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A Note on Three-Dimensional Probabilistic Finite Automata

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Abstract: We think that recently, due to the advances in many application areas such as computer graphics, computer vision, image processing, robotics, and so on, it is useful for analyzing computation of three-dimensional information processing to explicate the properties of three-dimensional automata. From this point of view, we have investigated many properties of three-dimensional automata and computational complexity. On the other hand, the class of sets accepted by probabilistic machines have been studied extensively. As far as we know, however, there is no results concerned with three-dimensional probabilistic machines. In this paper, we introduce three-dimensional probabilistic finite automata, and investigate some accepting powers of them.

Keywords: Accepting power, Alternation, Chunk, Probabilistic finite automaton, Three-dimensional input tape

1 INTRODUCTION

Computer science is the systematized field of knowledge and technology concerning computation. Its realistic beginnings can be traced back to the formalization of the concept of an effective procedure and the advent of excellent digital computers. In theoretical computer science, the Turing machine has played a number of important roles in understanding and exploiting basic concepts and mechanisms in computing and information processing. It is a simple mathematical model of computers which was introduced by Turing [17] in 1936 to answer fundamental problems of computer — 'What kind of logical work can we effectively perform ?' If the restrictions in its structure and move are placed on the Turing machine, the restricted Turing machine is less powerful than the original one. However, it has become increasingly apparent that the characterization and classification of powers of the restricted Turing machines should be of great important. Such a study was active in 1950's and 1960's. On the other hand, many researchers have been making their effects to investigate another fundamental problems of computer science — 'How complicated is it to perform a given logical work ?' The concept of computational complexity is a formalization of such difficulty of logical works. In the study of computational complexity, the complexity measures are of great importance. In general, it is well known that the computational complexity has originated in a study of considering how the computational powers of various types of automata are characterized by the complexity measures such as space complexity, time complexity, or some other related measures. After that, the growth of the processing of pictorial information by computer was rapid in those days. Therefore, the problem of computational complexity was also arisen in the two-dimensional information processing. Blum and Hewitt first proposed two-dimensional automata — two-dimensional finite automata and marker automata, and investigated their pattern recognition abilities in 1967 [1]. Since then, many researchers in this field have been investigating a lot of properties about automata on a two-dimensional tape [17]. By the way, the question of whether processing three-dimensional digital patterns is much difficult than two-dimensional ones is of great interest from the theoretical and practical standpoints. In recent years, due to the advances in many application areas such as computer graphics, computer-aided design / manufacturing, computer vision, image processing, robotics, and so on, the study of three-dimensional pattern processing has been of crucial importance. Thus, the study three-dimensional automata as the computational model of three-dimensional pattern processing has been meaningful. However, it is conjectured that the three-dimensional pattern processing has its own difficulties not arising in two-dimensional case. One of these difficulties occurs in recognizing topological properties of three-dimensional patterns because the three-dimensional neighborhood is more complicated than two-dimensional case. Generally speaking, a property or relationship is topological only if it is preserved when an arbitrary 'rubber-sheet' distortion is applied to the pictures. For example, adjacency and connectedness are topological; area, elongatedness, convexity, straightness, etc. are not. During the past thirty years, automata on a three-dimensional tape have been proposed and several properties of such automata have been obtained. We have also studied about three-dimensional automata, and introduced many computational models on three-dimensional input tapes. On the other hand, the classes of sets recognized by one- or two-dimensional probabilistic finite automata and probabilistic Turing machines have been studied extensively [2-15, 18-22]. As far as we know, however, there is no results concerning with three-dimensional probabilistic machines. In this paper, we introduce three-dimensional probabilistic finite automata, and investigate some their accepting powers.

