

Robust Digital Control of DC-DC Buck Converter with Low Frequency Samplings

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Abstract: Robust DC-DC converter which can covers extensive load change and also input voltage changes with one controller is needed. Then demand to suppress output voltage change becomes still severer, We propose an approximate 2 DOF digital controller which realized the startup response and dynamic load response independently. Controller make the control bandwidth wider, and at the same time make variations of the output voltage small at sudden changes of load and input voltage. In this paper a new approximate 2DOF digital control system with additional zeros is proposed. Using additional zeros second-order differential transfer characteristics between equivalent disturbances and output voltage are realized. Therefore the new controller makes variations of the output voltage smaller and sudden changes of load and input voltage. These controller is actually implement on a DSP and is connected to DC-DC converter. Experimental results demonstrate that this type of digital controller can satisfy given severe specifications with low frequency sampling.

Keywords: DC-DC converter, Approximate 2DOF, Second-order differential, Digital Robust control

1 INTRODUCTION

In many applications of DC-DC converters, loads cannot be specified in advance, i.e., their amplitudes are suddenly changed from the zeros to the maximum rating. Generally, design conditions are changed for each load and then each controller is re-designed. Then, a so-called robust DC-DC converter which can cover such extensive load changes and also input voltage changes with one controller is needed. Analog control IC is used usually for control of DC-DC converter. Simple integral control etc. are performed with the analog control IC. Moreover, the application of the digital controller to DC-DC converters designed by the PID or root locus method etc. has been recently considered [1]. However, it is difficult to retain sufficient robustness of DC-DC converter by these techniques. Various robust control methods [2] for improving start-up characteristics and load sudden changes characteristics of DC-DC converters. However, they take tens [ms] for the rising time of the startup response, and hundreds [mV] output voltage regulations, have arisen in the load sudden changes. The demand for suppressing output voltage changes becomes still severer, and the further improvements to startup characteristics and load sudden changes characteristics are required. In this paper, we propose a new approximate 2DOF digital controller which realizes second-order differential transfer characteristics. These characteristics are realized by introducing additional zeros into transfer functions between equivalent disturbances and the output. The new controller make the variations of the output voltage the almost same as the former controller [3]

at sudden changes of resistive loads and input voltages with low sampling frequency. A new DC-DC converter equipped with the proposed controller in DSP is actually manufactured. Some simulations and experiments show that this new DC-DC converter can satisfy given severe specifications.

2 DESIGN METHOD

2.1 DC-DC converter

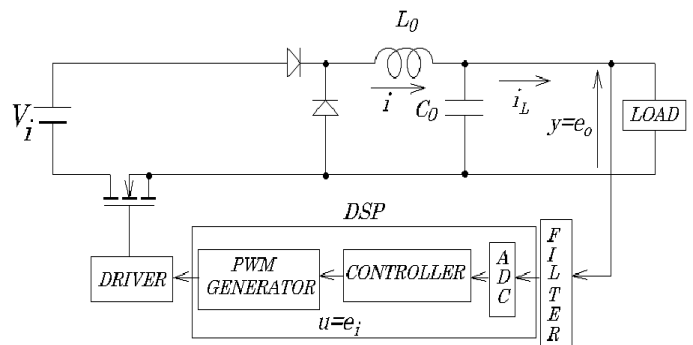


Fig. 1: DC-DC converter

The DC-DC converter are shown in Fig.1 has been manufactured. In order to realize the approximate 2DOF digital controller which satisfies given specifications, we use the DSP (TMS320F28335). This DSP has a builtin AD converter and a PWM switching signal generating part. The triangular wave carrier is adopted for the PWM switching signal. The

switching frequency is set at 100[KHz]. The LC circuit is a filter for removing carrier and switching noises. Where C_0 is 235[μF] and L_0 is 0.55[μH]. If the frequency of control signal u is smaller enough than that of the carrier, the state equation of the DC-DC converter at a resistive load in Fig.1 except for the controller in DSP can be expressed from the state equalizing method as follows:

$$\begin{cases} \dot{x} = A_c x + B_c u + B_{c_q} u \\ y = C_x + q_y \end{cases} \quad (1)$$

