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Dynamics of associative memory network with self-oscillatory and non-self-oscillatory oscillators

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Abstract: We investigate an associative memory model consisting of both self-oscillatory and non-self-oscillatory oscillators that store temporal patterns on relative phase differences between the oscillators. We numerically simulate this model and show that the speed of memory retrieval is enhanced with increase in the proportion of number of non-self-oscillatory elements. These results imply, from a viewpoint of neuroscience, that the presence of resting or down state of neurons facilitates an ability of memory retrieval.

Keywords: associative memory, aging transition, coupled oscillators, up-down state transition.

1 INTRODUCTION

There have been many associative memory models composed of various elements that store various types of patterns. Aoyagi proposed an associative memory network, each element of which is an oscillator, and represents patterns in their relative phase difference [1]. In this model, the Stuart-Landau equation is used as the model of oscillator. The oscillators are connected with Hopfield-like weight matrix [2].

Dynamics of coupled oscillators are well studied independently of the associative memory models. Daido and Nakanishi studied globally coupled oscillators which are composed of two types of oscillators: namely, active and inactive oscillators [3, 4]. The active oscillator shows oscillation without an external driving force (self oscillatory) and its underlying mechanism is characterized by a stable limit cycle. The inactive oscillator requires external driving forces to oscillate (non-self oscillatory) and is characterized by a stable equilibrium point. If the proportion of number of inactive oscillators in the coupled oscillator system is sufficiently small, all the oscillators show synchronous oscillation; the active oscillators oscillate around the limit cycle and the inactive ones oscillate around the stable equilibrium with small amplitude. If the proportion of number of inactive oscillators becomes large, on the other hand, the system shows a phase transition and stops the oscillation. This phase transition is called an aging transition [3, 4].

In the present study, we investigate an associative network model which stores phase information and is composed of both active and inactive oscillators. We first model the associative network model. Then, we analyze the dynamics of the system while the system retrieves stored pattern, i.e., how the increase in the proportion of number of inactive oscillators contributes to the initial quickness of retrieving the stored patterns. Finally, we discuss the underlying mechanism of this initial quick memory retrieval and an interpretation of the model from a viewpoint of neuroscience.

2 MODEL

Our model is based on the associative memory model proposed by Aoyagi [1], which is a coupled oscillator system that stores phase patterns in the coupling connections.

The coupled oscillator system is described by

$$\frac{\mathrm{d}z_i}{\mathrm{d}t} = f_i(z_i) + K(\sum_{j=1}^{N} C_{ij} z_j - z_i), \tag{1}$$

where z_i is the complex state variable of the ith element. The interactions between the elements are represented by the complex weight matrix C_{ij} and the coupling strength $K \geq 0$. As for the oscillation dynamics f_i, we follow Aoyagi [1] and use the Stuart-Landau equation as follows:

$$f_i(z_i) = (\alpha_i + \mathrm{i}\Omega - |z_i^2|)z_i, \tag{2}$$

where the parameter α_i determines the dynamical characteristics of the ith element; the ith oscillator is active if $\alpha_i > 0$ and inactive if $\alpha_i < 0$.

This coupled oscillator system is an associative memory model that stores relative phase differences between the elements. The μth phase pattern is represented by a complex vector $Z^\mu = (Z_1^\mu, \ldots, Z_N^\mu)$, where each element Z_i^μ with $|Z_i^\mu| = 1$ represents the phase of the ith element in the pattern. Note that multiplying by $e^{\mathrm{i}\theta}$ does not alter the embedded pattern, since only the phase differences are stored. Here, we assume the phase patterns $Z^1, \ldots, Z^P$ are orthogonal. The complex weight matrix C is determined from the

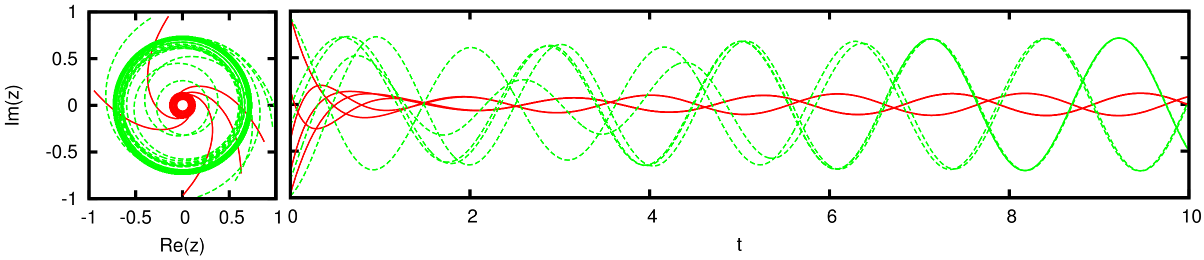

Fig. 1. Dynamics of the coupled oscillator system with $p = 0.6$. The trajectries of z_i in the complex plane (left) and the time evolutions of the imaginary part $\mathrm{Im} z_i$ (right) of five active oscillators (dashed lines) and five inactive oscillators (solid lines) are shown.

phase patterns $Z^1, \ldots, Z^P$ as follows:

$$C_{ij} = \frac{1}{N} \sum_{\mu} Z_i^\mu \cdot \overline{Z_j^\mu}. \tag{3}$$

Using the model whose all the elements are identical ($\alpha_i = 1$), Aoyagi [1] showed that the coupled oscillator system recalls the embedded phase patterns.

