Robust exponential stabilization criteria for uncertain linear systems with interval time-varying delay

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Abstract: In this paper, we study the problem of robust exponential stabilization for uncertain linear systems with interval time-varying delay. We first rewrite the original system into a new form, then by dividing the delay intervals into two equal subintervals, we construct the Lyapunov-Krasovskii functional with augmented vectors. By appropriate enlarging some terms that appeared in the derivative of the Lyapunov-Krasovskii functional and using a new lower bounds lemma, delay-dependent robust exponential stabilization criteria are obtained based on Lyapunov stability theory and free weighting matrix technic. For getting the design of controller, we fix some formations of the introduced free-weighting matrices with given parameters, thus the obtained criteria are in terms of Linear Matrix Inequalities(LMIs). Finally numerical examples are given to show the effectiveness and less conservativeness of the proposed method.

Keywords: Delay-dependent, Interval time-varying delay, LMI, Robust exponential stabilization

1 INTRODUCTION

Interval time-varying delay system is a type of time-delay system in which the lower bound of delay needn't to be 0. Since the existence of time delay can make system instable and degrade its performance, much effort has been done to study the stability and stabilization problem for such systems in recent years. Wang [1] research the exponential stabilization problem for interval time-delay system by using free-weighting matrix method, but in the derivative of Lyapunov functional there are some terms are enlarged improperly, which will lead conservative results. Botmart [2] get criteria to design robust exponential stabilization controller for such system by assuming the controller have some formation and using convex combination method, which get less conservative results compared to some existing papers.

When considering this problem, we find that by choosing specific Lyapunov functional and appropriate enlarging some terms appeared in its derivative, less conservative stabilization criteria can be obtained. Motivated by the above ideals, we research the problem of delay-dependent robust exponential stabilization for uncertain time-delay systems with interval time-varying delay in this paper. We divide the delay intervals into two subintervals, and construct the corresponding Lyapunov functional by using the augmented vectors. Based on Lyapunov stability theory and free weighting matrix methods, delay-dependent robust stabilization criteria are obtained, and the controller can be obtained by solving LMIs. Finally, several numerical examples are given to show the effectiveness of the obtained criteria.

2 PROBLEM STATEMENT

Consider the following uncertain linear system with time-varying delay

$$\begin{aligned}\dot{x}(t) &= (A+\Delta A(t))x(t) + (A_d+\Delta A_d(t))x(t-h(t)) \\ &\quad + (B+\Delta B(t))u(t), \\ x(t) &= \phi(t),\, t\in[-h_2,0], \qquad (1)\end{aligned}$$

where $x(t)\in R^n$ is the state vector, $u(t)\in R^m$ is the control input, the initial condition $\phi(t)$ is a continuously differentiable vector-valued function, A, A_d, and B are constant system matrices of appropriate dimensions, $\Delta A(t)$, $\Delta A_d(t)$ and $\Delta B(t)$ are unknown real matrices with appropriate dimensions representing the system's time-varying parameter uncertainties and satisfy

$$[\Delta A\ \Delta A_d\ \Delta B] = [E_1F_1(t)G_1\ E_2F_2(t)G_2\ E_3F_3(t)G_3] \quad (2)$$

with E_i, G_i $(i=1,2,3)$ are known real constant matrices. $F_i(t)$ is the time-varying nonlinear function which satisfies

$$F_i^T(t)F_i(t)\le I \quad \text{for } i=1,2,3,\ \forall t\ge 0. \qquad (3)$$

$h(t)$ is a continuous time-varying function satisfying

$$0\le h_1\le h(t)\le h_2, \qquad (4)$$

$$\dot{h}(t) \leq \mu, \tag{5}$$

where $h_1 < h_2$, and $\mu \geq 0$ are constants.

Definition 1 [3]**.** The original state $x^* = 0$ of time-delay system (1) with uncertainty and interval time-varying delay satisfying (2-5) is said to be robustly exponentially stabilizable if for given constants $\sigma \geq 1$ and $\rho > 0$, there exists state feedback controller such that the solution $x(t)$ to the resulting closed-loop of system (1) satisfies

$$\|x(t)\| \leq \sigma \|x(t_0)\|_\theta \mathrm{e}^{-\rho(t-t_0)}, \ \forall\, t \geq t_0, \tag{6}$$

where $\|x(t)\|_\theta$ is defined by

$$\|x(t)\|_\theta = \sup_{0\leq\theta\leq h_2}\{x(t-\theta),\ \dot{x}(t-\theta)\},$$

and ρ is called the exponential convergence rate.

