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A network consisting of phase adjusting units

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Abstract: We propose a one-step prediction method by using a simple network consisting phase adjusting units. The network is for learning a continuous time series. Each unit have a function of amplitude and phase adjusting. We tried to predict a point for nonlinear time series like as a logistic map by using the lasso algorithm.

Keywords: Network, Phase, Chaos, time series

1 INTRODUCTION

It is important to make a mathematical model for an observed time series which is complicated such as a deterministic chaos. Although some methods are proposed[1, 2, 3], it is still difficult problem to make a mathematical model for the deterministic chaos yet. As a preliminary step, we propose a method to predict the next point of a given time series by using the lasso algorithm[4]. Our method will be very fast for one-step prediction of complicated time series.

2 PROPOSAL METHOD

First, we prepare k units each of which outputs d dimensional polynomial equation $f(x) = \sum_{j=0}^{d} a_j x^j$. The given parameter k corresponds to the dimension of delay coordinates. This scheme can express many types of complicated time series including chaos.

The dimension of given time series is usually estimated by false nearest neighbor method. In future, we will consider about some types of networks consisted of these units. However in this report, we put the units linearly as shown in Fig. 1.

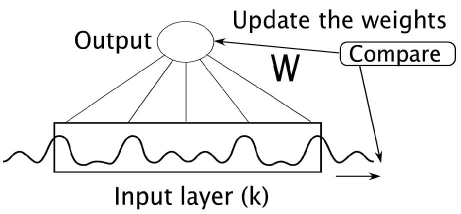

Fig. 1: Schematic diagram of inputs and output.

Next, we minimize the following error value E in Eq. (1) for an applicable prediction. We consider a decaying weight for using late x values.

$$E = \mathrm{E}_t \left(x_{t+1} - \sum_{i=0}^{k} \sum_{j=0}^{d} a_{ij} {x_{t-i}}^j \right)^2 + \lambda \sum_{i=0}^{k} \sum_{j=0}^{d} w_{ij} |a_{ij}|, \quad (1)$$

where x_i is i th data point of given time series, λ is the regularization parameter for the lasso, the weights w_{ij} means $\exp(\gamma_i)$ representing decaying weight, and γ is given value from 1.01 to 1.1. By using the lasso, we expect to get a sparse representation of the polynomial equation which is useful for checking a behavior of the dynamics.

Actually, we split the given time series by k th, and pack up into the following matrix X in Eq. (2).

$$\begin{cases} X = \begin{pmatrix} x_1, ..., x_k, x_1^2, ..., x_k^2, x_1^d, ..., x_k^d \\ x_2, ..., x_{k+1}, x_2^2, ..., x_{k+1}^2, x_2^d, ..., x_{k+1}^d \\ \vdots \\ x_{z-k}, ..., x_{z-1}, x_{z-k}^2, ..., x_{z-1}^2, x_{z-k}^d, ..., x_{z-1}^d \end{pmatrix} \\ Y = (x_{k+1} \cdots x_z)^T \end{cases} \quad (2)$$

We obtain the parameter sets satisfied the map $X \mapsto Y$ by using the lasso.

3 RESULTS

We used a logistic map which is generated a following equation (3) as a tested time series.

$$x_{n+1} = a x_n (1 - x_n), \quad (3)$$

where a characterized parameter a is fixed to 3.8, an initial value x_0 is set to 0.1, and the data size is 5000 points. The dimension of delay coordinates k is assumed to 2. The polynomial dimension of a model d is given to 2. We use the glmnet package[5] under the R system. A parameter in this package s corresponding to λ is set to 0.001. The results will be diverged when the parameter s is very small.

As the results, we obtained an estimated polynomial equation shown in Eq. (4). We completed the calculation within 1 sec.

$$x_{n+1} = 3.4911(x_n)^2 - 0.0160(x_{n-1}) - 3.5508(x_{n-1})^2 + 0.0881 \quad (4)$$

The convergence in a number of parameters and mean squared errors are shown in Figs. 2 and 3. We confirmed that the regularization parameter λ is an effective of the sparse representation. The results of one step prediction using obtained parameters are shown in Fig. 4. We confirmed that our method is effective for one step prediction of complicated time series. However, our method is inability for free-run with parameters fixed. The difference between given time series and obtained model are shown in Fig. 5.

We also checked the effects with an assumed dimension d and an assumed delay dimension k. The results in case of the dimension d assumed 3, are shown in Figs. 6, 7, and 8. The results in case of the delay k assumed 3, are shown in Figs. 9, 10, and 11.