In this paper, we consider a network consisting of both active ($\alpha_i = 1$) and inactive ($\alpha_i = -2$) oscillators. This kind of network with active and inactive oscillators has been investigated by Daido and Nakanishi [3, 4], though their model is not for associative memory. Specifically, they studied the system with uniform connection weights $C_{ij} = 1/N$ as follows:

$$\frac{\mathrm{d}z_i}{\mathrm{d}t} = f_i(z_i) + K\left(\frac{1}{N}\sum_{j=1}^{N} z_j - z_i\right). \tag{4}$$

If the proportion of inactive oscillator p is small, inactive elements oscillate by effect of oscillation of active elements. If p is increased, the oscillation stops at the critical value $p = p_c$, which is given by

$$p_c = \frac{K+2}{3K}. \tag{5}$$

This phase transition is called an aging transition.

Retrieval of the μth pattern embedded in the system is evaluated by the overlap M_μ with pattern Z^μ defined by

$$M_\mu = \frac{1}{N}\left|\sum_i \overline{Z_i^\mu} \cdot \frac{z_i}{|z_i|}\right|. \tag{6}$$

This value shows a concordance rate between the state of the system and the stored pattern Z_μ. The larger M is, the more accurately the pattern is retrieved.

3 SIMULATION RESULTS

We performed numerical simulations of the associative memory model composed of $N = 100$ elements with $\Omega = 3$. For simplicity, the number $P = 3$ of orthogonal patterns are chosen from $\{\pm 1\}^N$ and embedded in the weight matrix C_{ij}. The coupling strength is set to $K = 1$.

From N elements, pN elements are randomly chosen as inactive elements, and the rest $(1-p)N$ elements are set as active elements.

We set initial states close to a certain embedded pattern $Z^\mu \in \{\pm 1\}^N$. Specifically, we obtained initial phases from the chosen pattern by adding perturbations $\Delta\theta$ that follow the von Mises distribution, whose probability density function is given by

$$\mathrm{P}(\Delta\theta|\kappa) = \frac{1}{2\pi \mathrm{I}_0(\kappa)} \exp(\kappa \cos \Delta\theta), \tag{7}$$

with the measure of concentration $\kappa = 1$

Figure 1 shows the dynamics of the system with $p = 0.6$ in the simulation. Active oscillators and inactive oscillators show oscillations with large and small amplitude, respectively. Inactive oscillators converge to the corresponding limit cycle more quickly than active oscillators.

In the following, we calculate the overlap M_μ with the chosen pattern Z^μ averaged over 3000 realizations.

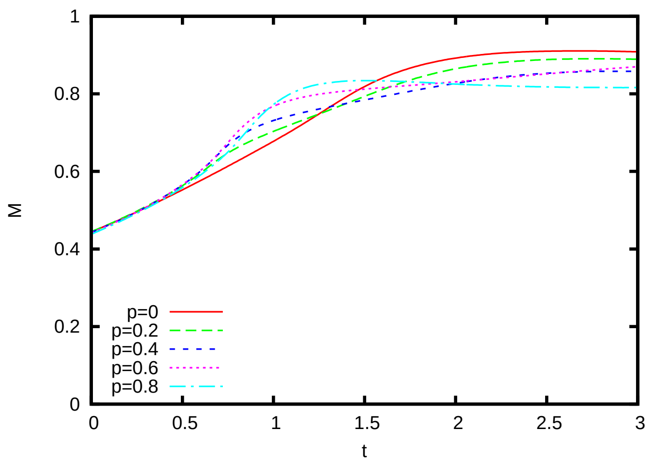

Fig. 2. The overlap M as a function of time t.

Figure 2 shows the overlap M as functions of time t for each value of p. We find that M is an increasing function in most of the time region. While the value of M for $p = 0$

is the smallest at time $t = 1$, it becomes the largest at time $t = 2$.

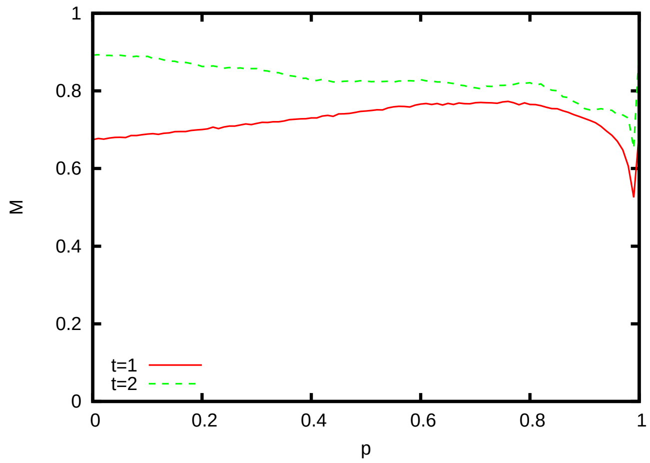

Fig. 3. The overlap M as a function of p at $t = 1$ and 2. In the range $0 \leq p \leq 0.8$, it is almost increasing for $t = 1$ (solid line) and almost decreasing for $t = 2$ (dashed line).

