The Seventeenth International Symposium on Artificial Life and Robotics 2012 (AROB 17th '12),
B-Con Plaza, Beppu, Oita, Japan, January 19-21, 2012

H_∞ consensus control for high-order multi-agent systems with disturbances

Ping Wang[1], Yingmin Jia[1,2], Junping Du[3] and Fashan Yu[4]

[1] The Seventh Research Division and the Department of Systems and Control, Beihang University (BUAA), Beijing 100191, China (e-mail: wplh@ss.buaa.edu.cn).
[2] Key Laboratory of Mathematics Informatics and Behavioral Semantics (LIMB), SMSS, Beihang University (BUAA), Beijing 100191, China (e-mail: ymjia@buaa.edu.cn).
[3] Beijing Key Laboratory of Intelligent Telecommunications Software and Multimedia, School of Computer Science and Technology, Beijing University of Posts and Telecommunications, Beijing 100876, China (e-mail: junpingdu@126.com).
[4] School of Electrical Engineering and Automation, Henan Polytechnic University, Jiaozuo 454000, Henan, China (e-mail: yufs@hpu.edu.cn).

Abstract: This paper is devoted to consensus problems in directed networks of high-order agents with disturbances. A new distributed protocol is proposed with the consideration of model uncertainty, which only depends on the agent's own information and it's neighbors' first state, an auxiliary variable is included to describe the effects of all-order derivatives' relative information. Based on Lyapunov theory, for three cases: (a) network with fixed topology and zero time-delay; (b) network with switching topology and zero time-delay; (c) network with fixed topology and non-zero time-delay, sufficient conditions are derived correspondingly to make all agents reach H_∞ consensus. Especially, the approach used in this paper does not need any model transformation. Finally, numerical simulations are provided to show the effectiveness of the obtained results.

Keywords: Consensus, directed graph, H_∞ control, high-order multi-agent systems

1 INTRODUCTION

Recently, consensus problems of multi-agent systems have attracted researchers from a wide range of disciplines, and numerous significant theoretical results have been obtained [1-9]. In engineering practice, multi-agent systems are often subjected to various disturbances such as time-delay, model uncertainty, and the variation of network topology. As we all know, these disturbances might degrade the system performance and even cause the network system to diverge or oscillate. Therefore, it is of great significance to improve the robustness of the multi-agent systems. However, to the best of our knowledge, little work has been done to consider high-order consensus problem with disturbances.

With this background, we investigate consensus problems in directed networks of high-order agents with disturbances. Firstly, a new distributed protocol is proposed with the consideration of model uncertainty. Then, based on Lyapunov theory, sufficient conditions are derived to make all agents reach consensus while satisfying desired H_∞ performance for three cases. Especially, the approach used in this paper does not need any model transformation.

2 GRAPH THEORY

Let $\mathscr{G}(\mathscr{V},\varepsilon,\mathscr{A})$ be a directed graph of order n with the set of nodes $\mathscr{V}=\{s_1,\cdots,s_n\}$, set of edges $\varepsilon\subseteq\mathscr{V}\times\mathscr{V}$, and a weighted adjacency matrix $\mathscr{A}=[a_{ij}]$. The node indexes belong to a finite index set $I=\{1,2,\cdots,n\}$. An edge of $\mathscr{G}$ is denoted by $e_{ij}=(s_i,s_j)$. The adjacency elements associated with the edges are positive, i.e. $e_{ij}\in\varepsilon\Leftrightarrow a_{ij}>0$. Moreover, we assume that $a_{ii}=0$ for all $i\in I$. The set of neighbors of node s_i is denoted by $N_i=\{s_j\in\mathscr{V}:(s_i,s_j)\in\varepsilon\}$. A diagonal matrix $D=diag\{d_1,\cdots,d_n\}$ is a degree matrix of $\mathscr{G}$, with $d_i=\sum_{j=1}^n a_{ij}$ for $i\in I$. Then the Laplacian of the weighted graph $\mathscr{G}$ is defined as $L=D-\mathscr{A}\in\mathscr{R}^{n\times n}$. A directed path is a sequence of ordered edges of the form $(s_{i_1},s_{i_2}),(s_{i_2},s_{i_3}),\cdots$, where $s_{i_j}\in\mathscr{V}$. A directed graph is said to have a spanning tree, if there exists a node such that there is a directed path from every other node to this node[10]. If the graph $\mathscr{G}$ has a spanning tree, then its Laplacian L satisfies: zero is a simple eigenvalue of L, and $\mathbf{1}_n$ is the corresponding eigenvector [4].