where

$$x = \begin{bmatrix} e_0 \\ i \end{bmatrix} A_c = \begin{bmatrix} -\frac{1}{C_0 R_L} & \frac{1}{C_0} \\ -\frac{1}{L_0} & \frac{R_L}{L_0} \end{bmatrix} B_c = \begin{bmatrix} 0 \\ \frac{k_p}{L_0} \end{bmatrix} c = [1 \quad 0]$$

$$u = e_i, y = e_o, k_p = -\frac{V_i}{C_m L_0}, G_p = \frac{R_L}{R_0 + R_L} \times \frac{V_{in}}{TBPRD}$$

and R_0 the total resistance of coil and ON resistance of FET, etc., whose value is 0.015[Ω]. Then the discrete-time state equation of the system eq.(1) with a zero-order hold is express as

$$\begin{cases} x(k+1) = A_d x(k) + B_d u(k) + B_{dq} u(k) \\ y(k) = Cx(k) + q_y(k) \end{cases} \quad (2)$$

where

$$A_c = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = e^{A_c T}$$

$$B_c = \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix} = \int_0^T e^{A_c t} B_c dt$$

2.2 Additional zeros method

The following equation is obtained by repeating the difference of the output of eq.(2):

$$Y = O^* x_d(k) + U \bar{u}(k) + U \bar{q}_u + \bar{q}_y \quad (3)$$

where

$$Y = \begin{bmatrix} y(k) \\ y(k+1) \\ y(k+2) \end{bmatrix} O^* = \begin{bmatrix} C \\ CA_d \\ CA_d^2 \end{bmatrix} U = \begin{bmatrix} 0 & 0 \\ CB_d & 0 \\ CA_d B_d & CB_d \end{bmatrix}$$

$$\bar{u} = \begin{bmatrix} u(k) \\ u(k+1) \end{bmatrix} \bar{q}_u = \begin{bmatrix} q_u(k) \\ q_u(k+1) \end{bmatrix} \bar{q}_y = \begin{bmatrix} q_y(k) \\ q_y(k+1) \\ q_y(k+1) \end{bmatrix}$$

If both sides of eq.(3) are multiplied by \bar{I}_2 from the left, x_d is obtained by the following equation:

$$\begin{aligned} x_d(k) &= (\bar{I}_2 O^*)^{-1} \bar{I}_2 Y - (\bar{I}_2 O^*)^{-1} \bar{I}_2 U \bar{u}(k) \\ &- (\bar{I}_2 O^*)^{-1} \bar{I}_2 U \bar{q}_u - (\bar{I}_2 O^*)^{-1} \bar{I}_2 \bar{q}_y \end{aligned} \quad (4)$$

where

$$\bar{I}_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

By substituting the above equation into eq.(3), the following equation is obtained:

$$(I_3 - (\bar{I}_2 O^*)^{-1} \bar{I}_2) U \bar{q}_u + (I_3 - (\bar{I}_2 O^*)^{-1} \bar{I}_2) \quad (5)$$

where I_3 is a 3 \times 3 unit matrix. That is, \bar{q}_u and \bar{q}_y can be replaced to Y and $\bar{u}(k)$. Eq.(5) is transformed as

$$\begin{aligned} &- (l_2 z + l_1) q_u(k) + (z^2 + m_2 z + m_1) q_y(k) \\ &= (l_2 z + l_1) u(k) + (z^2 + m_2 z + m_1) y(k) \end{aligned} \quad (6)$$

where

$$[l_1 \quad l_2] = -(I_3 - O^*(\bar{I}_2 O^*)^{-1} \bar{I}_2) U$$

$$[m_1 \quad m_2 \quad m_3] = (I_3 - O^*(\bar{I}_2 O^*)^{-1} \bar{I}_2) \quad (7)$$

