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Thermal Wave Effect for Living Tissue with Surface Heating Problems by Differential Transformation Method

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Abstract: The Pennes' bio-heat conduction equation is common used to simulate temperature distribution for bio-heat transfer problems, it adopted the classical Fourier heat conduction law that is obviously incompatible with physical reality when research on microscopic heat transfer, low-pressure gases, cryogenic engineering, etc. Applying the concept of finite heat propagation speed, a thermal wave model of bi-heat transfer has developed. In order to analyze the thermal wave effect on temperature distributions, the different boundary heating conditions are considered with thermal wave model of bi-heat transfer and also compare to the Pennes'. The differential transform method combined with the finite difference scheme is proposed to simulate the temperature distributions. From results show it takes a period of time for the surface heating to propagate to a desired point inside the living tissue by the effect of thermal wave.

Keywords: Thermal wave, Thermal relaxation time, Differential transformation method.

1. INTRODUCTION

Temperature predictions for living tissues have attracted a lot of interesting in the processes of hyperthermia, thermal diagnostics, cryosurgery, thermal comfort analysis, and thermal parameter analysis. There were many models have been developed for describing bio-heat transfer behavior [1]. Among these models, the well known Pennes' model is the most commonly adopted [2]. It used the classic Fourier's law for its conduction term and was given as

$$q(\vec{r},t) = -\kappa \nabla T(\vec{r},t) \tag{1}$$

where κ is conductivity. The thermal signal propagates in infinite velocity in Eq. (1) that is obviously incompatible with physical reality when studies the processes of microscopic heat transfer, low-pressure gases, cryogenic engineering, etc. Thus, the concept of finite heat propagation wave velocity was proposed to apply for the bio-heat transfer processes [3-4]. Vernotte and Cattaneo [5] proposed a modified unsteady heat condition equation with the thermal relaxation time and was given as

$$q(\vec{r},t+\tau) = q(\vec{r},t) + \tau \frac{\partial \vec{q}(\vec{r},t)}{\partial t} = -\kappa \nabla T(\vec{r},t) \tag{2}$$

here $\tau = \alpha / V^2$ is defined as the thermal relaxation time, and α is the thermal diffusivity, V is denoted as the heat propagation velocity in the medium.

In homogeneous substances, τ is in the range of 10^{-8} ~ 10^{-10}s as for gases, and 10^{-8} ~ 10^{-14}s for liquids and dielectric solids [6]. The time of heating process is much longer than the thermal relaxation time scale in homogeneous substances, so the phenomenon of heat wave is relatively more difficult to observe. In non-homogeneous materials such as living tissues, τ is the characteristic time that needs to take time to accumulate the thermal energy in order to transfer to the nearest element. Kaminski [7] reported the value of τ is 20~30 s in meat product. Liu et al. [8] used the thermal wave model of bio-heat transfer (TWMBT) to analysis the thermal signal wave of bio-heat transfer. Liu [9] performed the Laplace transform method to investigate the thermal propagation behaviors. Liu and Lin [10] investigated physiological parameters by the hybrid numerical scheme. It is difficult to obtain the fundamental solution of the thermal propagation wave model of bio-heat transfer for living tissue [11]. In this article, a differential transform method is proposed to predict temperature distributions for living tissues with different boundary heating conditions. The effects of thermal wave on temperature distribution are also investigated.

2. THE DIFFERENTIAL TRANSFORMATION METHOD

Zhou [12] proposed the concept of the differential transform and can be summarized below.

$$X(k;t_0) = M(k)\left[\frac{d^k}{dt^k}\big(q(t)x(t)\big)\right]_{t=t_0}, \quad k \in K \tag{3}$$

where k belongs to a set of non-negative integer denoted as K domain. $X(k;t_0)$ is the differential transformation of

$x(t)$ at $t=t_0$, *M(k)* ($M(k)\neq 0$) is called the weighting factor and *q(t)* ($q(t)\neq 0$)is a kernel corresponding to *x(t)*. Thus, if *q(t)x(t)* can be expressed in terms as Taylor's series, then *x(t)* can be presented by using the differential inverse transformation of $X(k;t_0)$ as

$$x(t)=\frac{1}{q(t)}\sum_{k=0}^{\infty}\frac{(t-t_0)^k}{k!}\frac{X(k;t_0)}{M(k)},\quad \forall t\in T \tag{4}$$

