . For this purpose, the tasks of target allocation and target enclosure must be performed simultaneously.

However, it seems difficult for most target enclosure models proposed so far to realize this requirement.

Several related studies deal with target enclosure[12][5][4][10]. Except for the study of Kobayashi et al.[5], all other studies require that a particular physical arrangement of the robots be maintained in order to build a target enclosure. For example, Yamaguchi[12] discussed a target capturing task in which the robots must maintain a chain structure. Kim et al.[4] discussed the target enclosure problem; in their solution, each robot needs information on the relative speed of one robot and relative geographical relation to its target to determine its behavior. If the relationship between a robot and its reference robot is considered as a link in graph theory, the graph of the group of robots must follow a Hamiltonian cycle.

When a robot changes the target to be enclosed, the following two events should be considered: withdrawal and accedence of the robot. In the former, the remaining robots in the group must maintain the constraint of the Hamiltonian cycle without the removed robot. In the latter, a group that satisfies the Hamiltonian cycle condition and the new member must form a new Hamiltonian cycle. As far as we know, discussion of these events is inadequate when there are no restrictions on the timing of withdrawal and accedence of robots.

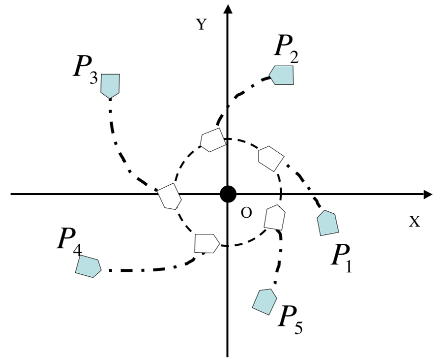

Fig. 1. Process of target enclosure using five robots.

Therefore, we investigated the relaxation of the condition of maintaining a Hamiltonian cycle to achieve target enclosure. In particular, we focused on the study of Takayama et al.[10]. In their model, each robot needs information of one neighbor and its target. As in other studies, this model also requires the Hamiltonian cycle constraint. However, in this paper, we show that this model can realize target enclosure without this constraint when each robot bases its behavior on information from its nearest neighboring robot. Therefore, in this model, robots can change targets without considering the above two events.

Note that the reference relationships among more than four robots in the proposed nearest neighbor model are often unconnected in the graph theory sense. Therefore, it is not

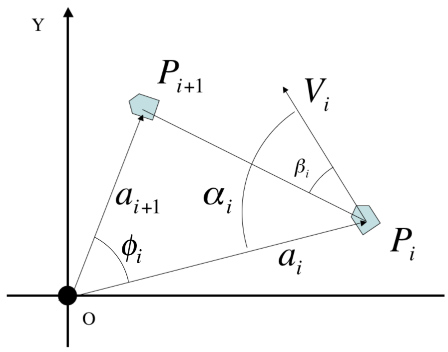

Fig. 2. Model of Takyama's target enclosure: α, β.

easy to discuss this issue using a graph Laplacian, which is the primary analytical approach used for multirobot systems. In this paper, the theory of switched systems[7] is adopted for analyzing groups of less than five robots. A series of computer simulations is used for larger groups.

This paper is organized as follows. First, Takayama's model is introduced. Next the proposed method based on using the nearest neighbor as a reference is presented, and the problems in verifying its ability to form a target enclosure are discussed. In section 4, the practical asymptotic stability of a small group is proven analytically. In addition, we use computer simulations to demonstrate the ability of a larger group to achieve target enclosure.

2 TAKAYAMA'S TARGET ENCLOSURE MODEL

Firstly, Takayama's target enclosure model is explained.

2.1 Takayama's target enclosure model

In this section, we assume that all robots choose the same target. We assume that on a two-dimensional (2D) plane, there is only one target O at the origin and n robots. Fig.1 illustrates the case of $n = 5$. Robots are numbered counterclockwise as $P_1, \ldots, P_n$, and r_i is the position vector of the robot P_i. In the target enclosure task, each robot moves to the corresponding white marker.