2 PRELIMINARIES

Let Σ be a finite set of symbols. A *three-dimensional tape* over Σ is a three-dimensional array of elements of Σ. The set of all three-dimensional tapes over Σ is denoted by $\Sigma^{(3)}$. Given a tape $x \in \Sigma^{(3)}$, for each integer $j (1 \leq j \leq 3)$, we let $l_j(x)$ be the length of x along the jth axis. The set of all $x \in \Sigma^{(3)}$ with $l_1(x)=n_1$, $l_2(x)=n_2$, and $l_3(x)=n_3$ is denoted by $\Sigma^{(n_1,n_2,n_3)}$. When $1 \leq i_j \leq l_j(x)$ for each $j(1 \leq j \leq 3)$, let $x(i_1,i_2,i_3)$ denote the symbol in x with coordinates (i_1,i_2,i_3). Furthermore, we define x [(i_1,i_2,i_3), (i'_1,i'_2,i'_3)], when $1 \leq i_j \leq i'_j \leq l_j(x)$ for each integer $j(1 \leq j \leq 3)$, as the three-dimensional input tape y satisfying the following conditions : (i) for each $j(1 \leq j \leq 3)$, $l_j(y) = i'_j - i_j + 1$; (ii) for each r_1, r_2, r_3, $(1 \leq r_1 \leq l_1(y)$, $1 \leq r_2 \leq l_2(y)$, $1 \leq r_3 \leq l_3(y)$, $y(r_1, r_2, r_3) = x(r_1 + i_1 - 1, r_2 + i_2 - 1, r_3 + i_3 - 1)$.

A *three-dimensional probabilistic finite automata* (denoted by 3-*PFA*) is a 6-tuple $M = (Q, \Sigma, \delta, q_0, q_a, q_r)$, where Q is a finite set of *states*, Σ is a finite set of *input symbols*, δ is a *transition function*, $q_0 \in Q$ is the *initial state*, $q_a \in Q$ is the *accepting state*, and $q_r \in Q$ is the *rejecting state*. An *input tape* for M is a three-dimensional tape over Σ surrounded by the *boundary symbols* $\#$'s (not in Σ). The transition function δ is defined on $(Q - \{q_a, q_r\}) \times (\Sigma \cup \{\#\})$ such that for each $q \in Q - \{q_a, q_r\}$ and each $\sigma \in \Sigma \cup \{\#\}$, $\delta[q, \sigma]$ is a *coin-tossing distribution* on $Q \times$ { East, West, South, North, Up, Down, Stay }, where East means ' moving east ', West ' moving west ', South ' moving south ', North ' moving north ', Up ' moving up ', Down ' moving down ', and Stay ' staying there '. The meaning of δ is that if M is in state q with the input head scanning the symbol σ, then with probability $\delta[q, \sigma]$ (q', d) the machine enters state q' and either moves the input head one symbol in direction d if $d \in$ { East, West, South, North, Up, Down } or does not move the input head if $d =$ Stay. Given an input tape $x \in \Sigma^{(3)}$ M starts in state q_0 with the input head on the upper northwest corner of x. The computation of M either accepts by entering the accepting state q_a or rejects by entering the rejecting state q_r. We assume that δ is denoted so that the input head never falls off an input tape out of the boundary symbols $\#$'s. M halts when it enters state q_a or q_r.

A *three-dimensional alternating finite automaton* (denoted by 3-*AFA*) is an alternating version of a three-dimensional finite automaton. See [1, 6, 7] for the formal definition of 3-*AFA*'s [16, 17].

Let $\mathcal{L}$[3-*PFA*] = $\{T \mid T = T(M)$ for some 3-*PFA* M $\}$. $\mathcal{L}$[3-*AFA*] is defined in the same way as $\mathcal{L}$[3-*PFA*].

3 RESULTS

This section shows that the 3-*PFA* is incomparable with 3-*AFA*. We first give several preliminaries to get our desired results. Let M be a 3-*PFA* and Σ be the input alphabet of M. For each $l, m, n \geq 1$, a three-dimensional type in $\Sigma^{l \times m \times n}$ is called an $(l, m, n) - chunk$ over Σ. For any (l, m, n)-chunk v with $l \geq 1$, $m \geq 1$, and $n \geq 2$, we denote by v($\#$) the pattern obtained from v by attaching the boundary symbols $\#$'s to v. Below, we assume without loss of generality that M enters or exits the pattern v($\#$). Thus, the number of the entrance points to v($\#$) (or the exit points from v($\#$)) for M is $4m + 8$. Let PT(v($\#$)) be the set of these entrance points (or exit points). .

