The Seventeenth International Symposium on Artificial Life and Robotics 2012 (AROB 17th '12),
B-Con Plaza, Beppu, Oita, Japan, January 19-21, 2012

Lane keeping control for 4WS4WD vehicles subject to wheel slip constraint

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Abstract: This paper proposes a lane keeping control scheme that prevents an autonomous 4WS4WD vehicle from wheel skidding in presence of road curvature and aerodynamic drag. The control objectives can be specified as various closed-loop specifications, such as lane departure avoidance, wheel slip constraint and disturbance attenuation. An LMI approach is used to deal with these objectives simultaneously, which combines the quadratic stabilization technique with constraints on inputs. Simulations show that the proposed controller effectively limits the combined wheel slip and improves lane keeping performance.

Keywords: Combined wheel slip, lane keeping, linear matrix inequality(LMI), wheel skidding

1 INTRODUCTION

Lane departure is one of the most important causes of car accidents. NHTSA estimated that running off the road caused about 28% of the fatal crashes in the United States in 2005. Most research on lane keeping system has focused on pure lateral control [1], [2]. However, it is known that the vehicle dynamics are not completely independent in both directions. The coupling effects become increasingly significant as maneuvers involve higher accelerations, larger tire forces, or reduced road friction [3], [4]. So many efforts have been made to merge the two control tasks into a single problem.

To avoid lane departure, the strategy of controlling the vehicle's worst displacement/offset to the guideline beneath the safety requirement in [5], [6]. The combined wheel slip can be used to characterize the vector of tire/road, particularly for the situation of path following [7], [8]. In this paper, we consider an issue of guiding an autonomous vehicle to follow the curve without wheel skidding in the presence of aerodynamic drag. The design task is synthesized as a multi-objective problem, which specifies the closed-loop objectives in terms of a common Lyapunov function. This still guarantees the desired specifications at the expense of conservation. As a benefit, controller design can be reduced to a convex optimization problem.

2 PROBLEM FORMULATION

2.1 Vehicle model

As shown in Fig. 1., the model considered here for simulation consists of 7 degrees of freedom (DOF), which includes longitudinal, lateral and yaw motion of the vehicle as well as the rotational dynamics of the four wheels. The vehicle body-fixed coordinate system is used to set up the model. The governing equations of motion for the vehicle can be expressed as follows:

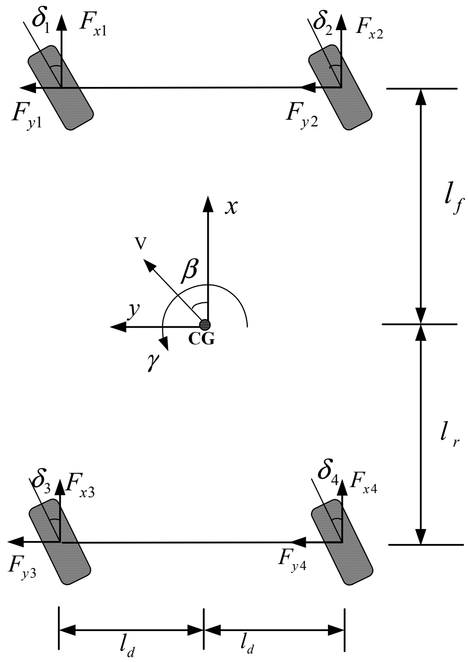

Fig. 1: Vehicle model

Longitudinal motion:

$$m(\dot{v}_x - \gamma v_y) = \sum F_x - C_{aero} v_x^2. \tag{1}$$

Lateral motion:

$$m(\dot{v}_y + \gamma) = \sum F_y. \tag{2}$$

Yaw motion:

$$J_z\dot{\gamma} = \sum M_z. \tag{3}$$

Wheel rotational equations of motion for wheels are as follows:

$$I_{wj}\dot{w}_j = T_j - r_j \begin{bmatrix}\cos\delta_j & \sin\delta_j\end{bmatrix} \begin{bmatrix}F_{xj} \\ F_{yj}\end{bmatrix}. \tag{4}$$

where m and J_z are the mass of vehicle and the inertia about z axis, respectively. F_{xj}, F_{yj}, and M_{zj} (j = 1, 2, 3, 4), defined in the body fixed $x-y-z$ coordinate system, are the external forces and yaw moments mainly resulting from tire/road friction. v_x and v_y stand for the longitudinal and lateral vehicle velocity, v the vehicle velocity, γ the yaw rate and β the vehicle side slip angle. $\sum F_x$, $\sum F_y$, $\sum M_z$ are the sum of the external forces and moments acting on the vehicle.