3 PROBLEM STATEMENT

Consider the multi-agent system consisting of n identical agents, each agent is regarded as a node in a directed graph $\mathscr{G}$. Suppose the ith agent has the dynamics as follows:

$$\begin{aligned}
\dot{x}_i^{(0)}(t) &= x_i^{(1)}(t)\\
&\vdots\\
\dot{x}_i^{(l-2)}(t) &= x_i^{(l-1)}(t)\\
\dot{x}_i^{(l-1)}(t) &= u_i(t)+\omega_i(t)\\
y_i(t) &= x_i^{(0)}(t)
\end{aligned}\tag{1}$$

where $x_i=[x_i^{(0)},x_i^{(1)},\cdots,x_i^{(l-1)}]^\top\in\mathbb{R}^l$ is the state of the ith agent, $u_i\in\mathbb{R}$ is the protocol, $\omega_i(t)\in L_2[0,\infty)$ is the external disturbance, $y_i(t)$ is the measured output that can be observed by its neighbors.

A protocol u_i is said to asymptotically solve consensus problem, if for any initial state, the states of all the agents satisfy

$$\lim_{t\to+\infty}[x_i(t)-x_j(t)]=0 \tag{2}$$

for all $i,j\in I$.

In order to solve the consensus problem of high-order multi-agent system (1), we propose the following consensus protocol

$$\begin{aligned} u_i &= k_0\sum_{s_j\in N_i}(a_{ij}+\Delta a_{ij}(t))(x_j^{(0)}(t-\tau)-x_i^{(0)}(t-\tau)) \\ &\quad -\sum_{j=1}^{l-1}k_jx_i^{(j)}(t)+p_i \\ \dot{p}_i &= -\gamma_1p_i-k_l\sum_{s_j\in N_i}(a_{ij}+\Delta a_{ij}(t))(x_j^{(0)}(t-\tau)-x_i^{(0)}(t-\tau)) \end{aligned} \tag{3}$$

where $k_i>0, i=0,1,\cdots,l,\ \gamma_1>0$ are protocol parameters to be designed, p_i is an auxiliary variable to describe the effects of all-order derivatives' relative information. τ denotes communication delay, $\Delta a_{ij}(t)$ denotes the uncertainty of a_{ij}.

A controlled output function

$$z_i(t)=[z_{i1}(t),z_{i2}(t),\cdots,z_{il}(t)]^\top\in\mathbb{R}^l \quad i\in I$$

is defined as an average of the relative displacements of all agents as follows

$$\begin{aligned} z_{i1}(t) &= x_i^{(0)}(t)-\frac{1}{n}\sum_{j=1}^{n}x_j^{(0)}(t) \\ z_{i2}(t) &= x_i^{(1)}(t)-\frac{1}{n}\sum_{j=1}^{n}x_j^{(1)}(t) \\ &\vdots \\ z_{il}(t) &= x_i^{(l-1)}(t)-\frac{1}{n}\sum_{j=1}^{n}x_j^{(l-1)}(t) \end{aligned} \tag{4}$$

Obviously, the multi-agent system (1) achieves consensus if and only if

$$\lim_{t\to+\infty}z_i(t)=0 \qquad i\in I \tag{5}$$

Let

$$A=\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & -k_1 & -k_2 & \cdots & -k_{l-1} & 1 \\ 0 & 0 & 0 & \cdots & 0 & -\gamma_1 \end{bmatrix}$$

$$B=\begin{bmatrix} 0_{l-1} & 0_{(l-1)\times l} \\ k_0 & 0_{1\times l} \\ -k_l & 0_{1\times l} \end{bmatrix} \quad B_1=\begin{bmatrix} 0_{l-1} \\ 1 \\ 0 \end{bmatrix}$$

$$B_2=\begin{bmatrix} I_l & 0_l \end{bmatrix}$$

$$C=\begin{bmatrix} \frac{n-1}{n} & -\frac{1}{n} & \cdots & -\frac{1}{n} \\ -\frac{1}{n} & \frac{n-1}{n} & \cdots & -\frac{1}{n} \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{1}{n} & -\frac{1}{n} & \cdots & \frac{n-1}{n} \end{bmatrix}$$