The purpose of this paper is to study the robust exponential stabilization problem for system (1) with uncertainty and interval time-varying delay satisfying (2-5) under the state feedback controller

$$u(t) = Kx(t). \tag{7}$$

Lemma 1 [4]**.** For scalars $\alpha, \beta \in [0,1], \alpha+\beta = 1$, and vectors η_1, η_2 satisfy $\eta_1 = 0$ with $\alpha = 0$ and $\eta_2 = 0$ with $\beta = 0$, matrices $P > 0$, $Q > 0$, there exists matrix T, satisfies

$$\begin{bmatrix} P & T \\ T^T & Q \end{bmatrix} \geq 0,$$

such that the following inequality holds

$$\frac{1}{\alpha}\eta_1^T P \eta_1 + \frac{1}{\beta}\eta_2^T Q \eta_2 \geq \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix}^T \begin{bmatrix} P & T \\ T^T & Q \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix}.$$

3 MAIN RESULTS

We rewrite the closed loop of system (1) as the following form

$$\dot{x}(t) = \bar{A}x(t) + A_d x(t-h(t)) + \bar{E}f(t,x), \tag{8}$$

with

$$\bar{A} = A + BK, \bar{E} = \begin{bmatrix} E_1 & E_2 & E_3 \end{bmatrix}, f(t,x) = \bar{F}(t)\bar{G}(x),$$

$$\bar{F}(t) = \mathrm{diag}\,\{F_1(t), F_2(t), F_3(t)\},$$

$$\bar{G}(x) = \begin{bmatrix} x^T(t)G_1^T & x^T(t-h(t))G_2^T & x^T(t)K^TG_3^T \end{bmatrix}^T.$$

Firstly we divide the delay intervals $[0, h_1]$ and $[h_1, h_2]$ into two subintervals separately, and denote $\delta_1 = \frac{h_1}{2}$, $\delta_2 = \frac{h_{12}}{2}$, $\delta = h_1 + \delta_2$, $\zeta_1(t) = \begin{bmatrix} x^T(t) & x^T(t-\delta_1) \end{bmatrix}^T$, $\zeta_2(t) = \begin{bmatrix} x^T(t) & x^T(t-\delta_2) \end{bmatrix}^T$. Then corresponding to the divisions and augmented vectors, we construct the following Lyapunov-Krasovskii functional

$$V(x_t) = V_1(x_t) + V_2(x_t) + V_3(x_t), \tag{9}$$

where

$$V_1(x_t) = x^T(t)Px(t),$$

$$\begin{aligned} V_2(x_t) &= \textstyle\int_{t-h(t)}^t x^T(s)\mathrm{e}^{\alpha(s-t)}Qx(s)\mathrm{d}s \\ &\quad + \textstyle\int_{t-\delta_1}^t \zeta_1^T(s)\mathrm{e}^{\alpha(s-t)}Q_1\zeta_1(s)\mathrm{d}s, \\ &\quad + \textstyle\int_{t-\delta}^{t-h_1} \zeta_2^T(s)\mathrm{e}^{\alpha(s-t)}Q_2\zeta_2(s)\mathrm{d}s, \end{aligned}$$

$$\begin{aligned} V_3(x_t) &= \delta_1 \textstyle\int_{-\delta_1}^0 \int_{t+\theta}^t \dot{x}^T(s)\mathrm{e}^{\alpha(s-t)}R_1\dot{x}(s)\mathrm{d}s\mathrm{d}\theta \\ &\quad + \delta_1 \textstyle\int_{-h_1}^{-\delta_1} \int_{t+\theta}^t \dot{x}^T(s)\mathrm{e}^{\alpha(s-t)}R_2\dot{x}(s)\mathrm{d}s\mathrm{d}\theta \\ &\quad + \delta_2 \textstyle\int_{-\delta}^{-h_1} \int_{t+\theta}^t \dot{x}^T(s)\mathrm{e}^{\alpha(s-t)}S_1\dot{x}(s)\mathrm{d}s\mathrm{d}\theta \\ &\quad + \delta_2 \textstyle\int_{-h_2}^{-\delta} \int_{t+\theta}^t \dot{x}^T(s)\mathrm{e}^{\alpha(s-t)}S_2\dot{x}(s)\mathrm{d}s\mathrm{d}\theta \end{aligned}$$