$$\begin{aligned}
\sum F_x &= F_{x1} + F_{x2} + F_{x3} + F_{x4} \\
\sum F_y &= F_{y1} + F_{y2} + F_{y3} + F_{y4} \\
\sum M_z &= l_f(F_{y1} + F_{y2}) - l_r(F_{y3} + F_{y4}) \\
&\quad + l_d(F_{x2} - F_{x1}) + l_d(F_{x4} - F_{x3}).
\end{aligned}$$

where l_f and l_r and l_d are the distances from the center of gravity to to the front, the rear axles, and to the wheel side. I_{wj} and r_j represent respectively the moment of inertia and the radius of wheel j; T_j and δ_j are the wheel torque and wheel steering angle used for the control scheme.

Because the sensors that measure the lateral deviation are not normally fixed on the vertical line through CG. Moreover, feedback based on error measured at the CG leads to bad ride comfort. Hence, it is natural to describe the vehicle dynamics in terms of the lateral displacement at the sensor y_l. The dynamics of path tracking can be expressed as

$$\begin{aligned}
\dot{\phi}_l &= \gamma - \rho v_x \\
\dot{y}_l &= v_x(\beta + \phi_l) + l_s(\gamma - \rho v_x).
\end{aligned} \tag{5}$$

Let ϕ_l be the angle between the road centerline and the vehicle longitudinal axis in radians, ρ the road curvature.

Because the wheel subsystem converges much faster, singular perturbation theory is used for model reduction. By linearizing the above nonlinear vehicle system around the operating point:

$$\rho_{ref} = 0,\ v_x = v_0,\ \beta = 0,\ \gamma = 0,\ y_l = 0,$$
$$\phi_l = 0,\ \delta_j = 0,\ T_j = 0,\ j = 1,2,3,4,$$

we finally arrive at the following design model:

$$\begin{aligned}
\frac{d}{dt}\begin{bmatrix}\partial v \\ \beta \\ \gamma \\ \phi_l \\ y_l\end{bmatrix} &= \begin{bmatrix}-2C_{aero}v_0/m & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & v_0 & l_s & v_0 & 0\end{bmatrix}\begin{bmatrix}\partial v \\ \beta \\ \gamma \\ \phi_l \\ y_l\end{bmatrix} \\
&+ \begin{bmatrix}1/m & 0 & 0 \\ 0 & 1/(mv_0) & 0 \\ 0 & 0 & 1/J_z \\ 0 & 0 & 0 \\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\sum F_x \\ \sum F_y \\ \sum M_z\end{bmatrix} \\
&+ \begin{bmatrix}-C_{aero}v_0^2/m \\ 0 \\ 0 \\ -v_0\rho_{ref} \\ -l_s v_0 \rho_{ref}\end{bmatrix}.
\end{aligned} \tag{6}$$

2.2 The combined wheel slip and friction forces

The combined wheel slip contains longitudinal and lateral components. The longitudinal slip S_L is defined in the direction of the wheel ground contact point velocity $v_j, j = 1,2,3,4$, and the lateral slip S_S at right angles to this.
When braking $(v_{Rj}\cos\alpha_j \le v_{Wj})$, the combined wheel slip is given by :

$$S_j = \begin{bmatrix}(v_{Rj}\cos\alpha_j - v_{Wj})/v_{Wj} \\ v_{Rj}\sin\alpha_j/v_{Wj}\end{bmatrix}. \tag{7}$$