Under the protocol (3), the network dynamics of the multi-agent system is

$$\begin{aligned} \dot{\varphi}(t) &= (I_n\otimes A)\varphi(t)-((L+\Delta L)\otimes B)\varphi(t-\tau) \\ &\quad +(I_n\otimes B_1)\omega(t) \\ z(t) &= (C\otimes B_2)\varphi(t) \end{aligned} \tag{6}$$

where $\varphi=[x_1^\top,p_1,\ \cdots\ x_n^\top,p_n]^\top$, $\omega=[\omega_1,\cdots,\omega_n]^\top$, $z=[z_1^\top,\cdots,z_n^\top]^\top$, L is the Laplacian of the graph $\mathscr{G}$, ΔL denotes the uncertainty Laplacian satisfying $\Delta L=E_1\Sigma(t)E_2$, where $E_1\in\mathbb{R}^{n\times|\varepsilon|}$, $E_2\in\mathbb{R}^{|\varepsilon|\times n}$ are specified constant matrices and $\Sigma(t)$ is a diagonal matrix satisfying $\Sigma^\top(t)\Sigma(t)\le I$.

Define the following H_∞ performance index

$$J=\int_0^\infty[z^\top(t)z(t)-\gamma^2\omega^\top(t)\omega(t)]dt<0 \tag{7}$$

where γ is a given positive constant.

Based on the above discussion, the H_∞ consensus problem to be addressed is stated as follows.

H_∞ **consensus problem**: For a given protocol u_i, we say the multi-agent systems reach H_∞ consensus if the following two conditions are satisfied simultaneously:
1). when $\omega(t)=0$, the multi-agent systems achieve consensus, i.e. $\lim_{t\to+\infty}z(t)=0$;
2). if $z_0=0$, the inequality (7) is satisfied.

4 MAIN RESULTS

In this section, we will give conditions to make all agents achieve H_∞ consensus. Before presenting the main results, we first introduce a lemma.

Lemma 1 [7]. Consider the matrix C. Then there exists an orthogonal matrix $U=[U_1\ \ \overline{U}_1]$ with $\overline{U}_1=\frac{1}{\sqrt{n}}\mathbf{1}_n$, such that

$$U^\top CU=\begin{bmatrix} I_{n-1} & \mathbf{0}_{(n-1)\times1} \\ \mathbf{0}_{1\times(n-1)} & 0 \end{bmatrix}$$

holds.

Theorem 1. Consider a directed network with fixed topology and zero time-delay. The multi-agent system (6) reaches H_∞ consensus, if there exist a symmetric positive-definite matrix $\overline{P}\in\mathbb{R}^{(l+1)(n-1)\times(l+1)(n-1)}$, and a scalar $\mu>0$ satisfying

$$M=\begin{bmatrix} M_{11} & M_{12} & M_{13} \\ * & -\mu I & \mathbf{0} \\ * & * & -\gamma^2I \end{bmatrix}<0 \tag{8}$$

where $M_{11}=\overline{P}(I_{n-1}\otimes A-\overline{L}\otimes B)+(I_{n-1}\otimes A-\overline{L}\otimes B)^\top\overline{P}+\mu\overline{E}+I_{n-1}\otimes B_2^\top B_2$, $M_{12}=\overline{P}(U_1^\top E_1\otimes B)$, $M_{13}=\overline{P}(U_1^\top\otimes B_1)$, $\overline{L}=U_1^\top LU_1$, $\overline{E}=(U_1^\top E_2^\top E_2U_1)\otimes I_{l+1}$.

Proof. Let

$$\begin{aligned} \delta(t) &= (U_1\otimes I_{l+1})^\top\varphi(t) \\ \overline{\delta}(t) &= (\overline{U}_1\otimes I_{l+1})^\top\varphi(t) \end{aligned} \tag{9}$$

where $\overline{\delta}(t)$ and $\delta(t)$ describe the average and disagreement states of all agents, respectively.

Define a Lyapunov function for system (6) as follows

$$V(t) = \varphi^\top(t)P\varphi(t)$$

where $P = P^\top \geq 0$ satisfies $P(\mathbf{1}_n \otimes I_{l+1}) = 0$ and $rank(P) = (l+1)n-(l+1)$.