with P, Q, Q_i, R_i and S_i are symmetric positive matrices. Denote $\xi(t) = \begin{bmatrix} x^T(t) & x^T(t-\delta_1) & x^T(t-h_1) & x^T(t-\delta) & x^T(t-h_2) & x^T(t-h(t)) & \dot{x}^T(t) & f^T(t,x) \end{bmatrix}^T$, and e_i $(i = 1, \cdots, 8)$ are block entry matrices such that $e_i\xi(t) = \xi_i(t)$.

Theorem 1. For given scalars $\alpha > 0$, $0 \leq h_1 < h_2$, $\mu \geq 0$, γ_1 and γ_2, the closed-loop system of (1) with uncertainty and time-delay satisfied (2-5) is robust exponential stabilization if there exist symmetric positive matrices $\bar{P}$, $\bar{Q}$, $\bar{Q}_i$, $\bar{R}_i$, $\bar{S}_i$, matrices $\bar{T}_i$ $(i = 1, 2)$, $\bar{N}$ and W, positive scalars $\bar{\epsilon}$ such that the following LMIs (10-11) hold for $i = 1, 2$, then the solution $x(t)$ of the closed-loop of system (1) satisfies (6) with convergence rate $\rho = \frac{\alpha}{2}$ and the robust stabilization controller is $K = W\bar{N}^{-1}$.

$$\begin{bmatrix} \bar{\Pi}(i) & \bar{\epsilon}\tilde{G} \\ \bar{\epsilon}\tilde{G}^T & -\bar{\epsilon}I \end{bmatrix} < 0, \tag{10}$$

$$\begin{bmatrix} \bar{S}_i & \bar{T}_i \\ \bar{T}_i^T & \bar{S}_i \end{bmatrix} \geq 0, \tag{11}$$

where

$$\bar{\Pi}(i) = \bar{\Pi}_0 + \bar{\Pi}_i + \bar{\Pi}_e,$$

$$\begin{aligned} \bar{\Pi}_0 &= e_1^T\bar{P}e_7 + e_7^T\bar{P}e_1 + \alpha e_1^T\bar{P}e_1 + e_1^T\bar{Q}e_1 + e_6^T\psi(\mu)\bar{Q}e_6 \\ &\quad + e_a^T\bar{Q}_1e_a - e_b^T\mathrm{e}^{-\alpha\delta_1}\bar{Q}_1e_b + e_c^T\mathrm{e}^{-\alpha h_1}\bar{Q}_2e_c - \bar{\epsilon}e_8^Te_8 \\ &\quad - e_d^T\mathrm{e}^{-\alpha\delta}\bar{Q}_2e_d + \textstyle\sum_{i=1}^2 e_7^T(\delta_1^2\bar{R}_i + \delta_2^2\bar{S}_i)e_7 \\ &\quad - (e_1-e_2)^T\mathrm{e}^{-\alpha\delta_1}\bar{R}_1(e_1-e_2) \\ &\quad - (e_2-e_3)^T\mathrm{e}^{-\alpha h_1}\bar{R}_2(e_2-e_3), \end{aligned}$$

$$\begin{aligned} \bar{\Pi}_1 &= -(e_4-e_5)^T\mathrm{e}^{-\alpha h_2}\bar{S}_2(e_4-e_5) \\ &\quad - \mathrm{e}^{-\alpha\delta}\begin{bmatrix} e_3-e_6 \\ e_6-e_4 \end{bmatrix}^T \begin{bmatrix} \bar{S}_1 & \bar{T}_1 \\ \bar{T}_1^T & \bar{S}_1 \end{bmatrix} \begin{bmatrix} e_3-e_6 \\ e_6-e_4 \end{bmatrix}, \end{aligned}$$