If $M(k)=H^k/k!$ and $q(t)=1$, then Eq. (3) and (4) become

$$X(k)=T[x(t)]=\frac{H^k}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=0},\quad k\in K \tag{5}$$

$$x(t)=T^{-1}[x(t)]=\sum_{k=0}^{\infty}\left(\frac{t}{H}\right)^k X(k),\quad \forall t\in T \tag{6}$$

3. BIOHEAT TRANSFER PROBLEMS

The Pennes' bio-heat transfer equation is described as

$$-\nabla\cdot\vec{q}+W_bC_b(T_b-T)+q_m+q_r=\rho C\frac{\partial T}{\partial t} \tag{7}$$

where, ρ C, and T denote the density, specific heat, and temperature of living tissue, C_b is the specific heat of blood, W_b blood perfusion rate, q_m and q_r are the heat generation from metabolism and the spatial heat source respectively, T_b is the artery temperature. Liu et al. [13] introduced a general model of thermal wave form of the bio-heat transfer in living tissue from Eq. (2) and (7) as

$$\begin{aligned}&\nabla\cdot(\kappa\nabla T)+W_bC_b(T_b-T)+q_m+q_r\\&+\tau(-W_bC_b\frac{\partial T}{\partial t}+\frac{\partial q_m}{\partial t}+\frac{\partial q_r}{\partial t})=\rho C(\tau\frac{\partial^2 T}{\partial t^2}+\frac{\partial T}{\partial t})\end{aligned} \tag{8}$$

Equation (8) is a general form of TWMBT in living tissue. When heat mainly propagates in the perpendicular direction to the living tissue surface, one-dimensional heat transfer can be a good approximation. With constant thermal properties, $q_m=$ constant, and $q_r=0$, Eq. (8) can be expressed as

$$\kappa\frac{\partial^2 T}{\partial x^2}+W_bC_b(T_b-T)+q_m-\tau W_bC_b\frac{\partial T}{\partial t}=\rho C(\tau\frac{\partial^2 T}{\partial t^2}+\frac{\partial T}{\partial t}) \tag{9}$$

By assuming $T_i(x,0)$ is the initial steady state temperature, then Eq. (9) is turned into a new form as

$$\kappa\frac{\partial^2 T_i}{\partial x^2}+W_bC_b(T_b-T_i)+q_m-\tau W_bC_b\frac{\partial T_i}{\partial t}=\rho C(\tau\frac{\partial^2 T_i}{\partial t^2}+\frac{\partial T_i}{\partial t}) \tag{10}$$

Define a transformation of $\theta(x,t)=T(x,t)-T_i(x,0)$ and combine Eq. (9) and (10), the final result becomes

$$\rho C\tau\frac{\partial^2\theta}{\partial t^2}+(\rho C+\tau W_bC_b)\frac{\partial\theta}{\partial t}+W_bC_b\theta-\kappa\frac{\partial^2\theta}{\partial x^2}=0 \tag{11}$$

By taking differential transformation of Eq. (11), then

$$\begin{aligned}&\rho C\tau\frac{(k+1)(k+2)}{H^2}U(k+2)+(\rho C+\tau W_bC_b)\frac{(k+1)}{H}U(k+1)\\&+W_bC_bU(k)=\kappa\frac{\partial^2 U(k)}{\partial x^2}\end{aligned} \tag{12}$$

where $U(k)=U(x,t)$ is the differential transformation function at $\theta(x,t)$. By dividing the coordinate of x into N equal internals and taking the finite difference approximation to Eq. (12)

$$\begin{aligned}&\rho C\tau\frac{(k+1)(k+2)}{H^2}U_i(k+2)+(\rho C+\tau W_bC_b)\frac{(k+1)}{H}U_i(k+1)\\&+W_bC_bU_i(k)=\kappa\frac{U_{i+1}(k)-2U_i(k)+U_{i-1}(k)}{(\Delta x)^2}\end{aligned} \tag{13}$$

3. NUMERICAL SIMULATION

When the surface of living tissue is heated by different boundary heating conditions, the bio-heat conduction equation and relative conditions are discussed.