Let $\overline{P} = (U_1 \otimes I_{l+1})^\top P(U_1 \otimes I_{l+1})$, then $V(t)$ can be rewritten as

$$\begin{aligned} V(t) &= \varphi^\top(t)P\varphi(t) \\ &= \begin{bmatrix}\delta(t)\\ \overline{\delta}(t)\end{bmatrix}^\top \begin{bmatrix}\overline{P} & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}\delta(t)\\ \overline{\delta}(t)\end{bmatrix} \\ &= \delta^\top(t)\overline{P}\delta(t) > 0 \end{aligned} \tag{10}$$

Differentiating $V(t)$ along the trajectory of (6), we have

$$\begin{aligned} \dot{V}(t) &= 2\delta^\top(t)\overline{P}\dot{\delta}(t) \\ &= 2\delta^\top(t)\overline{P}[I_{n-1}\otimes A - \overline{L}\otimes B]\delta(t) \\ &\quad -2\delta^\top(t)\overline{P}(\overline{\Delta L}\otimes B)\delta(t) + 2\delta^\top(t)\overline{P}(U_1^\top \otimes B_1)\omega(t) \end{aligned}$$

where $\overline{L} = U_1^\top LU_1$, $\overline{\Delta L} = U_1^\top \Delta LU_1 = U_1^\top E_1\Sigma(t)E_2U_1$.

And since $\Sigma^\top(t)\Sigma(t) \leq I$, we can obtain

$$\begin{aligned} &-2\delta^\top(t)\overline{P}(\overline{\Delta L}\otimes B)\delta(t) \\ &= -2\delta^\top(t)\overline{P}(U_1^\top E_1 \otimes B)(\Sigma(t)E_2U_1 \otimes I_{l+1})\delta(t) \\ &\leq \tfrac{1}{\mu}\delta^\top(t)\overline{P}(U_1^\top E_1\otimes B)(U_1^\top E_1 \otimes B)^\top\overline{P}\delta(t) \\ &\quad + \mu\delta^\top(t)\overline{E}\delta(t) \end{aligned}$$

where $\mu > 0$, $\overline{E} = (U_1^\top E_2^\top E_2U_1)\otimes I_{l+1}$.

By Lemma 1, we have

$$\begin{aligned} z(t) &= (C\otimes B_2)(U\otimes I_{l+1})(U\otimes I_{l+1})^\top\varphi(t) \\ &= (CU\otimes B_2)\begin{bmatrix}\delta(t)\\ \overline{\delta}(t)\end{bmatrix} \\ &= (U_1\otimes B_2)\delta(t) \end{aligned}$$

we have $\lim_{t\to+\infty} z(t) = 0$ if $\lim_{t\to+\infty}\delta(t) = 0$. Thus whether the system (6) can reach consensus is only related to the component $\delta(t)$.

Denote

$$\begin{aligned} N &= \overline{P}(I_{n-1}\otimes A - \overline{L}\otimes B) + (I_{n-1}\otimes A - \overline{L}\otimes B)^\top\overline{P} \\ &\quad + \mu\overline{E} + \tfrac{1}{\mu}\overline{P}(U_1^\top E_1\otimes B)(U_1^\top E_1\otimes B)^\top\overline{P} \end{aligned}$$

when $\omega(t) = 0$, we can obtain

$$\dot{V}(t) \leq \delta^\top(t)N\delta(t) \tag{11}$$

$N < 0$ holds when $M < 0$, it follows from (10) and (11) that consensus can be achieved asymptotically.

To study the H_∞ performance for the multi-agent system, assume zero initial condition, that is $V(0) = 0$. Therefore, we have

$$\begin{aligned} J &= \textstyle\int_0^\infty[z^\top(t)z(t) - \gamma^2\omega^\top(t)\omega(t) + \dot{V}(t)]dt - V(\infty) + V(0) \\ &\leq \textstyle\int_0^\infty[z^\top(t)z(t) - \gamma^2\omega^\top(t)\omega(t) + \dot{V}(t)]dt \\ &\leq \textstyle\int_0^\infty \xi^\top(t)\overline{M}\xi(t)dt \end{aligned}$$

where

$$\begin{aligned} \xi(t) &= [\delta^\top(t)\ \ \omega^\top(t)]^\top \\ \overline{M} &= \begin{bmatrix}\overline{M}_{11} & \overline{P}(U_1^\top\otimes B_1)\\ * & -\gamma^2 I\end{bmatrix} \\ \overline{M}_{11} &= M_{11} + \tfrac{1}{\mu}\overline{P}(U_1^\top E_1\otimes B)(U_1^\top E_1\otimes B)^\top\overline{P} \end{aligned}$$

By Schur complement, $M < 0$ is equivalent to $\overline{M} < 0$. That is, (8) guarantees $J < 0$. Therefore, under the condition $M < 0$, all agents reach H_∞ consensus.

