# The modeling and implementation of tri-rotor flying robot

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**Abstract:** The objective of this study is going to develop a tri-rotor flying robot, which adopts the Y-shaped three-rotor structure. In order to balance the yaw torque produced by the three rotors, it installs the RC servo motor and linkage on the tail axis, so as to improve the angle of the rolling axis of the tail motor. Moreover, through the torque generated by the horizontal component of the lift from the inclined motor on the tail axis, it balances the yaw torque of the three rotors.

The dynamic equations of the tri-rotor flying robot were determined in this paper. The relationship between motor thrust, angular acceleration and voltage input were also studied in this research. In order to study the effect of control parameters on the flight stability completely, this study develops a universal stability experimental platform to help tuning the control parameters safely. Based on this, the tri-rotor flying robot can rapidly change flying gesture and avoid oscillation.

Finally, we made some indoor and outdoor flight tests. From the experimental results, the tri-rotor flying robot can fly and hover stably in the sky.

Keywords: tri-rotor, tricopter, flying robot, dynamic equations, stability experiment, hover

# I. INTRODUCTION

The unmanned aerial vehicles (UAV) are widely applied in aerial photography, marine or air pollution detection, aerial security surveillance and disaster response. The lifting force of a rotorcraft is mainly the air reaction force produced by the rolling of one or multiple rotors. However, when the rotor provides lift, the fuselage will also tend to roll towards the opposite direction due to the effect of reaction torque. For the multi- rotor helicopter, it mostly uses the opposite rolling between the rotors to balance the effect of the reaction torque.

Fig. 1 is the real view of the tri-rotor flying robot. The yaw control in this study adopts the variable inclination angle mechanism of the back rotor. The rolling axis of the back rotor uses the RC servo motor and the linkage mechanism to control the variable declination angle. The rolling axis is fixed by using two pillow block ball bearings. Yaw angular speed is detected by a MEMS (Micro Electro Mechanical Systems) based Gyro, which is also used to adjust the inclination angle of the rolling axis of the back propeller. These are then taken as the compensation of the yaw torque produced by the three rotors.



Fig. 1 Real view of the tri-rotor flying robot

# II. FLIGHT PRINCIPLE OF THE Y-SHAPED TRI-ROTOR FLYING ROBOT

The tri-rotor flying robot in this study adopts the Y-shaped three-rotor structure (as shown in Fig. 2). The three rotors are divided into two right-handed and one left-handed, one right-handed and two left-handed, or three right-handed. In order to balance the yaw torque produced by the three rotors, it installs the RC servo motor and linkage structure on the tail axis, so as to improve the angle of the rolling axis of the motor on the tail axis. Moreover, through the torque generated by the horizontal component of the lift from the inclined motor (motor 3) on the tail axis, it balances the yaw torque of the three rotors.



Fig. 2 Rolling directions of the propellers of the Y-shaped tri-rotor flying robot

Below will introduce the flight principle of the tri-rotor flying robot:

• Fly forwards:

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When flying forwards, motor 1 and 2 must decelerate, while motor 3 on the tail axis must accelerate. As a result, the fuselage of the tricopter is inclined forwards, so it flies towards the same direction. On the contrary, when flying backwards, motor 1 and 2 must accelerate, while motor 3 must decelerate.

Fly to the right direction:

When the tri-rotor flying robot flies to the right direction, motor 1 on the left side must accelerate, while motor 2 on the right side must decelerate, so as to allow the fuselage incline to the right side and make the tricopter fly to the right direction.

Clockwise yaw

When the tri-rotor flying robot yaws in the clockwise direction, it needs to use the RC servo motor and linkage to drive the propeller of the motor 3 inclined in the left side. When the motor 3 rolls, it will generate the clockwise yaw torque, so as to make the tri-rotor flying robot yaw in the clockwise direction.

# III. KK MULTICOPTER FLIGHT CONTROL SYSTEM

The KK multicopter controller (as shown in Fig. 3) is a kind of flight control system, which can be applied in the multi-rotor aircraft with different axes, including: single-axis, dual-axis, tri-axis, quad-axis, hex-axis, eight-axis, as well as the aircraft with fixed wings. The KK multicopter flight controller has an Atmega micro-processor, and tri-axis Gyro that can detect the angular velocity of roll, pitch and yaw directions, as well as 8-channel PWM signal output. It can control 8 motors or RC servos at most, so that the aircraft can fly stably.