The delay time for AD conversion time etc., replacing current feedback and zeros addition are connected to input of system(2). The state equation of a new controlled object connecting can be expressed as

$$\begin{aligned} x_{dw}(k+1) &= A_{dw} x_{dw}(k) + B_{dw} v(k) \\ y(k) &= C_{dw} x_{dw}(k) \end{aligned} \quad (8)$$

where

$$x_{dw}(k) = \begin{bmatrix} x_d(k) \\ \xi_1(k) \\ \xi_2(k) \\ \xi_3(k) \end{bmatrix} A_{dw} = \begin{bmatrix} A_d & B_d & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B_{dw} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} C_{dw} = [C \quad 0 \quad 0 \quad 0] \xi_1(k) = u(k)$$

Applying the following feedforwards from q_u , q_y and r , and state feedback from x_{dw} for model matching to the system in eq.(8), the system shown in Fig.2 is obtained. From Fig.2,

$$\begin{aligned} v(k) &= -k_q(l_2 z + l_1) q_u(k) + k_q(z^2 + m_2 z + m_1) q_y(k) \\ &+ (z^2 + g_2 z + g_1) r(k) + [f_1 \ f_2 \ f_3 \ f_4 \ f_5] x_{dw}(k) \end{aligned} \quad (9)$$

In Fig.2, the parts surrounded by dotted lines are the feedforward coefficients from q_u and q_y and the part surrounded by a chain line is the estimated part of current. From eq.(6), the feedforwards of eq.(9) are changed as

$$\begin{aligned} v(k) &= k_q(l_2 z + l_1) u(k) + k_q(z^2 + m_2 z + m_1) y(k) + \\ &(z^2 + g_2 z + g_1) r(k) + [f_1 \ f_2 \ f_3 \ f_4 \ f_5] x_{dw}(k) \end{aligned} \quad (10)$$

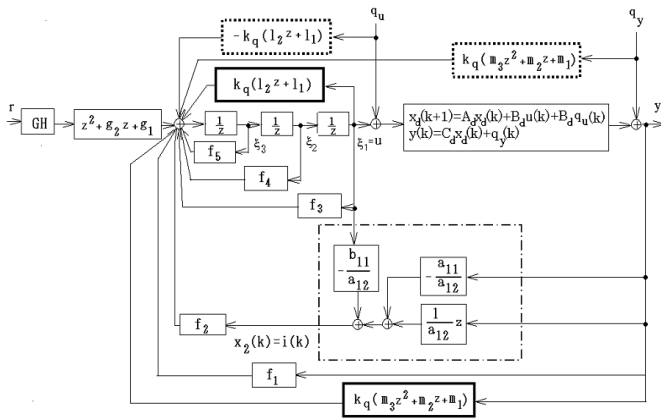


Fig. 2: Feedforward from equivalent disturbances q_u and q_y , and model matching with state feedback

That is, the parts surrounded by the dotted lines are replaced by the parts surrounded by solid lines from u and y . The system expect for the parts surrounded by the dotted lines in Fig.2 can be transformed equivalently as shown in Fig.3. In Fig.3,

$$\begin{aligned}
 ff_1 &= f_1 - f_2(a_{11}/a_{12}) + k_q m_1 + f_5(k_q m_2 + f_2/a_{12}) \\
 &\quad + f_5^2 k_q m_3 + (f_4 + k_q l_2) k_q m_3 \\
 ff_2 &= k_q m_2 + f_2/a_{12} + f_5 k_q m_3 \\
 ff_3 &= f_3 - f_2(b_{11}/a_{12}) + k_q l_1 \\
 ff_4 &= f_4 + k_q l_2 \\
 ff_5 &= f_5 \\
 ff_6 &= k_q m_3 \\
 gg_1 &= f_5^2 + f_5 g_2 + k_q l_2 + f_4 + g_1 \\
 gg_2 &= f_5 + g_2
 \end{aligned} \tag{11}$$