When driving $(v_{Rj}\cos\alpha_j > v_{Wj})$, the combined wheel slip is given by:

$$S_j = \begin{bmatrix}(v_{Rj}\cos\alpha_j - v_{Wj})/v_{Rj}\cos\alpha_j \\ \tan\alpha_j\end{bmatrix}, \tag{8}$$

where the tire side slip angle α_j is the angle between the wheel plane and the velocity of the wheel ground contact point

$$\alpha_j = \delta_j - \beta_j,\ \beta_j = \arctan(v_{yj}/v_{xj}), \tag{9}$$

and

$$\begin{aligned}
v_{W1} &= (v_x - l_d\gamma)\vec{e}_x + (v_y + l_f\gamma)\vec{e}_y \\
v_{W2} &= (v_x + l_d\gamma)\vec{e}_x + (v_y + l_f\gamma)\vec{e}_y \\
v_{W3} &= (v_x - l_d\gamma)\vec{e}_x + (v_y - l_r\gamma)\vec{e}_y \\
v_{W4} &= (v_x + l_d\gamma)\vec{e}_x + (v_y - l_r\gamma)\vec{e}_y.
\end{aligned}$$

The resultant wheel slip is the geometrical sum of the longitudinal and lateral slip $S_{Res} = \sqrt{S_L^2 + S_S^2}$, and the resultant slip S_{Res} must always be between -1 and 1.

The friction forces in the body co-ordinate system (x,y) are given by

$$\begin{bmatrix}F_{xj} \\ F_{yj}\end{bmatrix} = F_{zj}\frac{\mu_{Res}(\|S_j\|,\chi)}{\|S_j\|}\begin{bmatrix}\cos\beta_j & \sin\beta_j \\ -\sin\beta_j & \cos\beta_j\end{bmatrix}\begin{bmatrix}1 & 0 \\ 0 & k_s\end{bmatrix}\begin{bmatrix}S_{Lj} \\ S_{Sj}\end{bmatrix}$$

$\mu_{Res}(\|S_j\|,\chi)$ is the resultant friction co-efficient. It is a scalar saturation function depending on the magnitude of resultant slip $\|S_j\|$ and road condition χ. Define

$$k_j \triangleq \frac{\partial \mu_{Res}}{\partial S_{Res}} \tag{10}$$

where the slope k_j in equation (10) depends mainly on road conditions. A better road condition gives a larger slope k_j and in turn provides a larger friction force. Then

$$\begin{bmatrix} F_{xj} \\ F_{yj} \end{bmatrix} = F_{zj} \begin{bmatrix} \cos\beta_j & \sin\beta_j \\ -\sin\beta_j & \cos\beta_j \end{bmatrix} \begin{bmatrix} k_j & 0 \\ 0 & k_S k_j \end{bmatrix} \begin{bmatrix} S_{Lj} \\ S_{Sj} \end{bmatrix}. \tag{11}$$

In this paper, we assume that the vehicle runs on a uniform road condition, more specifically, k_j has the same value as k.

Consider yaw rate γ, lateral derivation y_l as the outputs, and $z_\infty = x$, $z_1 = y_l$ the variable to be regulated. By combining (6) and (11), we can arrive at

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u \\ z_\infty &= C_\infty x \\ z_1 &= C_1 x \\ y &= C_2 x \end{aligned} \tag{12}$$

where

$$\begin{aligned} x &= \begin{bmatrix} \partial v & \beta & \gamma & \phi_l & y_l \end{bmatrix}^T, \\ u &= \begin{bmatrix} S_{L1} & S_{S1} & S_{L2} & S_{S2} \end{bmatrix}^T, \\ w &= \begin{bmatrix} f_w & \rho_{ref} \end{bmatrix}^T, f_w = C_{aero} v_0^2/m. \end{aligned}$$

The design objective is to design control laws to achieve the following objectives: 1) minimize the H_∞ norm of the transfer function matrix from w to z_∞ so as to reject disturbance; 2) keep the resultant wheel slip $S_{Res} \le s_{limi}$ to avoid wheel skidding; 3) keep z_1 bounded to satisfy the displacement constraints.

3 CONTROL LAW DESIGN

Theorem 1. *Consider system (12) with $S_j = C_u^i u$ for $i = 1,2$. Given some desired level of performance $\gamma_\infty > 0$, $\gamma_1 > 0$ and $\gamma_* = s_{limi}/w_{max}$ associated with each input, if there exist a constant matrix $Q > 0$ and a scalar $\nu > 0$ for some $\alpha > 0$ such that the following LMIs are feasible:*