To achieve this task, Takayama et al.[10] proposed the following model. Each robot determines its control input, speed v_i, and angular velocity ω_i using two aspects of angular information: relative angles with respect to the target and an anterior neighboring robot,denoted as α_i and β_i, respectively. As a result, rotational movement occurs with a central focus on the target.

$$v_i = f\beta_i \tag{1}$$

$$\omega_i = v_i/\bar{r} \quad k\cos\alpha_i, \tag{2}$$

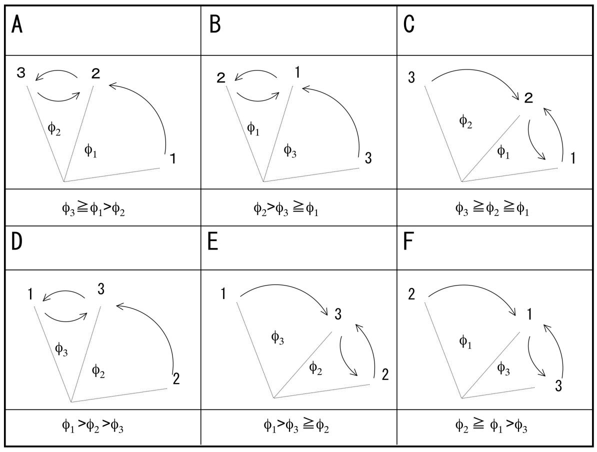

Fig. 3. Patterns of reference relationships among robots in a three-robot group.

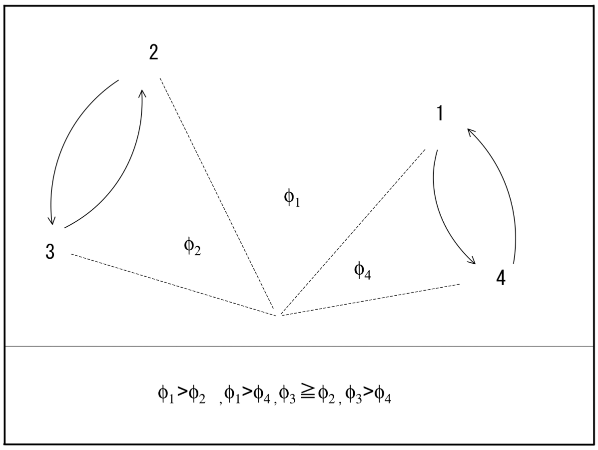

Fig. 4. Unconnected pattern of reference relationships in a four-robot group.

where the parameters $\bar{r}, k$, and $f > 0$ specified beforehand. P_{i+1} is the robot to which P_i refers, and $\bar{r}$ is the expected distance to the target. In Takayama's model, the i-th robot refers to the $i + 1$-th robot, and the n-th robot refers to the first robot P_1. That is, if the relationship between a robot and its reference robot is considered as a link in graph theory, the graph of the group of robots must be a Hamiltonian cycle. The authors proved the convergence to the goal state of the target enclosure under this constraint.

Takayama et al. reported the following three characteristics of their model. (E1) The distance between the target and each robot converges to $\bar{r}$. (E2) The speed vector V_i and the vector $(O \quad P_i)$ are orthogonal. (E3) The gaps between a robot and its neighbors are equalized, i.e., $\ _i = \frac{2\pi}{n}$.

3 NEW REFERENCE RULE PROPOSAL OF TAKAYAMA'S MODEL

3.1 Takayama's model considering the nearest neighboring robot as the reference

In this paper, the robots observed by the i-th robot are considered to be its reference robots. In the original Takayama's model, the i-th robot P_i's reference robot is the $i+1$-th robot P_{i+1}. This relationship forms a Hamiltonian cycle. As mentioned above, this constraint makes target allocation behavior difficult. It also causes the scalability problem. Furthermore, each robot must identify its reference robot from the group of robots. This typically becomes difficult as the group size increases.

Therefore, we examine a new reference robot scheme in which each robot considers its anterior neighboring robot as its reference robot. Each robot controls itself as described in equations 1 and 2, but it chooses its nearest neighbor as its reference robot. If possible, the robots can change their target during the target allocation task. Such a system also has higher scalability because individual robots need not be identified to observe the nearest robot.

3.2 Problems in verification of the proposed reference model

In the work of Takayama et al.[10], the model was proven analytically by two approaches: convergence of the distance from the target and convergence of the distance between robots. The former convergence holds true for our proposed model.

In contrast, the result of the latter approach in which the angle between each adjacent robot converges to $2 \quad /n$ does not apply in our proposed model because Takayama's proof assumes that the relationship between the robot and its reference robot is static and robots are connected in the graph theory sense, as in references [8], [8], [9], and [10]. However, this assumption is inadequate for the following reasons.

1. The reference relationship in the proposed model changes dynamically. For example, there are six graphical patterns for the three-robot group (see Fig.3.).

2. The graph is not connected when $n > 3$. If $n \quad 3$, the graph is dynamic but connected (at least, it is weakly connected as a digraph). However, when $n > 3$, unconnected patterns appear, as shown in Fig.4.

Because of these two differences, alternative approaches of verifying the proposed model are required.