Fig. 3 KK multicopter flight controller

### 3.1 Basic functions of KK multicopter flight controller:

Tri-axis Gyro-stabilized system has contra-rotating Gyro chips, with the functions of electronic adjustment and calibration of the accelerator pedal and locking protection. Fig. 4 is the hardware architecture of the flight controller applied on the tricopter, in which the remote control receiver is used to receive the remote control signal sent from the radio controller. And the MEMS gyro can be used to detect the angular velocity of the directions of roll, pitch and yaw for the three rotors. The simulated voltage output by the gyro can be directly read by the single chip Atmega 168-20AU on the flight controller, based on which the

axis declination angles of the three rotors are calculated. The single chip on the flight controller can control the speed difference between the three motors, and the declination angle of the RC servos, so as to maintain the balance gesture of the three rotors.



Fig. 4 Hardware architecture of the KK multicopter flight controller applied on the tri-rotor flying robot

### IV. MATHEMATICAL MODEL

Fig. 5 is diagram of dynamic equation of the tricopter, in which the lower right side is the diagram of dynamic equation of the Yaw control. That is because in Yaw control, RC servo motor drives the tail axis to change the declination angle of the tail axis.



Fig. 5 Diagram of dynamic equation of the tri-rotor flying robot

#### 4.1Moment of Inertia of tricopter

Fig. 6 is the diagram of the moment of inertia of the tri-rotor flying robot. When calculating the inertia torque of the tri-rotor flying robot, we assume the fuselage is rectangular shape, the three motors are cylinder shape, and moreover, the inertias of the round rods of the axes are neglected.



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Fig. 6 moment of inertia of the tri-rotor flying robot

#### Moment of inertia of each axis:

• Moment of inertia about axis X:

$$I_{xx} = \frac{3}{2}m\ell^2 + \frac{1}{12}m_0b^2 + \frac{1}{12}m(3r^2 + h^2)$$
(1)

Moment of inertia about axis Y:

Th

$$I_{yy} = \frac{3}{2}m\ell^2 + \frac{1}{12}m_0a^2$$
 (2)

• Moment of inertia about axis Z:

The total moment of inertia about Z-axis:

$$I_{zz} = \frac{1}{12}m_0(a^2 + b^2) + 3m\ell^2$$
(3)

#### 4.2 Relationship between motor thrust and voltage

The thrust of the motors on the rotorcraft is the most important variables to determine the load and lift of the entire craft. The pre-condition to select the suitable motor is the net weight and expected payload of the entire tricopter. After that, it needs to determine the cruising ability of the craft, namely, how long it can fly. Next, it will select the suitable battery based on the voltage and power consumption of the motor. Therefore, the relation between the motor and voltage needs to be discussed.

The air speed generated by the propeller:

$$V_h = \sqrt{\frac{F}{2A\rho_a}} \tag{4}$$

$$P_h = F_{\sqrt{\frac{F}{2A\rho_a}}} \tag{5}$$

where:  $P_h$ : the power induced in air

F: thrust of the propeller

 $\rho_a$ :air density

A: area scanned when the properller spins

Ν

$$\tau_m = K_t F \tag{6}$$

voltage input can be obtained as follows:

$$F = 2\rho_a A \left[ \frac{f\eta \kappa_t}{\kappa_q} \right]^2 V^2 \tag{7}$$

where: K<sub>t</sub>:constant of torque

**4.3 Relationship between angular acceleration of the motor and the voltage** 

$$V = \frac{JZ\dot{\Omega}}{K_{q}} + K_{e}\Omega + \frac{ZD\Omega^{2}}{K_{q}}$$
(8)





# 4.4 Relationship between the declination angle of the Yaw motor and the Yaw control

The yaw control of the tricopter is realized by the declination © ISAROB 2012

angle of the motor  $M_3$ . The declination angle is determined by a RC servo motor and a linkage. Assume the thrust force of the tail motor  $M_3$  is  $F_3$ , and the declination angle is  $\alpha$ , as shown in Fig. 7. The vertical component of  $F_3$  is  $F_3 \cos \alpha$ , and the horizontal component of  $F_3$  is  $F_3 \sin \alpha$ .

#### 4.5 The principle of rotor torque

The torque rotors have a circular are profile. With this profile the torque forces increase as the angle of attack increases. While the rotor blades are fixed in place, they are quite flexible and probably change attack angle when they are accelerated.

$$F_{drag} = \frac{\rho C_d U^2 S}{2} = K_d \rho_{mp}^2 \tag{9}$$

Where : $\rho$  is the fluid density, U is the flow velocity,  $C_d$  is the torque coefficient and S = span × chord

# 4.6 Relationship between the angular acceleration and voltage of the motors

When the tricopter rolls, pitches, yaws and moves vertically, it is directly related with the motor's angular velocity or voltage input. Rolling to axis X is called Roll, to axis Y is Pitch, to axis Z is Yaw, moving along the Z-axis is vertical motion.