3.1. Case 1：Constant surface temperature heating

The skin surface is heated for constant temperature and the temperature distributions have simulated.

$$\rho C\tau\frac{\partial^2\theta}{\partial t^2}+(\rho C+\tau W_bC_b)\frac{\partial\theta}{\partial t}+W_bC_b\theta-\kappa\frac{\partial^2\theta}{\partial x^2}=0 \tag{14}$$

Assume there is no heat flux at x=L [8], then the initial and boundary conditions are described as

$$\theta(x,0)=0 \tag{15}$$

$$\frac{\partial\theta(x,0)}{\partial t}=0 \tag{16}$$

$$\theta(0,t)=\theta_0 \tag{17}$$

$$\frac{\partial\theta(L,t)}{\partial x}=0 \tag{18}$$

The differential transformation of Eq. (15)-(18) are

$$U_i(0)=0 \tag{19}$$

$$U_i(1)=0 \tag{20}$$

$$U_1(0)=\theta_0\ ,\ U_1(k)=0\ \ k\neq 0 \tag{21}$$

$$U_N(k)-U_{N-1}(k)=0 \tag{22}$$

The differential transformation equation from Eq. (14) can be resulted in tow situations.

For $\tau=0$

$$U_i(k+1)=\frac{H}{\rho C(k+1)}\left\{\begin{aligned}&-W_bC_bU_i(k)\\&+\frac{\kappa}{(\Delta x)^2}\left[\begin{aligned}&U_{i+1}(k)-2U_i(k)\\&+U_{i-1}(k)\end{aligned}\right]\end{aligned}\right\} \tag{23}$$

For $\tau\neq 0$

$$U_i(k+2)=\frac{H^2}{\rho C\tau(k+1)(k+2)}\left\{\begin{aligned}&-(\rho C+\tau W_bC_b)\frac{(k+1)}{H}U_i(k+1)\\&-W_bC_bU_i(k)\\&+\frac{\kappa}{(\Delta x)^2}\left[\begin{aligned}&U_{i+1}(k)\\&-2U_i(k)+U_{i-1}(k)\end{aligned}\right]\end{aligned}\right\} \tag{24}$$

3.2. Case 2：Constant surface flux heating

For a constant heat flux heat at the living surface, the corresponding boundary conditions typically can be described as

$$\begin{cases} -\kappa \dfrac{\partial \theta(0,t)}{\partial x} = q(t) \quad 0 < t < t_s \\ -\kappa \dfrac{\partial \theta(0,t)}{\partial x} = 0 \quad t > t_s \end{cases} \tag{25}$$

$$\frac{\partial \theta(L,t)}{\partial x} = 0 \tag{26}$$

Where t_s is defined as the duration of heating period and $q(t)$ denotes the heat flux that is time variable. Considering the heating time is short then the heat flux is approximately assumed as a constant, q_0. In the practices the heat flux is 83.2kw/m^2 as for the flash fire on the human skin surface.

3.3 Case 3：Constant temperature pulse surface heating

In the cases of eye surgery by using laser pulse or skin subjects to hot plate for a short period of time, the boundary conditions are expressed as

$$\begin{cases} \theta(0,t) = \theta_0 \quad 0 \le t \le t_s \\ \theta(0,t) = 0 \quad t > t_s \end{cases} \tag{27}$$

$$\frac{\partial \theta(L,t)}{\partial x} = 0 \tag{28}$$

4. RESULT AND DISCUSSION

The thermal property for homogeneous tissue are taken as $\rho = 1000 kg/m^3$, $C = C_b = 4200 J/kg°C$, $W_b = 0.5 kg/m^3$, $T_b = 32.5°C$, and $\kappa = 0.2 W/m$ [5]. As shows in Fig. 1, the computation domain is taken as L=0.01208m and the value of θ_0 is specified as 12℃ [5] and temperature distributions were analytically estimated at x=0.00208m inside the body.

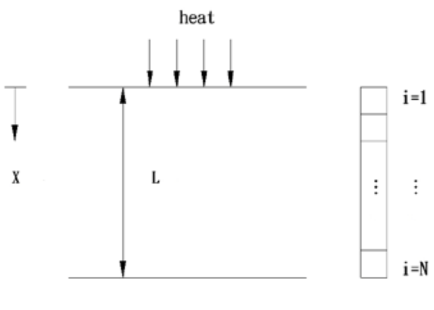

Fig. 1. The physical model

4.1. Case 1：Constant surface temperature heating

In Fig. 2, the temperature distributions predicted from Pennes' and TWMBT equation were different. As $\tau = 0$, the Pennes' bio-heat equation is used to characterize the thermal conduction, the thermal gradient has no jump discontinuity because of the infinite speed of thermal wave and the thermal signal can arrive the positions instantaneously. For $\tau \neq 0$, the influence of thermal relaxation time can result in a finite thermal wave propagation velocity and a travel time for thermal heat to distribute. The time for thermal heat to arrive at x=0.00208m is evaluated by using $t = L/\sqrt{\alpha/\tau}$. For $\tau = 20$ s and $\tau = 30$ s, the thermal wave reach x=0.00208m in $t = 0.00208/\sqrt{0.2/(1000\times 4200\times 20)} = 42.627(s)$ and $t = 0.00208/\sqrt{0.2/(1000\times 4200\times 30)} = 52.21(s)$, respectively.