4 VERIFICATION OF THE PROPOSED NEAREST NEIGHBOR REFERENCE MODEL

In this section, switched systems theory is adopted to verify the convergence of the angle between a pair of neighboring robots in the three- and four-robot groups.

First, the target enclosure problem is defined.

4.1 Definition of enclosure task

In this paper, the target enclosure task for an n-robot group is defined as follows. The task consists of determining the distance to the target and equalizing the gap angle.

The distance task is

$$E_d = \sum_{i=1}^{n} (r_i \quad \bar{r})^2. \tag{3}$$

The angle equalization task is

$$E_a = \sum_{i=1}^{n} (\quad_i \quad \frac{2 \quad}{n})^2. \tag{4}$$

Because of these two requirements, the robots are deployed evenly on a circle having a radius of $\bar{r}$.

4.2 Verification using switched system

The results of the switched system are used here. Instead of the graph Laplacian, the Poincaré-Bendixson theorem[3] can be used, but this theorem is generally applicable only to systems with two variables. In contrast, the results of the switched system adopted here can be used to examine the convergence property of a small group of robots.

4.3 Switched systems

A switched system is defined as[7, 11]

$$\dot{x} = f_s(x), \tag{5}$$

where $x \in R^n$ is a continuous state variable, and $\dot{x}$ is its derivative. Furthermore, S is a set of discrete values s, and s is static even if t and/or x change. In this case, reference [11] proves the sufficient condition for the practical asymptotic stability of the switched system. Let $V(x)$ be a continuous differentiable positive definite function. In addition, we assume that a set of positive values $\Omega_\rho = \{x \in R^n : V(x) \quad \rho\}$ is bounded. In this case, the switched system exhibits practical asymptotic stability for any $D \subset \Omega_\rho$ when the following conditions are satisfied.

$$a) \quad \min_{s \in S} \frac{\partial V}{\partial x} f_s(x) < 0, \ \ \forall x \in \Omega_\rho \quad \{0\} \tag{6}$$

$$b) \quad 0 \in Int(C), \tag{7}$$

where $Int(C)$ is the interior of (C). C is given as

$$C = conv(\{f_s(0) : s \in S\}) \\ = \{\sum_{s \in S} \lambda_s f_s(0) : \lambda s \quad 0, \sum_{s \in S} \lambda_s = 1\}. \quad (8)$$

We assume that a sufficient time has passed so that all the robots are near their common target. Furthermore, we assume that the distance between the robots and the target is $\bar{r}$, and each robot determines its nearest neighbor using only the angle with respect to its neighbor. In this case, the angle $_i$ between the i-th robot and its reference robot is expressed as follows.

Case 1: $_{i+1} \quad _i, \; _i \quad _{i \; 1}$

$$\frac{d \; _i}{dt} = \frac{b}{2}(\quad _i + \; _{i \; 1}) \quad (9)$$

Case 2: $_{i+1} < \; _i, \; _i \quad _{i \; 1}$

$$\frac{d \; _i}{dt} = \frac{b}{2}(\; _{i+1} + \; _{i \; 1} \quad 2 \;) \quad (10)$$

Case 3: $_{i+1} \quad _i, \; _i < \; _{i \; 1}$

$$\frac{d \; _i}{dt} = b(\quad _i) \quad (11)$$

Case 4: $_{i+1} < \; _i, \; _i < \; _{i \; 1}$

$$\frac{d \; _i}{dt} = \frac{b}{2}(\; _{i+1} \quad _i) \quad (12)$$

where $b = f/\bar{r}$. In this case, the dynamics of their angles is considered to represent a switched system according to each robot's three angles $_{i \; 1}, \; _i, and \; _{i+1}$.The heading direction d_i of the i-th robot can be described by $_{i \; 1}$ and $_i$ as follows.

$$d_i = \begin{cases} 1 & (\; _i \quad _{i \; 1}) \\ 0 & (otherwise) \end{cases} \quad (13)$$

where "0" and "1" indicate a counterclockwise and clockwise heading direction, respectively. By using equation 13, equations 9-ref(84) are written as follows.

$$\dot{} = A_s \quad + B_s \quad (14)$$

$$= [\; _1 \cdots \; _i \cdots \; _n]^T \quad (15)$$

$$A_{s,i,j} = \begin{cases} \frac{b}{2}d_i & (j = i \quad 1) \\ \frac{b}{2}(d_{i+1} \quad d_i + 1) & (j = i) \\ \frac{b}{2}(1 \quad d_{i+1}) & (j = i + 1) \\ 0 & (otherwise)) \end{cases} \quad (16)$$

$$B_i = \quad (d_{i+1} \quad d_i) \quad (17)$$

$$s = \{d_1, \ldots, d_i, \ldots, d_n\} \in \{0,1\}^n \quad (18)$$

where $_n = 2 \quad \sum_{i=1}^{n \; 1} \; _i$. For simplicity, let $b = 1$ in the remainder of this paper. In the next subsection, we prove the practical asymptotic stability of the system represented by equation 14.