The transfer function between r and y , q_u and y , and q_y and

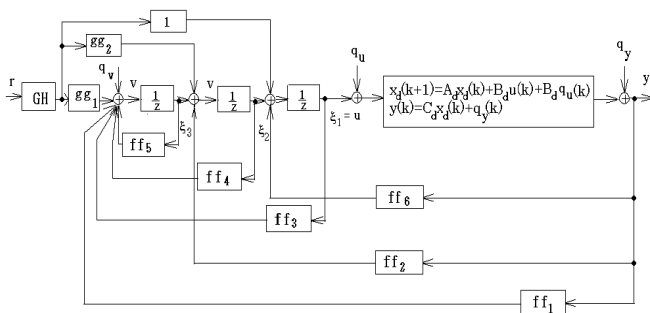


Fig. 3: Model matching system using only voltage (output) feedback

y in Fig.3 are described as

$$W_{ry}(z) = N_{ry}(z)/D(z) \tag{12}$$

$$W_{quy}(z) = N_{quy}(z)/D(z) \tag{13}$$

$$W_{qyy}(z) = N_{qyy}(z)/D(z) \tag{14}$$

where

$$N_{ry}(z) = GH(z^2 + g_1 z + g_0)(b_{11} z + b_{21} a_{12} - a_{22} b_{11})$$

$$N_{quy}(z) = N_{qz} N_p$$

$$N_{qyy}(z) = N_{qz} D_p$$

$$\begin{aligned}
 N_{qz} &= (a_{12} z^3 - a_{12} f_5 z^2 + (a_{12} b_{11} k_q - a_{12} f_4) z \\
 &\quad - f_3 a_{12} + f_2 b_{11} - a_{12} a_{22} b_{11} k_q + a_{12}^2 b_{21} k_q)
 \end{aligned}$$

$$\begin{aligned}
 D(z) &= z^5 + (-f_5 - a_{22} - a_{11}) z^4 + (a_{11} f_5 \\
 &\quad + a_{11} a_{22} - a_{21} a_{12} + a_{22} f_5 - f_4) z^3 \\
 &\quad + (a_{21} a_{12} f_5 - f_3 + a_{11} f_4 - a_{11} a_{22} f_5 \\
 &\quad + a_{22}^2 2 f_4) z^2 + (a_{22} f_3 + a_{21} a_{12} f_4 + a_{11} f_3 \\
 &\quad - b_{11} f_1 - f_2 b_{21} - a_{11} a_{22} f_4) z + f_2 a_{11} b_{21} \\
 &\quad - a_{21} f_2 b_{11} + a_{21} a_{12} f_3 + f_1 a_{22} b_{11} \\
 &\quad - f_1 a_{12} b_{21} - a_{11} a_{22} f_3
 \end{aligned}$$

From $D(z)$, the poles of the overall system can be arranged arbitrarily by f_1, f_2, f_3, f_4 and f_5 . From $N_{ry}(z)$, two zeros of r - y can be arranged arbitrarily by g_0 and g_1 . Moreover, from common N_{qz} in $N_{quy}(s)$ and $N_{qyy}(s)$, one zeros of $q_u - y$ and $q_y - y$ can be arranged arbitrarily at the same place by k_q . That is, one zeros can be added arbitrarily to W_q .

2.3 Design of the model matching system

We consider transfer function between the reference input r and the output y specified as below.

$$W_{ry}(z) = \frac{(1 + H_1)(1 + H_2)(1 + H_3)(z - n_1)(z + n_2)(z + n_4)(z + n_5)}{(1 - n_1)(1 - n_2)(1 + H_1)(z + H_2)(z + H_3)(z + H_4)(z + H_5)} \tag{15}$$

This transfer function is realized by the model matching system shown in Fig.5. The robust system is constructed as shown in Fig.4. The transfer function r to y and q of Fig.5 are as follows

$$y \approx \frac{1+H_2}{z+H_2} \quad y \approx \frac{(z-1)^2}{z-1+k_z} W_{Qy} Q \tag{16}$$

From eqs. (16) it turn out that the characteristics from r to y can be specified with H_2 , and the characteristics from q_u and q_y to y can be independently specified with k_z . That is, the system in Fig.4 is an approximate 2DOF, and its sensitivity against disturbances becomes lower with the increase of k_z . If an equivalent conversion of the controller in Fig.4 is carried out, the approximate 2DOF digital integral-type control system will be obtained as shown in Fig.5.

3 SIMULATION AND EXPERIMENTAL RESULTS

The sampling period T are set $10[\mu s]$. The nominal value of R_L is $0.33[\Omega]$. We design a control system so that H_1 ,