Lemma 3.1. *Let* $L_1 = \{x \in \{0,1\}^{(3)} \mid l_3(x) \geq 2$ $\exists\ k\ (2 \leq k \leq l_3(x))$ $[x[(1,1,1),(l_1(x),l_2(x),1)] = x[(1,1,k),(l_1(x),l_2(x),k)]]$ *(i.e., the top plane of* x *is identical with some another plane of* x)]. *Then,* $L_1 \in$ 3-*AFA* — 3-*PFA*.

Proof: L_1 is accepted by the 3-*AFA* M with acts as follows. Given an input tape x with $l_3(x) \geq 2$, M existentially tries to check that, for each $i, j (1 \leq i \leq l_1(x)$, $1 \leq j \leq l_1(x)$ $x(i, j, k) = x(i, j, 1)$. That is, ont the kth plane of $x(1 \leq i \leq l_1(x)$, $1 \leq j \leq l_2(x)$, $1 \leq k \leq l_3(x)$, M enters a universal statoe to choose one of two further actions. One action is to pick up the symbol $x(i, j, k)$, move up the symbol store in the finite control, compare the stored with the symbol $x(i, j, 1)$, and enter an accepting state if both symbols are identical. The other action is to continue to move next tape cell (in order to pick up the symbol $x(i + 1, j + 1, k)$ and compare it with the symbol $x(i + 1, j + 1, k)$ and compare it with the symbol $x(i + 1, j + 1, 1)$. It will be obvious that M accepts L_1.

We next show that $L_1 \notin$ 3-*PFA*. Suppose to the contrary that there exsts a 3-*PFA* M' recognizing L_1 with error probability $\epsilon < \frac{1}{2}$. For large m, let V(n) be the set of all the (2^n, 2^n, n)-chunks over $\{0, 1\}$. We shall below consider the computations of M' on the input tapes x with $l_1(x) = l_2(x) = 2^n$ and $l_3(x) = n$. Let c be the number of states of M'. Consider the chunk probabilities p(v, σ, τ) defined above. For each (2^n, 2^n, n)-chunk v in V(n), there are a table of

$$d(n) = c \times \mid \mathrm{PT(v(\#))} \mid \times (c \times \mid \mathrm{PT(v(\#))} \mid + 5) = O(n^2)$$

chunk probabilities, where fo any S, $\mid S \mid$ denotes the number of elements of S. Fix some ordering of the pairs (σ, τ) of starting and stopping conditions and let **p**(v) be the vector of these d(n) probabilities according to this ordering. By using the counting argument and reduction to absurdity, we can derive the following lemma. □

Lemma 3.2. *Let* $L_2 = \{x \in \{0,1\}^{(3)} \mid l_3(x) = 1\& \; (x$ *is of the form* $0^n 1^n$ *for some* $n \geq 1) \}$. *Then,* $L_2 \in$ 3-*PFA* — 3-*AFA*.

Proof: It is showed the L_2 is recognized by a two-way probabilistic finite automaton with error probability ϵ for any $\epsilon < \frac{1}{2}$ [1, 3]. On the other hand, it is showed that alternating finite automata accept only regular sets. Thus $L_2 \in \mathcal{L}[$3-*PFA* — 3-*AFA*] by using the same technique. □

From Lemmas 3.1 and 3.2, we have the following theorem.

Theorem 3.1. 3-*PFA is incomparable with* 3-*AFA*.

4 CONCLUSION

It was introduced three-dimensional probabilistic finite automata 3-*PFA*^{c}'s and shown their some properties in this paper. We conclude this paper by giving the following open problems.

(1) Let 3-*PFA*^{c} (resp. 3-*AFA*^{c}) be the class of sets of cubic tapes recognized by 3-*PFA*^{c}'s with error probability less than $\frac{1}{2}$ (resp., accepted by 3-*AFA*^{c}'s). Is 3-*PFA*^{c} incomparable with 3-*AFA*^{c} ?

(2) Let T^c be all the three-dimensional connected tapes. Is Tc recognized by 3-*PFA*^{c}'s ?

(3) It will be interesting to investigate the properties of various three-dimensional probabilistic Turing machines.

(4) It will be also interesting to deal with the closure properties of 3-*PFA*^{c}'s.

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