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# Coherence patterns in neural fields at criticality

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**Abstract:** Phase synchronization is a mechanism that plays a crucial role for information processing in the brain, and coherence is one of methods that are used to evaluate pairwise degree of phase synchronization. Coherence is also an important measure for examining brain functions because it can indicate communication and cooperation among neurons. In this work, we study the coherence patterns of spontaneous activity in the neural field model at criticality, which is a region where a second order phase transition occurs. The results are summarized as follows. First, at high frequency bands, the system outside the critical regime cannot communicate via phase synchronization at all. Second, the dynamical coherence patterns in the critical regime show switching between high and low coherent states. Finally, we found that in a very brief period of time, there is the high broadband coherence between some pairs of spatial points. This phenomenon can be observed only in the critical regime.

**Keywords:** criticality, neural fields, coherence, synchronization, brain spontaneous activity

## 1 INTRODUCTION

Critical phenomena occur at a second order or continuous phase transition (a critical point). This point is at the boundary between a stable and ordered state and a disordered state. Critical phenomena can be found frequently in various natural systems, such as earthquakes and avalanches. This kind of systems is characterized by scale invariance, which is a power-law distribution of some variables, and by the divergence of both spatial and temporal correlation scales.

Recently, there is a proposal by many authors suggesting that neuronal networks probably operate on the critical regime [1]. Some researches also show that at the critical point, the dynamic range [2], memory capacity [3], and computational power [4] are optimal in neural network simulations. The critical neural network also provides important factors for learning, such as flexible response and adaptation to input [5]. Furthermore, there are some evidences from experimental data pointing that criticality might underlie neural network behaviour, such as neuronal avalanches *in vitro* [6], $1/f$ type power spectrum of local field potential in visual neocortex [7] implying long-range correlations, and $1/f^{\beta}$ power spectrum of electroencephalograms (EEG) [8] and functional magnetic resonance imaging (fMRI) [9]. In addition, functional brain networks exhibit a power-law degree distribution [10].

Even when someone receives no explicit sensory stimulus and does not perform any task, which is called a resting state, his brain is still active. This activity is brain spontaneous activity or so-called ongoing activity. In the most brain experiments, spontaneous activity is filtered or averaged out and treated as a baseline activity or noise because experimenters are interested only activity induced by performing tasks or given stimuli. However, it was found that this spontaneous activity is not random noise but specifically organized [11]. Until now, the function which spontaneous activity in the brain serves for is still unclear. After all, the importance of spontaneous activity is stressed by the fact that most of the energy consumed by the brain is accounted for this activity, and task-induced activity increases only a small portion ($< 5\%$) of brain energy consumption [11].

Phase synchronization plays a crucial role for brain information processing. It is one of the methods the brain uses to code information and to allow distributed neural populations to communicate. Phase synchronization occurs not only when external stimuli are presented but can also occur in spontaneous activity of the brain during the resting state [12]. Coherence is one of methods that are used to evaluate pairwise degree of phase synchronization and is an important measure for examining brain functions and structures. It can indicate communication and cooperation between neurons.

Here, we use the neural field model tuned to the critical region and regions outside to simulate the brain spontaneous activity and studied some dynamical aspects of coherence by adopting the analysis using a moving time window. The results from the field at criticality are compared with those from the fields far from critical point.

## 2 METHODS

### 2.1 Neural field model

Neural fields are the mesoscopic models used to describe spatio-temporal dynamics of neural activity in the brain at a tissue level, where there are many neural populations interacting with each other. Because a brain cortex tissue is composed of a large number of neurons, we can take the continuum limit and characterize activity of neural populations as a field [13]. Here, we use this model to describe activity in the brain's cortex tissue.

In neural field models, the dynamics of each point in a field depends on its internal dynamics, activity of other points, and an external stimulus. A common form of neural field equation [14] is the integro-differential equation

$$\frac{1}{\ }\frac{\partial u(x,t)}{\partial t} = -u(x,t) + \int_{-\infty}^{\infty} w(x-y) f(u(y,t)) dy, \quad (1)$$

where $u(x,t)$ is the neural activity at point $x$ and time $t$. It corresponds to the local potential in this work.     is the decay rate of activity. $w(x)$ is the weight function, which describes the strength of connections between points in the field. $f(x)$ is the firing rate function, which determines firing rate output transmitted to postsynaptic neurons. The first term in the right-hand side of equation (1) represents neurons' internal dynamics, while the second term is a convolution between $w(x)$ and $f(x)$ representing influence of output from other neurons weighted by synaptic efficacy between points.

There are many choices of proper $w(x)$ and $f(x)$. In this research, we choose

$$w(x) = (2 - |x|)\frac{e^{-|x|}}{2}. \quad (2)$$

It is called a Mexican hat function because its shape looks like a Mexican hat. The lateral-inhibition connectivity used by Amari [14] is the function of this type, too. The shape of this weight function displays local (short-range) excitation and distant (long-range) inhibition, which are typical cortical connections. It is also assumed that this neural field is homogeneous and isotropic, i.e., the weight function is identical for all neurons and depends only on distance between interacting neurons. For the firing rate function $f(x)$, we use a half sigmoidal function,

$$f(x) = \frac{2}{1 + e^{-ax\Theta(x)}} - 1, \quad (3)$$

where $\Theta(x)$ is the Heaviside step function. The negative input cannot send out the output at all. Here, $a$ is a parameter controlling steepness of the function when $x > 0$.