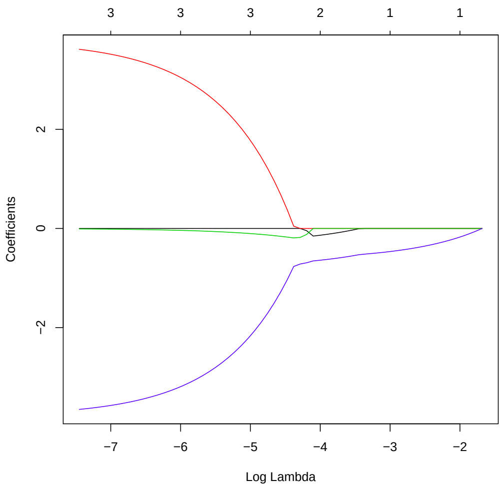

Fig. 2: The convergence in a number of parameters with lasso strength λ.

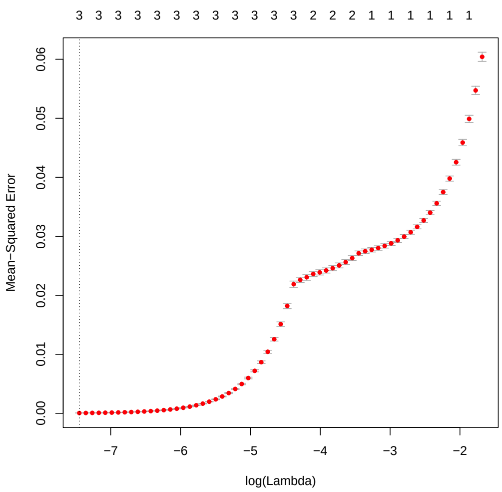

Fig. 3: MSE with the λ.

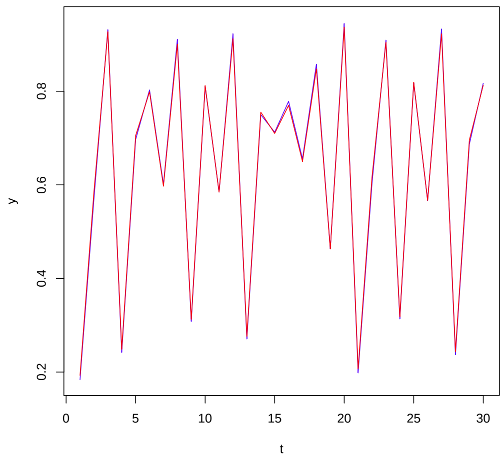

Fig. 4: Results of one step prediction.

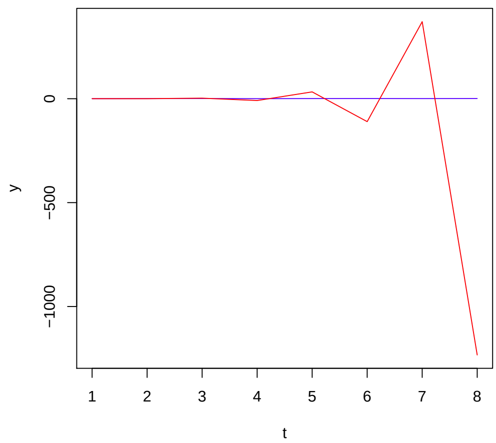

Fig. 5: Results of free run.

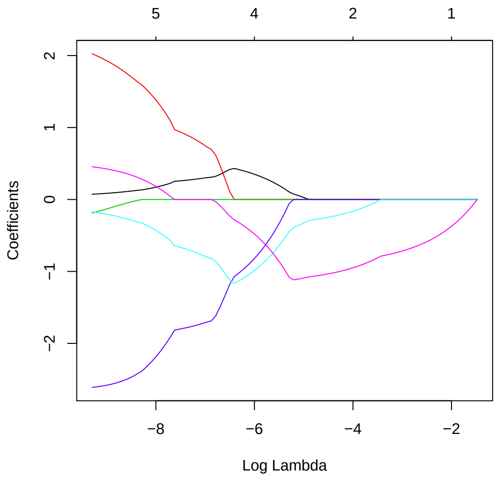

Fig. 6: The convergence in a number of parameters with the λ in case of $d = 3$.