The dependence of the overlap on p is shown in Fig. 3. The solid and dashed lines correspond to $t = 1$ and $t = 2$, respectively. At $t = 1$, M is an almost increasing function in the range between $p = 0$ and $p = 0.8$. At $t = 2$, M is an almost decreasing function in the range.

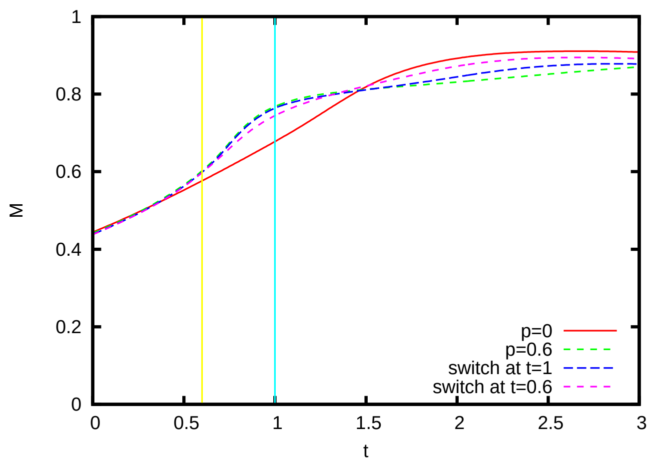

Fig. 4. The overlap M as a function of time t. The value of p is changed from 0.6 to 0 at time $t = 0.6$ and $t = 1$. The cases of $p = 0$ and $p = 0.6$ without the change are also shown.

These result show that small p enhances speed of retrieving and decrease the value M after certain period of time. We investigate the case in which the value of p is changed at a certain time. Figure 4 shows the overlap M as functions of t when the value of p changes from 0.6 to 0 at $t = 0.6, 1$. The value of M at time $t = 2$ in the cases with the change are larger than the case of $p = 0.6$ without the change. While the value of M at $t = 1$ in the case with the change at $t = 0.6$ is smaller than the case without the change, it becomes larger at $t = 2$.

4 DISCUSSION

In the previous section, we observed that the system with inactive oscillators shows quicker initial responses than without inactive oscillators. This can be intuitively explained as follows. Let r_i and θ_i be the amplitude and the phase of the state z_i of the ith element, respectively; i.e., $z_i = r_i \exp \mathrm{i}\theta_i$. Then the dynamics of θ_i is given by

$$\frac{\mathrm{d}\theta_i}{\mathrm{d}t} = \Omega + K\frac{r_i^\star}{r_i}\sin(\theta_i^\star - \theta_i), \tag{8}$$

where $r_i^\star$ and $\theta_i^\star$ are the amplitude and the phase of the input to the ith element through C_{ij}, respectively, as follows:

$$\sum_j C_{ij} z_j = r_i^\star \exp \mathrm{i}\theta_i^\star. \tag{9}$$

The second term of Eq. (8) describes the effect that attracts θ_i to $\theta_i^\star$. Thus its strength is inversely proportional to r_i. Since the amplitude r_i is large for active elements and small for inactive elements, the phases of inactive elements move more quickly to an embedded pattern than active elements.

In the context of neuroscience, temporal aspects of neural activities have been considered to be important [6]. In a neural network, spikes (action potentials) are responsible for the mutual interaction and communication between neurons. Multi-channel recording of neural activities often shows a repetition of certain temporal patterns of spike train, which may be caused by interactions among neurons. Such temporal aspects of neural activities are often modeled with coupled oscillators as shown in the present paper. In Aoyagi's model, temporal patterns are represented as the phase differences among oscillators and embedded in the complex weight matrix.

Another intriguing observation of neural activities is about the two-state dynamics [7], which is a characteristic switching of the membrane potential between two preferred levels, namely the more polarized level (down state) and the more depolarized level (up state). Similarly, the transition between the resting state and the sustained oscillatory state is observed and is modeled as a bistable system with a stable equilibrium and a stable limit cycle [8]. Such a model is closely related to Daido's model in the sense that active and inactive element coexist.

In the present study, to elucidate the contribution of coexistence of active and inactive elements in the associative network, we consider non-uniformity of the elements instead of the bistability. We found that the proportion of inactive oscillator is important for speed of retrieving a temporal pattern. This indicates that the increase in proportion of neurons in resting state facilitates retrieval process of memories. Although the proportion of inactive oscillators is fixed or time-invariant in the present network model, real neurons dynamically switch their state between up and down state. Contribution of this dynamical aspect of switching to the ability of memory retrieval should be evaluated in future.

5 CONCLUSION

We investigated the dynamics of pattern embedded oscillatory networks which include non-self-oscillatory elements. The property of non-self-oscillatory element determines the speed of retrieving stored patterns. This mechanism may play some functional role in neural networks of the brain.

ACKNOWLEDGEMENT

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