The real brain tissue is noisy and also has spontaneous activity, so we add a noise term $\xi$ and small external field $h$ representing spontaneous activity to the equation (1). Now, we have a neural field model of the form

$$\frac{1}{\ }\frac{\partial u(x,t)}{\partial t} = -u(x,t) + \int_{-\infty}^{\infty} w(x-y) f(u(y,t)) dy + h(x,t) + \xi(x,t). \quad (4)$$

$\xi$ is a multiplicative white noise with $\langle \xi(x,t)\xi(x',t')\rangle = \ u(x,t)\delta(x-x')\delta(t-t')$, and     is noise intensity. For the spontaneous activity, we use a constant $h(x,t) = h$.

## 2.2 Critical point

Our neural field model (equation (1), (2), and (3)) can feature second order phase transition and a critical point. The steepness variable $a$ in equation (3) works as a control parameter. To identify a critical point, we want to know at which value of $a$, a uniform stationary solution $u(x) = 0$ for all $x$ begins to be unstable, or the order parameter $\langle u \rangle = \frac{1}{2L}\int_{-L}^{L} u(x)dx$, where $L$ is a half size of the neural field, becomes nonzero.

Then, we did simulations of one dimensional neural field and consider the model's bifurcation graph between the order parameter $\langle u \rangle$ versus the control parameter $a$ (Fig. 1.). According to the diagram, we can see that many possible neural activity patterns can emerge. However, the point that a non-uniform stationary solution begins to appear is $a_c \approx 1.782$, which is the position of a critical point. From now, we will call the left side of the critical point ($a < a_c$) a subcritical region and the right side of the critical point ($a > a_c$) a supercritical region, while a critical region is placed in the vicinity of $a_c$.

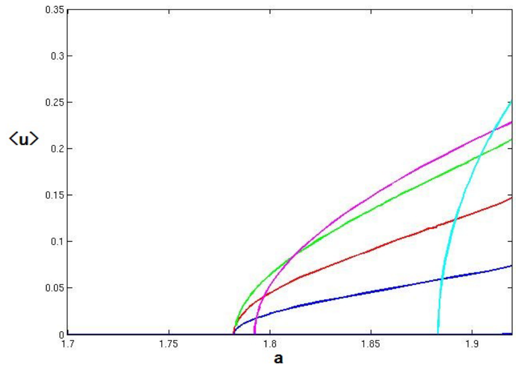


Fig. 1. A bifurcation diagram of the neural field model. A blue, red, green, pink, and light blue lines represent a pattern of 1, 2, 3, 4, and no bump, respectively.

In our simulation, we used $a = 1.782$, $1.982$, and $1.582$ to represent critical, supercritical, and subcritical region, respectively. We used $L = 20$ and applied periodic boundary condition to the field. Other parameters were set as follows: decay rate,     $= 0.2$; noise intensity,     $= 1$; and spontaneous activity, $h = 0.00001$.

## 2.3 Dynamical coherence

The coherence or magnitude-squared coherence is used in spectral analysis for measuring of the phase consistency and one of methods to determine a degree of phase synchronization between a pair of signals. It does not only determine the dependency between simultaneous values of two time series but also considers leading and lagging relationships. The coherence between signals $x(t)$ and $y(t)$ at given frequency $f$

 

is defined by

$$C_{xy}(f) = \frac{|P_{xy}(f)|^2}{P_{xx}(f)P_{yy}(f)}, \tag{5}$$

where $P_{xy}$ is the cross power spectral density (CPSD) between $x(t)$ and $y(t)$, which is the Fourier transform of cross correlation, and $P_{xx}$ and $P_{yy}$ are power spectral densities of $x(t)$ and $y(t)$, which are the Fourier transform of autocorrelation. Also, note that the coherence is a function of frequency $f$, and signals are regarded as the superposition of each frequency component.

The coherence value ranges from 0 to 1. The coherence value of 1 or perfectly coherent signals indicate that the phase difference between two signals is fixed with time, while the value of 0 means that the two signals are completely unrelated. However, noise in the systems can contribute to the coherence greater than zero.

We were interested in dynamical coherence patterns, which mean that coherence values can change in time. We begin with defining the time window $\Delta t$ as a length of time series at each time step (here, $\Delta t = 2000$). Then, move this frame forward by 1 step in each time step until it reaches the terminal time point. Consequently, dynamical time window is constructed, and we obtain a set of time series at each time step in which we can measure dynamical quantities of our interest, coherence in this case.