Assume the rolling torque is  $\tau_{xx}$ , angular acceleration is  $\ddot{\theta}$ , moment of inertia of axis X is  $I_{xx}$ , unit vector of axis X is  $\hat{\iota}$ , therefore, the rolling torque is:

$$\tau_{xx} = I_{xx} \ddot{\theta} \hat{\imath} \tag{10}$$

Rolling about X-axis

The rolling motion equation finally:

$$\ddot{\theta} = \frac{\sqrt{3}\rho_a A\ell}{l_{xx}} \left[ \frac{f\eta \kappa_t}{\kappa_q} \right]^2 (V_2^2 - V_1^2) \tag{11}$$

#### • Pitching about Y-axis

The pitching motion equation finally:

$$\ddot{\phi} = \frac{2\rho_a A\ell}{l_{yy}} \left[ \frac{f\eta K_t}{K_q} \right]^2 \left[ \frac{(V_1^2 + V_2^2)}{2} - V_3^2 \cos \alpha \right]$$
(12)

Yawing about Z-axis

The yawing motion equation finally:

$$\ddot{\psi} = \frac{F_{drag}}{I_{zz}} - \frac{2\rho_a A\ell}{I_{zz}} \left[ \frac{f\eta K_t}{K_q} \right]^2 V_3^2 \sin \alpha \tag{13}$$

#### Vertical acceleration

The vertical lift is generated by the three propellers, in which the thrust of the vertical component force of motor  $M_3$  is  $F_3 \cos \alpha$ . The resultant force of three motors is vertical to the plane of three propellers. If the pitch angle ( $\phi$ ) and roll angle ( $\theta$ ) are not equal to zero. The Z-axis component of the resultant force is  $F_{total} \cos \theta \cos \phi$ , which is applied on the centroid of the tricopter.

The lift in the centroid is:  

$$F_{all} = Ma_z = (F_1 + F_2 + F_3 \cos \alpha) \cos \theta \cos \phi - Mg$$
 (14)

Substitute it into equation (7) to get the relationship between the vertial acceleration and input voltage of motor: The Seventeenth International Symposium on Artificial Life and Robotics 2012 (AROB 17th '12), B-Con Plaza, Beppu, Oita, Japan, January 19-21, 2012

$$\frac{2\rho_a A}{M} \left[ \frac{f\eta \kappa_t}{\kappa_q} \right]^2 \left( V_1^2 + V_2^2 + V_3^2 \cos \alpha \right) \cos \theta \cos \phi - g \quad (15)$$

# V. EXPERIEMENTS OF THE TRI-ROTOR AERIAL VEHICLE

#### 5.1 The universal test platform:

 $a_{z} =$ 

The gains of the response parameters of pitch, roll and yaw on KK multicopter flight controller need to be slightly adjusted, so we made a universal test platform to adjust the gains of pitch, roll and yaw for the tri-rotor flying robot, with the expectation to rapidly return to the stable gesture without oscillation state. The universal test platform (as shown in Fig. 8) is designed to test the stability and balance of the tricopter. The biggest advantage of this test platform is to conduct the balance test of pitch, roll and yaw in all directions, and the error between the frictional force and the inertia of the test platform is the smallest.



Fig. 8 Mount the tricopter on the universal test platform

#### 5.2 Outdoor flight test

In terms of outdoor flight test, comparing the generation I tri-rotor flying robot (without KK multicopter installed ) with the generation II tri-rotor flying robot with KK multicopter installed, it is found the latter one performs better than the former one in terms of auto-hovering and stability. Fig. 9 is the actual test of outdoor flight for the generation II tri-rotor flying robot with KK multicopter installed.



Fig. 9 Outdoor flight test of the tri-rotor flying robot with KK multicopter installed

VI.CONCLUSION

The rolling speed of the rotor on the fueling helicopter is not suitable for indoor flight, and there is potential injury risk of the operator in case of high rolling speed. Moreover, the waste gas produced by burning the fuel will cause environmental pollution. Therefore, using the brushless motor as the resource of driven force is safe and environmental-friendly.

At present, the stable hovering of the tri-rotor flying robot has been completed, which is quite good flight platform. When studying this copter, most cases are the application of chemical, biological and radioactive (C.B.R.) detection, reconnaissance combat, disaster prevention and criminal prevention. The purpose of this study expects the stability of the tri-rotor flying robot can reach some level, and the tri-rotor flying robot can fly indoor or outdoor with hovering in the fixed indoor point, so as to accomplish the assigned tasks.

## ACKNOWLEDGMENT

This work is sponsored by the National Science Council, Taiwan, Republic of China under grant number NSC 99-2221-E-150-063.

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