$$\begin{aligned} \bar{\Pi}_2 &= -(e_3-e_4)^T\mathrm{e}^{-\alpha\delta}\bar{S}_1(e_3-e_4) \\ &\quad - \mathrm{e}^{-\alpha h_2}\begin{bmatrix} e_4-e_6 \\ e_6-e_5 \end{bmatrix}^T \begin{bmatrix} \bar{S}_2 & \bar{T}_2 \\ \bar{T}_2^T & \bar{S}_2 \end{bmatrix} \begin{bmatrix} e_4-e_6 \\ e_6-e_5 \end{bmatrix}, \end{aligned}$$

$$\bar{\Pi}_e = \bar{\Omega} + \bar{\Omega}^T,$$

$$\bar{\Omega} = (e_1 + \gamma_1e_6 + \gamma_2e_7)^T(-\bar{N}e_7 + A\bar{N}e_1 + BWe_1$$

$$+ A_d\bar{N}e_6 + \bar{\epsilon}\bar{E}e_8),$$

$$\psi(\mu) = \begin{cases} (\mu-1)\mathrm{e}^{-\alpha h_1} \ (\mu \geq 1) \\ (\mu-1)\mathrm{e}^{-\alpha h_2} \ (\mu < 1), \end{cases}$$

$e_a = [e_1^T, e_2^T]^T$, $e_b = [e_2^T, e_3^T]^T$,

$e_c = [e_3^T, e_4^T]^T$, $e_d = [e_4^T, e_5^T]^T$,

$\tilde{G} = \bar{G}_1\bar{N}^{-1}e_1 + \bar{G}_2\bar{N}^{-1}e_6 + \bar{G}_3We_1$,

$\bar{G}_1 = \left[\begin{array}{ccc} G_1^T & 0 & 0 \end{array}\right]^T$, $\bar{G}_2 = \left[\begin{array}{ccc} 0 & G_2^T & 0 \end{array}\right]^T$,

$\bar{G}_3 = \left[\begin{array}{ccc} 0 & 0 & G_3^T \end{array}\right]^T$, $\sigma = \sqrt{\frac{\bar{n}}{\bar{m}}}$, $\bar{N}_1 = \mathrm{diag}\{\bar{N}, \bar{N}\}$,

$\bar{m} = \lambda_{min}(\bar{N}^{-1}\bar{P}\bar{N}^{-T})$,

$$\begin{aligned}\bar{n} &= \lambda_{max}(\bar{N}^{-1}\bar{P}\bar{N}^{-T}) + (\tfrac{1-\mathrm{e}^{-\alpha h_2}}{\alpha})\lambda_{max}(\bar{N}^{-1}\bar{Q}\bar{N}^{-T}) \\ &+ (\tfrac{1-\mathrm{e}^{-\alpha\delta_1}}{\alpha})\lambda_{max}(2\bar{N_1}^{-1}\bar{Q}_1\bar{N_1}^{-T}) \\ &+ (\tfrac{\mathrm{e}^{-\alpha h_1}-\mathrm{e}^{-\alpha\delta}}{\alpha})\lambda_{max}(2\bar{N_1}^{-1}\bar{Q}_2\bar{N_1}^{-T}) \\ &+ \delta_1(\tfrac{\mathrm{e}^{-\alpha\delta_1}+\alpha\delta_1-1}{\alpha^2})\lambda_{max}(\bar{N}^{-1}\bar{R}_1\bar{N}^{-T}) \\ &+ \delta_1(\tfrac{\mathrm{e}^{-\alpha h_1}+\alpha h_1-\mathrm{e}^{-\alpha\delta_1}-\alpha\delta_1}{\alpha^2})\lambda_{max}(\bar{N}^{-1}\bar{R}_2\bar{N}^{-T}) \\ &+ \delta_2(\tfrac{\mathrm{e}^{-\alpha\delta}+\alpha\delta-\mathrm{e}^{-\alpha h_1}-\alpha h_1}{\alpha^2})\lambda_{max}(\bar{N}^{-1}\bar{S}_1\bar{N}^{-T}) \\ &+ \delta_2(\tfrac{\mathrm{e}^{-\alpha h_2}+\alpha h_2-\mathrm{e}^{-\alpha\delta}-\alpha\delta}{\alpha^2})\lambda_{max}(\bar{N}^{-1}\bar{S}_2\bar{N}^{-T}).\end{aligned}$$