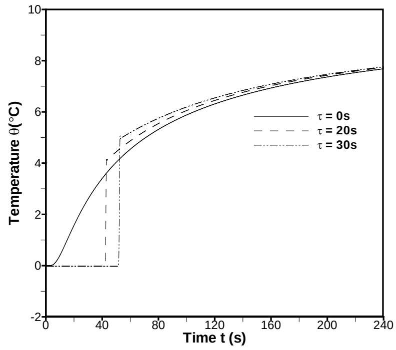

Fig. 2. Temperature distribution during constant surface temperature heating at x=0.00208m

4.2. Case 2：Constant surface flux heating

When there is a constant heat flux on the living skin surface for a period of heating time, as shows in Fig. 3. A substantial deviation was found between temperature predictions from the Pennes' and TWMBT equation. The temperature distributions predicted by the Pennes' equation increase at the initial then quickly decrease when the surface heat flux becomes zero. As in TWMBT equation, a period of time is needed for the thermal signal to travel from the surface to the particular position, the temperature distribution increase with a slope for the period of heating time then decrease. In Fig. 4, the different heat time were carried out to analyze the effect of thermal wave. The longer the heating time is the higher temperature is.

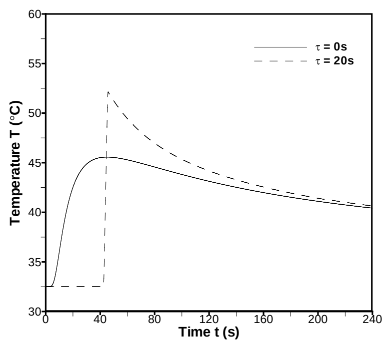

Fig. 3. Temperature distribution for t_s =3s during constant surface flux heating at x=0.00208m

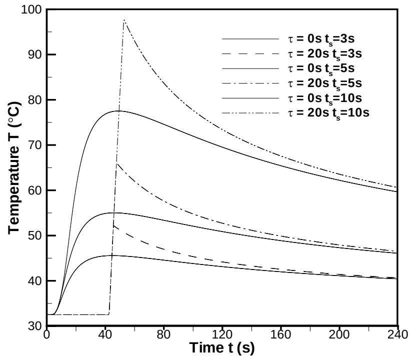

Fig. 4. Temperature distribution for various t_s during constant surface flux heating at x=0.00208m

4.3 Case 3：Constant temperature pulse surface heating

When the living tissues heat by the constant temperature pulse heating, such as eye surgery by laser irradiation or a flash fire on skin. As shows in Fig. 5, the temperature distributions predict by Pennes' and TWMBT equations for 3s, 5s, and 10s heating time. There is a travelling time for the thermal wave propagates from the heating living tissue surface to the particular and the temperature increase for the period of temperature pulse heating time. The longer heating time is, the higher temperature is. As for $\tau = 0$, the temperature increases at the period of temperature pulse heating time from begin, then gradually decreases at the end of heating.

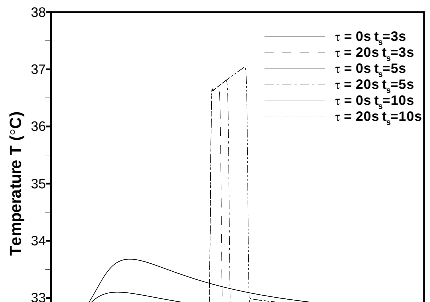

Fig. 5. Temperature distribution for various t_s, $\theta_0 = 12°C$ during constant temperature surface flux heating at x=0.00208m

5.CONCLUSION

The paper presents the effect of thermal wave in the living tissues for different surface heating problems. We simulate the bio-heat conduction problems with different values of the thermal relaxation time and also compares the results simulated by Pennes' and TWMBT equations. The results show that the heat wave speed under the effects of various thermal relaxation time and heat transfer wave propagation, can be expressed as $V = \sqrt{\alpha / \tau} = \sqrt{\kappa / \rho C \tau}$ and the travelling time also be calculated. In the non-homogeneous substance, the thermal wave propagates in a finite speed and cause a delay time to heat transfer compare with the temperature predicted by the Pennes' equation.

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