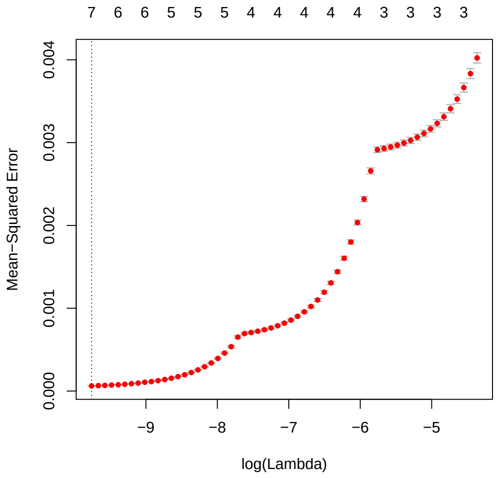

Fig. 7: MSE with the λ in case of $d = 3$.

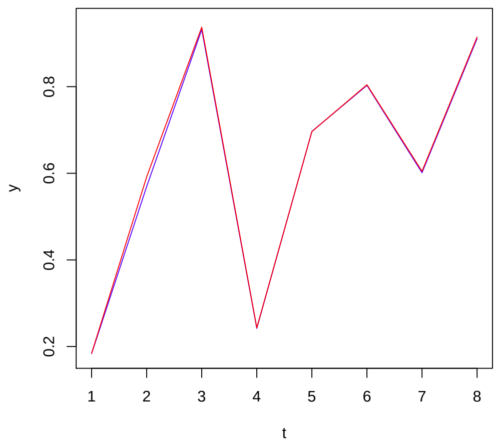

Fig. 8: Results of one step prediction in case of $d = 3$.

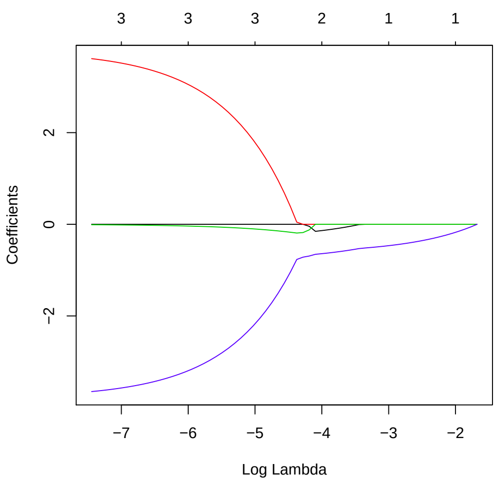

Fig. 9: The convergence in a number of parameters with the λ in case of $k = 3$.

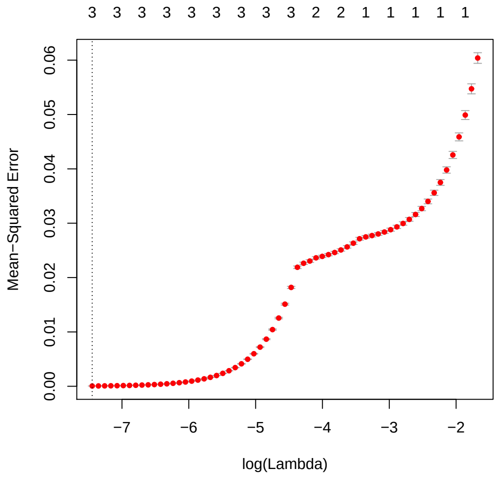

Fig. 10: MSE with the λ in case of $k = 3$.

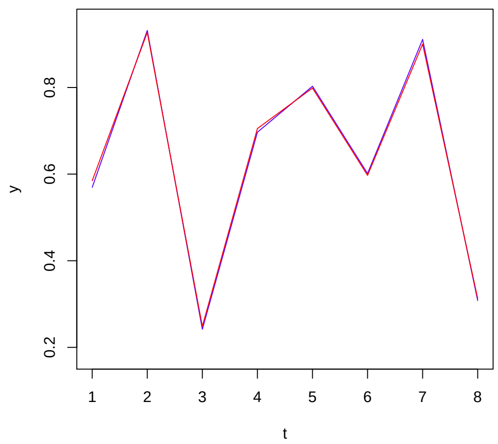

Fig. 11: Results of one step prediction in case of $k = 3$.

4 CONCLUSION

We proposed a one-step prediction method for chaotic time series using the lasso. As the results, we got good predictions for the logistic map. In future we will try to make a mathematical model which is capable of free run, based on this method.

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