Proof. Taking the time derivative of Lyapunov functional (9) along the trajectory of closed-loop of system (1), we have

$$\begin{aligned}\dot{V}_1(x_t) &= 2x^T(t)P\dot{x}(t) + \alpha x^T(t)Px(t) - \alpha V_1(x_t), \\ \dot{V}_2(x_t) &= x^T(t)Qx(t) + \zeta_1^T(t)Q_1\zeta_1(t) - \alpha V_2(x_t) \\ &- (1-\dot{h}(t))x^T(t-h(t))\mathrm{e}^{-\alpha h(t)}Qx(t-h(t)) \\ &- \zeta_1^T(t-\delta_1)\mathrm{e}^{-\alpha\delta_1}Q_1\zeta_1(t-\delta_1) \\ &+ \zeta_2^T(t-h_1)\mathrm{e}^{-\alpha h_1}Q_2\zeta_2(t-h_1) \\ &- \zeta_2^T(t-\delta)\mathrm{e}^{-\alpha\delta}Q_2\zeta(t-\delta), \\ \dot{V}_3(x_t) &= \textstyle\sum_{i=1}^2 \dot{x}^T(t)(\delta_1^2R_i + \delta_2^2S_i)\dot{x}(t) - \alpha V_3(x_t) \\ &- \delta_1\textstyle\int_{t-\delta_1}^{t} \dot{x}^T(s)\mathrm{e}^{\alpha(s-t)}R_1\dot{x}(s)\mathrm{d}s \\ &- \delta_1\textstyle\int_{t-h_1}^{t-\delta_1} \dot{x}^T(s)\mathrm{e}^{\alpha(s-t)}R_2\dot{x}(s)\mathrm{d}s \\ &- \delta_2\textstyle\int_{t-\delta}^{t-h_1} \dot{x}^T(s)\mathrm{e}^{\alpha(s-t)}S_1\dot{x}(s)\mathrm{d}s \\ &- \delta_2\textstyle\int_{t-h_2}^{t-\delta} \dot{x}^T(s)\mathrm{e}^{\alpha(s-t)}S_2\dot{x}(s)\mathrm{d}s.\end{aligned}$$

Then by using Jensen inequality [5] and enlarging some terms appropriately, we get

$$(\dot{h}(t)-1)\mathrm{e}^{-\alpha h(t)} \leq \psi(\mu), \tag{12}$$

$$\begin{aligned}&-\delta_1\textstyle\int_{t-\delta_1}^{t} \dot{x}^T(s)\mathrm{e}^{\alpha(s-t)}R_1\dot{x}(s)\mathrm{d}s \\ &\leq -\delta_1\mathrm{e}^{-\alpha\delta_1}\textstyle\int_{t-\delta_1}^{t} \dot{x}^T(s)R_1\dot{x}(s)\mathrm{d}s \\ &\leq -(\textstyle\int_{t-\delta_1}^{t} \dot{x}(s)\,\mathrm{d}s)^T\mathrm{e}^{-\alpha\delta_1}R_1(\textstyle\int_{t-\delta_1}^{t} \dot{x}(s)\,\mathrm{d}s) \\ &= -\xi^T(t)(e_1-e_2)^T\mathrm{e}^{-\alpha\delta_1}R_1(e_1-e_2)\xi(t),\end{aligned} \tag{13}$$

$$\begin{aligned}&-\delta_1\textstyle\int_{t-h_1}^{t-\delta_1} \dot{x}^T(s)\mathrm{e}^{\alpha(s-t)}R_2\dot{x}(s)\mathrm{d}s \\ &\leq -\xi^T(t)(e_2-e_3)^T\mathrm{e}^{-\alpha h_1}R_2(e_2-e_3)\xi(t),\end{aligned} \tag{14}$$

for the case time delay $h(t) \in [h_1, \delta]$, by Lemma 1, there exists matrix T_1, such that

$$\left[\begin{array}{cc} S_1 & T_1 \\ T_1^T & S_1 \end{array}\right] \geq 0, \tag{15}$$