4.4 A four-robot group

In this section, we discuss the practical asymptotic stability of a four-robot system. A four-robot group has 14 control inputs, $s = \{1,0,1,0\}$, $\{1,0,0,1\}$, $\{0,1,1,0\}$, $\{0,1,0,1\}$, $\{0,0,1,1\}$,
$\{1,1,1,0\}$, $\{1,1,0,1\}$, $\{1,0,1,1\}$, and $\{0,1,1,1\}\}$.

When $s = \{1,0,0,0\}$, the result of the left of equation 6 in this case by using equation 16 is $w(\;) = \quad 4 \; ^2 \quad 3 \; _1^2 \quad 4 \; _1 \; _2 \quad 3 \; _2^2 + 6 \; (\; _1 + \; _2) + 8 \; \; _3 \quad 6 \; _1 \; _3 \quad 4 \; _2 \; _3 \quad 4 \; _3^2$. Therefore, the maximum of $w(\;)$ in the given range is calculated by a Lagrange multiplier. We rewrite $z(\;) = \quad w(\;)$ as a minimization problem.

The set of constraints representing the control input $s = \{1,0,0,0\}$ is $_1 > \; _4 \wedge \; _2 < \; _1 \wedge \; _3 < \; _2 \wedge \; _4 < \; _3 \wedge \; _1, \; _2, \; _3 > 0, \; _4 > 0$. We define the following functions from this condition by adding equal conditions for convenience.

$$\begin{aligned} g_1(\;) = 2 \quad 2 \; _1 \quad _2 \quad _3 \quad 0 \\ g_2(\;) = \; _2 \quad _1 \quad 0 \\ g_3(\;) = \; _3 \quad _2 \quad 0 \\ g_4(\;) = \; _1 \quad 0 \\ g_5(\;) = \; _2 \quad 0 \\ g_6(\;) = \; _3 \quad 0 \\ g_7(\;) = \quad 2 \quad + \; _1 + \; _2 + \; _3 \quad 0 \end{aligned} \quad (19)$$

$$\begin{aligned} \nabla g_1 = [\quad 2 \quad 1 \quad 1]^T, \nabla g_2 = [\quad 1 \; 1 \; 0]^T \\ \nabla g_3 = [0 \quad 1 \; 1]^T, \nabla g_4 = [\quad 1 \; 0 \; 0]^T \\ \nabla g_5 = [0 \quad 1 \; 0]^T, \nabla g_6 = [0 \; 0 \quad 1]^T \\ \nabla g_7 = [1 \; 1 \; 1]^T \end{aligned} \quad (20)$$

Then, the following Karush-Kuhn-Tucker conditions are obtained from $\nabla z(\;) + \sum_i^7 \nabla g_i(\;) = 0$.

$$\begin{aligned} 6 \; _1 + 4 \; _2 \quad 6 \quad + 6 \; _3 \quad 2u_1 \quad u_2 \quad u_4 + u_8 = 0 \\ 4 \; _1 + 6 \; _2 \quad 6 \quad + 4 \; _3 \quad u_1 + u_2 \quad u_3 \quad u_5 + u_7 = 0 \\ 6 \; _1 + 4 \; _2 \quad 8 \quad + 8 \; _3 \quad u_1 + u_3 \quad u_6 + u_7 = 0 \\ u_1(2 \quad 2 \; _1 \quad _2 \quad _3) = 0, \; u_1 \quad 0 \\ u_2(\; _2 \quad _1) = 0, \; u_2 \quad 0 \\ u_3(\; _3 \quad _2) = 0, \; u_3 \quad 0 \\ u_4(\quad _1) = 0, \; u_4 \quad 0 \\ u_5(\quad _2) = 0, \; u_5 \quad 0 \\ u_6(\quad _3) = 0, \; u_6 \quad 0 \\ u_7(\quad 2 \quad + \; _1 + \; _2 + \; _3) = 0, \; u_7 \quad 0 \end{aligned} \quad (21)$$

This equation reveals that the maximum of $w(\;)$ in the given range is $w(\;) = 0$ at $_1 = \; _2 = \; _3 = \; _4 = \frac{\pi}{2}$.