$$\begin{aligned} \phi_S &\triangleq \begin{bmatrix} AQ + QA^T - \nu B_2 B_2^T & B_1 & QC_\infty \\ B_1^T & -\gamma_\infty^2 I_2 & 0 \\ C_\infty Q & 0 & -I_5 \end{bmatrix} < 0 \\ \Omega_S &\triangleq \begin{bmatrix} AQ + QA^T + \alpha Q - \nu B_2 B_2^T & B_1 \\ B_1^T & -\alpha I_2 \end{bmatrix} \le 0 \\ \theta_{Si} &\triangleq \begin{bmatrix} 4Q & \nu B_2 C_u^{iT} \\ \nu C_{ui} B_2 & \gamma_*^2 I_2 \end{bmatrix} > 0,\ i = 1,2 \\ \Pi_S &\triangleq \begin{bmatrix} Q & QC_1^T \\ C_1 Q & \gamma_1^2 / w_{max}^2 \end{bmatrix} > 0. \end{aligned} \tag{13}$$

then the state feedback controller

$$u = -\frac{\nu}{2} B_2^T Q^{-1} x \tag{14}$$

guarantees quadratic stability with L_2-gain, from w to z_∞. Furthermore, within the ellipsoid $\xi_F = \{x : x^T Q^{-1} x \le w_{max}^2\}$ $\|S_j\|_\infty = \| -\frac{\nu}{2} C_u^i B_2^T Q^{-1} x\|_\infty \le s_{limi}$, $i = 1,2$.

Theorem 2. *For some desired level of performance $\gamma_\infty > 0$, $\gamma_1 > 0$ and $\gamma_* > 0$ associated with each input, assume there exist a constant matrix $P = Q^{-1} > 0$ and a scalar $\nu > 0$ for some $\alpha > 0$ such that the LMIs given in theorem 1 are satisfied. If there exist constant matrices $S > 0$ and W such that the following LMIs are feasible,*

$$\begin{aligned} \phi_L &\triangleq \begin{bmatrix} \Sigma_{11} & \frac{\nu}{2} P B_2 B_2^T P & PB_1 \\ * & \Sigma_{22} & SB_1 \\ * & * & -\gamma_\infty^2 I_2 \end{bmatrix} < 0 \\ \Omega_L &\triangleq \begin{bmatrix} \Sigma_{11}' & \frac{\nu}{2} P B_2 B_2^T P & PB_1 \\ * & \Sigma_{22} & SB_1 \\ * & * & -\alpha I_2 \end{bmatrix} \le 0 \\ \theta_{Li} &\triangleq \begin{bmatrix} P & 0 & -\frac{\nu}{2} P B_2 C_u^{iT} \\ 0 & S & \frac{\nu}{2} P B_2 C_u^{iT} \\ * & * & \gamma_*^2 I_2 \end{bmatrix} > 0 \\ \Pi_L &\triangleq \begin{bmatrix} P & C_1^T \\ C_1 & \gamma_1^2 / w_{max}^2 \end{bmatrix} > 0. \end{aligned} \tag{15}$$

where $*$ represents a block matrix referred by symmetry.

$$\begin{aligned} \Sigma_{11} &= PA + A^T P - \nu P B_2 B_2^T P + C_\infty^T C_\infty \\ \Sigma_{22} &= SA + A^T S - WC_2 - C_2^T W^T \\ \Sigma_{11}' &= PA + A^T P + \alpha P - \nu P B_2 B_2^T P. \end{aligned}$$

then the observer-based output controller given by

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + B_2 u + L(y - C_2 \hat{x}) \\ u &= -\frac{\nu}{2} B_2^T Q^{-1} \hat{x} \end{aligned} \tag{16}$$

with $L = S^{-1} W$ guarantees quadratic stability with L_2-gain, from w to z_∞. Furthermore, within the ellipsoid $\{\tilde{x} : \tilde{x}^T \hat{P} \tilde{x} <= w_{max}^2\}$, $\|\hat{S}_j\|_\infty = \| -\frac{\nu}{2} C_u^j B_2^T Q^{-1} \hat{x}\|_\infty <= s_{limi}$, $j = 1,2$, where $\hat{P} \triangleq blockdiag\{P,S\}$, $\tilde{x}^T = \begin{bmatrix} x^T & e^T \end{bmatrix}$.