$$\begin{aligned}&-\delta_2\textstyle\int_{t-\delta}^{t-h_1} \dot{x}^T(s)\mathrm{e}^{\alpha(s-t)}S_1\dot{x}(s)\mathrm{d}s \\ &\leq -\delta_2\mathrm{e}^{-\alpha\delta}\textstyle\int_{t-d(t)}^{t-h_1} \dot{x}^T(s)S_1\dot{x}(s)\mathrm{d}s \\ &\quad - \delta_2\mathrm{e}^{-\alpha\delta}\textstyle\int_{t-\delta}^{t-d(t)} \dot{x}^T(s)S_1\dot{x}(s)\mathrm{d}s \\ &\leq -\mathrm{e}^{-\alpha\delta}\xi^T(t)\left[\begin{array}{c} e_3-e_6 \\ e_6-e_4 \end{array}\right]^T\left[\begin{array}{cc} S_1 & T_1 \\ T_1^T & S_1 \end{array}\right]\left[\begin{array}{c} e_3-e_6 \\ e_6-e_4 \end{array}\right]\xi(t).\end{aligned}$$

Similarly to above procedure, we can cope with the situation $h(t) \in [\delta, h_2]$.

According to (3), there exists scalar $\epsilon > 0$ such that

$$\epsilon f^T(t,x)f(t,x) \leq \epsilon\bar{G}^T(x)\bar{G}(x),$$

thus we have

$$\epsilon\xi^T(t)\bar{G}^T\bar{G}\xi(t) - \epsilon\xi^T(t)e_8^Te_8\xi(t) \geq 0 \tag{16}$$

with $\bar{G} = \bar{G}_1e_1 + \bar{G}_2e_6 + \bar{G}_3Ke_1$.

By using system's information we add the left side of following equation into the derivative of Lyapunov functional:

$$\begin{aligned}&2(x^T(t)N_1 + x^T(t-h(t))N_2 + \dot{x}^T(t)N_3)(-\dot{x}(t) \\ &+\bar{A}x(t)+A_dx(t-h(t))+\bar{E}f(t,x)) = 0,\end{aligned} \tag{17}$$

where N_i are matrices with appropriate dimensions.

For getting LMI criteria and obtaining the controller, we set $N_1 = N$, $N_2 = \gamma_1N$, $N_3 = \gamma_2N$ and assume N^{-1} exist, and from the above equations we can get

$$\dot{V}(x_t)+\alpha V(x_t) \leq \xi^T(t)(\Pi(i)+\bar{G}^T\bar{G})\xi(t), \tag{18}$$

with some transformation of the matrices. By Schur Complement, the negative of (18) imply

$$\left[\begin{array}{cc} \Pi_i & \epsilon\bar{G} \\ \epsilon\bar{G}^T & -\epsilon I \end{array}\right] < 0. \tag{19}$$

Pre-multiplying and post-multiplying both sides of (19) by $\mathrm{diag}\{\sum_{i=1}^7 e_i^TN^{-1}e_i + \epsilon^{-1}e_8^Te_8, \epsilon^{-1}I\}$ and its transpose, denoting $N_1 = \mathrm{diag}\{N,N\}$, $\bar{P} = N^{-1}PN^{-T}$, $\bar{Q} = N^{-1}QN^{-T}$, $\bar{Q}_i = N_1^{-1}Q_iN_1^{-T}$, $\bar{R}_i = N^{-1}R_iN^{-T}$ $\bar{S}_i = N^{-1}S_iN^{-T}$, $\bar{T}_i = N^{-1}T_iN^{-T}$ $(i = 1,2)$, $\bar{N} = N^{-1}$, $W = K\bar{N}^T$ and $\bar{\epsilon} = \epsilon^{-1}$, then the hold of (19) is equivalent to (10). By applying the similar transformation to (15), and notice that from (9) and (18), we have

$$V(x_t) \geq \bar{m}\|x(t)\|^2, V(x_{t_0}) \leq \bar{n}\|x(t_0)\|_\theta^2, \tag{20}$$

$$V(x_t) < \mathrm{e}^{-\alpha(t-t_0)}V(x_{t_0}), \tag{21}$$

by some computation we can get Theorem 1, thus complete the proof.