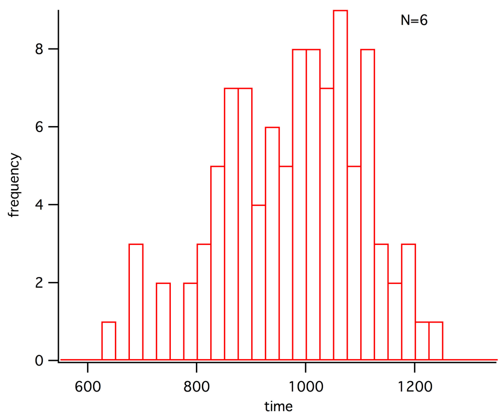

Fig. 5. Time to achieve enclosure task by 6 robots group

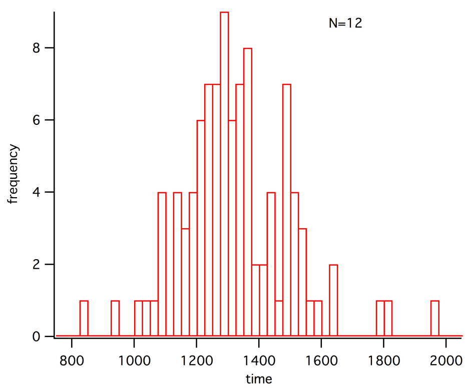

Fig. 6. Time to achieve enclosure for 12-robot group.

Therefore, equation 6 is satisfied when this control signal s is activated.

We verify the maximum of $w(\quad)$ for all other s values in a similar manner.

When $s = \{0,1,0,0\}$, $w(\quad) = \quad 4\ ^{2} \quad 4\ _{1}^{2} \quad _{2}^{2}$ $2\ _{2}\ _{3} \quad 3\ _{3}^{2} \quad 2\ _{1}(\ _{2}+2\ _{3})+2\ (4\ _{1}+\ _{2}+2\ _{3})$. The maximum of $w(\quad)$ is 0 at $(\ _{1},\ _{2},\ _{3}) = (\frac{\pi}{2}, \frac{\pi}{2}, \frac{\pi}{2})$.

In the same manner as these cases, the maximum of $w(\quad)$ in all of the remaining cases, is 0.

Therefore, equation 6 is satisfied for any control input s. Furthermore, $C = \{0\}$ satisfies $0 \in Int(C)$ because $\ _{1} = \ _{2} = \ _{3} = \ _{4} = \frac{\pi}{2}$ is a fixed point for which $f_s(\quad) = 0$ for all f_s. Therefore, equation 7 is satisfied.

4.5 Verification of target enclosure task for larger groups

The above discussion shows that the proposed model can achieve angle equalization for a small group. However, we did not provide the proof of the distance task represented by equation 3. In addition, we did not verify the performance for groups of more than four robots. Therefore, in this section, we discuss the ability to achieve target enclosure by using computer simulations.

We examined 3-, 4-, 6-, and 12-robot groups. There was only one target at the origin, and it was assumed that $\bar{r} = 20$. The initial position of a robot was specified inside a 100 100 rectangular region by a 2D uniform random number generator. We counted the time to achieve target enclosure as the time until $E_d + E_a < 0.5$ in equations 3 and 4. This simulation was repeated 100 times for each group size.

Fig.5,6 show the results for the 3-, 4-, 6-, and 12-robot groups, respectively. The x-axis of each graph indicates the time required to achieve enclosure, and the y-axis denotes the frequency. For the three-robot system, the average time required for enclosure is 813.123, and the standard deviation is 125.737. For the four-robot system, the average time is 813.123 and the standard deviation is 125.737. For the six-robot system, the average time is 874.143 and the standard deviation is 96.921. For the 12-robot system, the average time is 1044.371 and the standard deviation is 115.408.

Thus, as the number of robots increases, the time required to achieve target enclosure increases. However, in all the simulations, groups of any size can achieve this task. Therefore, we conclude that any group of fewer than 13 robots can achieve target enclosure.

5 CONCLUSION

In this paper, to achieve a highly scalable target enclosure model, we examined a new reference model based on that of Takayama et al., in which each robot determines its actions according to its nearest neighboring robot. We demonstrated the model's performance using an analytical discussion and a set of computer simulations. Conventional research on target enclosure assumes that a robot can recognize predefined team-mates from among many robots. However, this recognition becomes difficult as the group size increases. In the proposed model, a robot does not need this recognition capability. The results of switched systems theory were applied instead of the graph Laplacian to prove the convergence to an enclosure state because the connectivity of the reference relationship among robots is not maintained. We analytically proved that a group containing fewer than five robots can enclose a target. Computer simulations with n = 3, 4, 6, and 12 suggest that a group of 12 or fewer robots can enclose a target.

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