Remark 1. The approach used in Theorem 1 does not need to perform any model transformation.

Remark 2. It is worth pointing out that a necessary condition for (8) is that the graph $\mathcal{G}$ has a spanning tree.

Theorem 2. Consider a directed network with switching topology $\mathcal{G}_\sigma$ and zero time-delay. The multi-agent system (6) reaches H_∞ consensus, if there exist a common symmetric positive-definite matrix $\overline{P}\in\mathbb{R}^{(l+1)(n-1)\times(l+1)(n-1)}$, and positive scalars μ_σ for each possible communication graph $\mathcal{G}_\sigma$ satisfying

$$\hat{M}_\sigma = \begin{bmatrix}\hat{M}_{11} & \hat{M}_{12} & \hat{M}_{13}\\ * & -\mu_\sigma I & \mathbf{0}\\ * & * & -\gamma^2 I\end{bmatrix} < 0 \tag{12}$$

where $\hat{M}_{11} = \overline{P}(I_{n-1}\otimes A - \overline{L}_\sigma\otimes B) + (I_{n-1}\otimes A - \overline{L}_\sigma\otimes B)^\top\overline{P} + \mu_\sigma\overline{E}_\sigma + I_{n-1}\otimes B_2^\top B_2$, $\hat{M}_{12} = \overline{P}(U_1^\top E_{1\sigma}\otimes B)$, $\hat{M}_{13} = \overline{P}(U_1^\top\otimes B_1)$, $\overline{L}_\sigma = U_1^\top L_\sigma U_1$, $\overline{E}_\sigma = (U_1^\top E_{2\sigma}^\top E_{2\sigma}U_1)\otimes I_{l+1}$, and σ denotes the switching signal that determines the topology.

Theorem 3. Consider a directed network with fixed topology and non-zero time-delay. The multi-agent system (6) reaches H_∞ consensus, if there exist symmetric positive-definite matrices $\overline{P},\overline{Q},\overline{R}\in\mathbb{R}^{(l+1)(n-1)\times(l+1)(n-1)}$, and positive scalars $\mu_1,\mu_2,\mu_3,\mu_4,\mu_5$ satisfying

$$\Gamma = \begin{bmatrix}\Gamma_{11} & \Gamma_{12}\\ * & \Gamma_{22}\end{bmatrix} < 0 \tag{13}$$

where Γ_{11},Γ_{12} and Γ_{22} are defined in (14), $\psi_1 = \overline{P}(I_{n-1}\otimes A - \overline{L}\otimes B) + (I_{n-1}\otimes A - \overline{L}\otimes B)^\top\overline{P} + \overline{Q}$, $\overline{E} = (U_1^\top E_2^\top E_2U_1)\otimes I_{l+1}$, and $\hat{E} = (U_1^\top E_2^\top E_2U_1)\otimes BB^\top$.

Proof. This theorem can be proved following the lines of the proof of Theorem 1 and hence omitted. It deserves pointing out that the Lyapunov function adopted here is as follows

$$\begin{aligned} V(t) &= \varphi^\top(t)P\varphi(t) + \textstyle\int_{t-\tau}^t \varphi^\top(s)Q\varphi(s)ds \\ &\quad + \textstyle\int_{-\tau}^0\int_{t+\theta}^t \dot{\varphi}^\top(s)R\dot{\varphi}(s)dsd\theta \end{aligned}$$

5 SIMULATIONS

The simulation is given to illustrate the effectiveness of the obtained results. Fig.1. shows a directed graph with 0-1 weights. Each agent has three-order dynamics. Suppose that the uncertainty of each edge satisfies $|a_{ij}| \leq 0.01$, the communication delay $\tau = 0.1s$, and the initial conditions $\varphi(0) = [0.5\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ -0.5\ 0\ 0\ 0\ -1\ 0\ 0\ 0]^\top$.