Remark 1. The term $(\dot{h}(t)-1)\mathrm{e}^{-\alpha h(t)}$ appeared in the derivative of Lyapunov functional can be enlarged as $\mu e^{-\alpha h_1} - e^{-\alpha h_2}$, to reduce the conservativeness of the criteria, we consider the situations that $\mu \geq 1$ and $\mu < 1$, respectively, and enlarge it as $\psi(\mu)$, while the equation (24) in Wang [1] only holds when $\mu \geq 1$ but have some conservativeness when $\mu < 1$.

4 NUMERICAL EXAMPLES

In this section, we consider three examples to show the effectiveness of the obtained criteria.

We compare our methods to that of Botmart [2] in Example 1. The obtained results are listed in Table 1 below, which show the less conservativeness of our criteria.

Example 1. Consider the system (1) with

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, A_d = \begin{bmatrix} -2 & -0.5 \\ 0 & -1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Table 1. Admissible upper bound h_2 for unknown μ and various (ρ, h_1) with $(\gamma_1, \gamma_2) = (0, 1.9)$.

	(ρ, h_1)	(0.1, 0)	(0.1, 0.5)	(0.2, 0)	(0.2, 0.5)
[2]	h_2	0.406	0.590	0.381	0.545
T1	h_2	1.2152	1.4650	1.0859	1.3172

Remark 2. Following the same method, we can get better results by dividing the time intervals into more subintervals while more computational burden will be taken.

Remark 3. To get the controller matrix K, we fix the formation of free-weighting matrices S_i, to get less conservative results the values of γ_1 and γ_2 need to be appropriate chosen, we will do further research to this problem.

In Example 2, we compare our method to that of Wang [1] with the robust exponential stabilization problem.

Example 2. Consider the uncertain system (1) with

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, A_d = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} \sqrt{0.2} & 0 \\ 0 & \sqrt{0.05} \end{bmatrix}, E_2 = \begin{bmatrix} \sqrt{0.1} & 0 \\ 0 & \sqrt{0.3} \end{bmatrix}, E_3 = \begin{bmatrix} \sqrt{0.01} & 0 \\ 0 & \sqrt{0.1} \end{bmatrix}, G_1 = E_1, G_2 = E_2, G_3 = \begin{bmatrix} 0.01 \\ 0 \end{bmatrix}.$$

For $\mu = 1.1$, $\rho = 0.1$, $h_1 = 0.3$, and $h_2 = 0.7$, in Wang [1] the state convergence rate is estimated as

$$\|x(t)\| \leq 2.0463 e^{-0.1(t-t_0)} \|x(t_0)\|_\theta,$$

while with $(\gamma_1, \gamma_2) = (0.1, 1)$, from Theorem 1 we get the stabilization controller is $K = [-0.2273, -3.0087]$, and with a smaller $\sigma = 1.5947$, the state convergence rate satisfy

$$\|x(t)\| \leq 1.5947 e^{-0.1(t-t_0)} \|x(t_0)\|_\theta.$$

Furthermore, by Theorem 1 we can get the allowable upper bound h_2 is 1.5815, and the stabilization controller is $K = [-49.5411, -157.8835]$ with $\sigma = 5.4790$. From Fig.1. we can see that the state of controlled closed-loop system is exponential stable under initial condition $\varphi(s) = [3, -1]$, interval time-varying delay $[h_1, h_2] = [0.3, 1.5815]$ and $\mu = 1.1$ with the convergence rate $\rho = 0.1$.

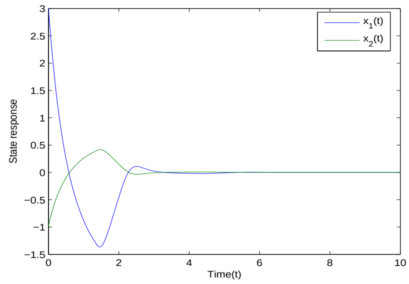

Fig.1. State response with controller $u(t)$

5 CONCLUSION

In this paper, the problem of robust exponential stabilization for interval time-varying delay systems have been investigated. By dividing the delay intervals into two equal subintervals and using free-weighting matrix technic, robust exponential stabilization criteria are obtained in terms of LMIs. Numerical examples are given to show the effectiveness and less conservativeness of our obtained criteria.

6 ACKNOWLEDGMENTS

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