Using the quasi-steady-state combined wheel slip, the wheel torque T_j and steering angle δ_j are derived

$$\begin{bmatrix} T_j \\ \delta_j \end{bmatrix} = \begin{bmatrix} 0 \\ \beta + l_j \gamma / v_0 \end{bmatrix} + \begin{bmatrix} F_{zj} r_j k & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} S_{Lj} \\ S_{Sj} \end{bmatrix},$$

where $l_1 = l_2 = l_f$, $l_3 = l_4 = -l_r$, $j = 1,2,3,4$. When the states of the system are unmeasurable, we substitute β_j, γ with their estimate.

4 SIMULATIONS

To examine the effectiveness of the control scheme, simulation tests are carried out as follows. The desired speed for traveling is set as $v_0 = 16.7\ m/sec$ and the reference path is assumed to be a circular path of curvature $\rho_{ref} = 1/100\ m^{-1}$. We employ the following tire model [7]

$$\mu_{Res}(\|S_j\|) = 1.1973(1 - exp(25.168\|S_j\|)) - 0.5373\|S_j\|$$

for simulating the dry concrete condition and thus obtain the related initial slope (10) as $k_j = k \simeq 30$. The data of the vehicle system is given as follows; $C_{aero} = 0.3743\ kg/m$, $m = 1480\ kg$, $J_z = 1950\ kgm^2$, $l_f = 1.421\ m$, $l_r = 1.029\ m$, $l_d = 0.751\ m$, $k_s = 0.9$, $g = 9.81\ m/s^2$. Based on the dry-concrete-covered road condition and the vehicle data, we choose the wheel slip constraint as $s_{limi} = 0.8$.

The vehicle control system is assumed to start with the following initial state: $V_0 = 16.7m/sec$, $\beta(0) = 0\ deg$, $\gamma(0) = 0\ deg/sec$, $y_l(0) = 0.3\ m$, $\phi_l(0) = 0\ deg$. Fig. 2. and Fig. 3. illustrate the time responses of the lateral displacement and the longitudinal slip S_{L1} and lateral slip S_{S1} of the four wheels based on state feedback and output feedback. It can be seen that the maximum displacement can be kept less than $0.30\ m$, and the magnitudes of the resultant wheel slip S_{Res} are constrained below the pre-specified constraint $s_{limi} = 0.8$.

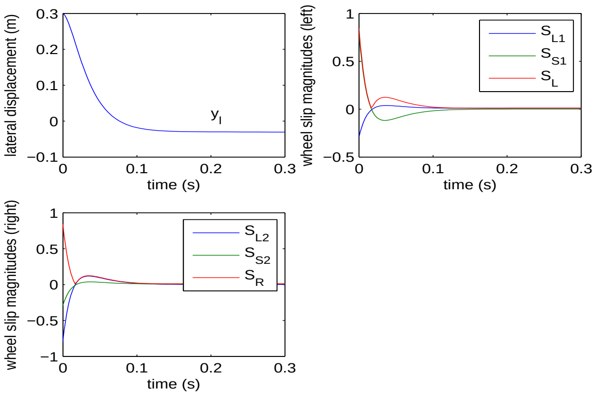

Fig. 2: displacement and magnitudes of the wheel slip

5 CONCLUSIONS

In this paper a robust controller is presented for an autonomous 4WS4WD vehicle to avoid lane departures and wheel skidding. The control strategy can be constructed as a multiobjective optimization problem. Simulation results are presented to validate the approach.

6 ACKNOWLEDGE

This work was supported by the National 973 Program (2012CB821200) and the NSFC (61134005, 60921001, 90916024, 91116016).

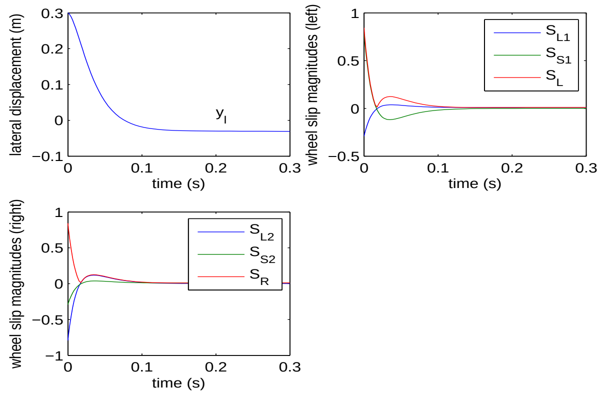

Fig. 3: displacement and magnitudes of the wheel slip based on observer

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