## 3 RESULTS

The spatial averages of dynamical coherence between the point $x = 0$ and all other points in the field for four frequency bands ($f_0$, $2f_0$, $3f_0$, and $4f_0$) from one sample of simulations are shown in Fig. 2 (Here, $f_0 = 5$ Hz, and the sampling rate determined from the time constant (decay rate) is 2000 points). The dynamical coherence of random noise is also shown for being a reference of insignificant coherence level. It is apparent that the highest coherence appears in the critical region at all frequency bands. In the other regions, the coherence is very low, particularly in higher frequency bands. Actually, at higher frequencies ($\geq 2f_0$), only coherence values in the critical region are significantly higher than the coherence level of noise, though, sometimes the coherence level in supercritical region can be little higher than noise's but still lower than the critical region's. However, there are some periods of time where the coherence in the critical region drops to the same level with or lower than noise's.

Another interesting phenomenon observed only in the critical region is broadband coherence. For the coherence of some pairs of spatial points, there are some short periods of time where high coherence appears in many frequency bands. For example, Fig. 3. shows dynamical coherence in all frequencies between the point $x = 0$ and $x = -20$ at the critical region in the same simulation of Fig. 2.

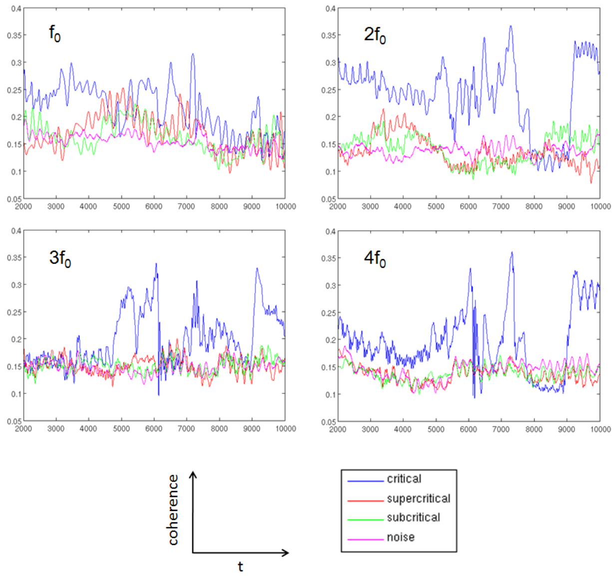


Fig. 2. Spatially average values of dynamical coherence between the point $x = 0$ and the others of the three regions in $f_0$, $2f_0$, $3f_0$, and $4f_0$ frequency bands including that of random noise.

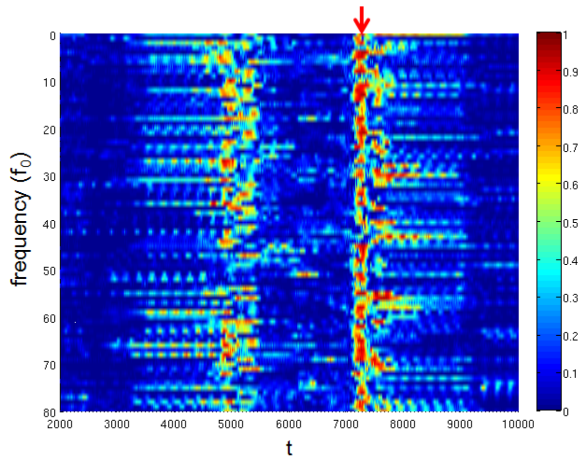


Fig. 3. The pairwise dynamical coherence between points $x = 0$ and $x = -20$. The arrow points the brief periods of high broadband coherence.

## 4 DISCUSSION & CONCLUSION

The dynamical coherence is highest in the critical region. Actually, at frequencies higher than $2f_0$, only coherence in the critical region is significantly higher than coherence of noise. This result shows that outside the critical region, neurons in the field communicate to each other by high frequency bands much less efficiently than those at criticality. Also, it is at criticality that neurons can communicate most efficiently by any frequency, although there are some periods that the coherence level drops to a value of a non-coherent state. The rise and drop of coherence can be regarded as coherence switching. Archerman and Borbély demonstrated the similar switching in human EEG during NREM-REM (non-rapid eye movement and rapid eye movement) sleep cycles [15].

Furthermore, we found that only at criticality, there are some periods in time that the coherence of some pairs become broadband, i.e., coherence occurs in almost all frequencies at the same time. One reason for this phenomenon is that power spectral density and squared CPSD's cross-frequency correlations are very high in the critical region (the results are not shown). However, its origination and functions still need further investigation. Gervasoni and his colleagues found the high transient broadband coherence occurring at global brain state transitions in rats' LFP (local field potential) [16]. They also suggested that this transient coherence may construct distributed structures of functional connectivity and allow information flow between neurons.

In summary, we used the neural field model to simulate spatio-temporal evolution of spontaneous neural activity in the critical regime, namely, the region around a critical point. Then, dynamical coherence patterns were studied in the critical region compared with regions outside. Our study suggests that communication between neurons in the field is optimized in the critical region. Also, in this region, we observed many phenomena that are consistent with the empirical experiments of the brain.

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