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Simple Model of Economic Stability and Control

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Abstract: Stability is one of the important issues in the system studies of economic structures, investment's decisions, and government expenditures. We simplify the model of the stability and control of economic systems in this paper. Two important variables, the GDP and investment, are included in economic systems. We build a model to identify the stability in economic systems including these two variables. The stable control shows that the control should have inverse relation to the state of the systems. A system will be exploded if an immense control shock makes it reach an unstable equilibrium state. It indicates that the government investments or expenditures should decrease but not increase when the economic or government expenditure continues increasing.

Keywords: Government Investment or Expenditure, Euler Equation, Approximate Control

I. INTRODUCTION

Stability is very crucial for the system studies, such as economic structures, investment's decisions, and government expenditures.

Only recently has performance, which is based on graph theory, been extended and analyzed to assess structural change of network organizations.

Intuitively speaking, the stability is that the systems will keep some characteristics of its structure when its form or initial value varies in different ways. Hence, it is necessary to control the systems in order to keep it stable. In other words, the stability is to design a control such that a small perturbation does not have a great influence on the system [1].

Many researchers suggest different perturbation techniques in mathematics, physics and engineering. How should we, however, consider the stability to design a control in the economic systems?

Two very important variables, the GDP and investment, are included in the economic systems. Considering a system at any time *t*, its state can be described as a differential equation,

$$\frac{du}{dt} = g(u,v), u(0) = c \qquad (1)$$

where $u(t)$ represents a function for the GDP, du/dt=u', that is the derivative of $u(t)$, can be thought of as the investment, *v* is a control variable, and *c* is the initial state of $u(t)$. Since the function *g* is undetermined, this gives us a few troubles for analytically dealing with the stability and control. Therefore, we may let the function *g* be a simple form, for example, $g(u,v) = v$. Although this simple form can not quite illustrate the complex of the real economic systems, this is enough for our purpose of research

The main contribution of this paper is to suggest that for stability of the above simple form, the control is $u' = -u$, that is, the investment (for example, the government investment to rescue the market) should have inverse relation to the state of the GDP. The second contribution shows that the simple economic structure would become unstable when a big control perturbation (say, the government investment) happened.

The structure of this paper is as follows. In section 2, we build a new model and show the solution of the stability in economic systems. Section 3 analyzes stability of control. In section 4, we make a brief discussion about economic systems including government investment and expenditures. Section 5 contains some concluding remarks.

II. MODEL, SET-UP, SOLUTION and APPROXIMATE CONTROL

1. Model

Assume an economic system is described as equation (1), where we make $du/dt = g(u,v) = v$. We choose control variable $v(t)$ such that the variations of u and u' are very small on average. For instance, we want to determine a $v(t)$ such that

$$\min_{u'}\left\{\int_0^T [u^2+(u')^2]dt\right\} \qquad (2)$$

We may denote the functional $J(u)$ by $J(u) := \int_0^T [u^2+(u')^2]dt$.

2. Set-up

The basic problem is to determine the functional $J(u)$ well-defined. Our set-up here is $u'(t)\in L^2[0,T]$.

To make equation (2) have solution u, it is necessary to satisfy the Euler equation

$$-u''+u=0\,, u(0)=c\,, u'(T)=0 \qquad (3)$$

We see that this variation problem (2) becomes the Euler equation.

3. Solution and approximate control

Solute equation (3) and get the analytical solution

$$u=c\left(\frac{e^{t-T}+e^{-(t-T)}}{e^{-T}+e^{T}}\right)=c\frac{ch(t-T)}{chT} \qquad (4)$$

What we sometimes want to know is the characteristics of the control v (or u') when $T\to\infty$.

By equation (4), we obtain

$$u'=c\frac{sh(t-T)}{shT} \qquad (5)$$

After algebraic calculation, we further have $|u'+ce^{-t}|\le 2|c|e^{-T}$. In equations (4) and (5), we may explain the parameter c as the initial value of the equation system, that is, c is also the constraint or initial endowment for the system. If T is very large, we can get the approximate control $u'=-u$ [2].

III. MSTABILITY of CONTROL

C To illustrate the stability of control, we consider the following equation of pendulum

$$u''+au'+\sin u=g(t) \qquad (6)$$

where $g(t)$ is a control variable. In fact, the behavior of economic is somewhat similar to the motivation of pendulum. We know that $u(t)=0$ is one of its solutions, which correspond to its equilibrium if $g(t)=0$.

Could this system keep stable when a large control perturbation $g(t)$ is imposed on the system such that the system even reaches another equilibrium state, $u(t)=\pi$. After $g(t)$ disappears, then the new equation becomes

$$u''+au'+\sin(\pi+u)=0 \qquad (7)$$

In addition, assume that $u(0)=e_1$ and $u'(0)=e_2$.

$\sin(\pi+u)=-\sin u\approx -u$ when u is small. Therefore, we approximately have

$$u''+au'-u=0,\ u(0)=e_1,\ u'(0)=e_2 \qquad (8)$$

Characteristic roots of this differential equation is

$$\lambda_1,\lambda_2=\frac{-a\pm(a^2+4)^{0.5}}{2}$$

One of characteristic roots is positive and the other negative for whatever value of a. Hence $u(t)$ will explode when $t\to\infty$, that is, the system is unstable at $u(t)=\pi$ [3].

IV. DISCUSSION

In order to get a stable control, Section 2 shows that approximate control, for example, the government investment, should have inverse relation to the state of the GDP. Section 3 suggests that the system likely becomes more volatile if imposing an immense shock on it.

As a national GDP always continue highly increasing, decreasing instead of increasing government investment may stabilize the economic systems. There is surprisingly increasing in Chinese economic system in last decade as we know. 4 trillion Chinese Yuen (?) of economic stimulus plans, however, launched out into business in various government investments from 2008 to 2011. Does this stabilize or rescue the market? The answer seems to be obvious because high inflation, which could destroy the stability of economic systems, had occurred in 2011 in China.

Another example is London riots in 2011. Between 6 and 10 August 2011, many London districts and some other cities and towns in England suffered widespread rioting, looting and arson. Possible causes for the riots of many, however, it is not to be neglected that while a great deal of domestic expenditures had to pay, England government has paid a lot of war bills, such as Afghan, Iraq, and Libya wars. This violates the rule of stable control, which means that the control should have inverse relation to the state of the system, $u'=-u$.

V. CONCLUSION

We simply model the stability and control for economic systems. By Euler equation of variation method, we obtain the analytical solution for this system and the approximate control rule for stability. The stable control shows that the control should have inverse relation to the state of the system. Furthermore, by the analysis of pendulum's motivation, we see that a system will explode after an immense control shock makes the system reach an unstable equilibrium state.

These indicate that the government investments or expenditures should decrease but not increase when the economic or government expenditure continues increasing.

REFERENCES

[1] Brockett R. W., 1966, The status of stability theory for deterministic system, *IEEE Trans. Automatic Control*, *11 (3)*; 596-606
[2] Bellman R., 1967, *Introduction to the mathematical theory of control processes*, New York: Academic Press
[3] Bellman R., 1964, Perturbation techniques in mathematics physics and engineering, Holt, New York: Dover Publications Inc