$$\Gamma_{11}=\begin{bmatrix}\psi_1+I_{n-1}\otimes B_2^\top B_2 & -\tau(I_{n-1}\otimes A)^\top\overline{R}(\overline{L}\otimes B) & \overline{P}(U_1^\top\otimes B_1)+\tau(I_{n-1}\otimes A)^\top\overline{R}(U_1^\top\otimes B_1)\\ * & -\overline{Q}+\mu_1\overline{E}+\tau(\mu_2+\mu_3+\mu_5)\overline{E}+\tau\mu_4\hat{E} & -\tau(\overline{L}\otimes B)^\top\overline{R}(U_1^\top\otimes B_1)\\ * & * & -\gamma^2 I\end{bmatrix}$$

$$\Gamma_{12}=\begin{bmatrix}\overline{P}(\overline{L}\otimes B) & \overline{P}(U_1^\top E_1\otimes B) & \tau(I_{n-1}\otimes A)^\top\overline{R} & (I_{n-1}\otimes A)^\top\overline{R}(U_1^\top E_1\otimes B) & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \tau(\overline{L}\otimes B)^\top\overline{R} & (\overline{L}\otimes B)^\top\overline{R}(U_1^\top E_1\otimes B) & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \tau(U_1^\top\otimes B_1)^\top\overline{R} & (U_1^\top\otimes B_1)^\top\overline{R}(U_1^\top E_1\otimes B)\end{bmatrix} \quad (14)$$

$$\Gamma_{22}=diag\{-\frac{\overline{R}}{\tau},-\mu_1 I,-\tau\overline{R},-\frac{\mu_2}{\tau}I,-\tau\overline{R},-\frac{\mu_3}{\tau}I,\begin{bmatrix}-\tau\overline{R} & \overline{R}(U_1^\top E_1\otimes I)\\ * & -\frac{\mu_4}{\tau}I\end{bmatrix},-\tau\overline{R},-\frac{\mu_5}{\tau}I\}$$

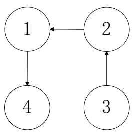

Fig. 1. A directed graph

Take $k_0=3, k_1=7, k_2=3, k_3=1, \gamma_1=1, \gamma=1$, and the external disturbance as $\omega(t)=[2\ -3\ 2.5\ -1]^\top\overline{\omega}(t)$, where

$$\overline{\omega}(t)=\begin{cases}1 & 0\le t\le 1\\ 0 & otherwise\end{cases}$$

is a pulse signal.

Fig.2. and Fig.3. give the state trajectories of the network, and energy trajectories of $z(t)$ and the disturbance $\omega(t)$.

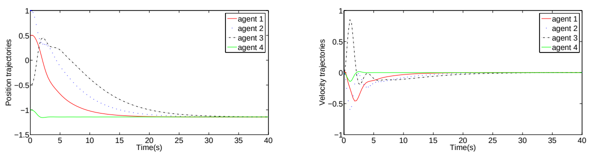

Fig. 2. Left: Position trajectories of the network

Right: Velocity trajectories of the network

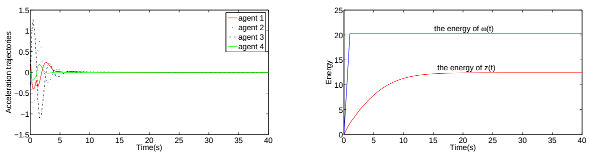

Fig. 3. Left: Acceleration trajectories of the network

Right: Energy trajectories of $z(t)$ and $\omega(t)$

Clearly, we can see that all agents achieve consensus while satisfying desired H_∞ performance.

6 CONLUSIONS

In this paper, we investigate the consensus problems in directed networks of high-order agents with disturbances. A new protocol is proposed with the consideration of model uncertainty. Sufficient conditions are derived to make all agents reach H_∞ consensus for three cases. Especially, the approach used in this paper does not need any model transformation. Finally, simulations are provided to show the effectiveness of the obtained results.

7 ACKNOWLEDGMENTS

This work was supported by the National 973 Program (2012CB821200) and the NSFC (61134005, 60921001, 90916024, 91116016).

REFERENCES

[1] Vicsek T, Czirok A, and Ben-Jacob E, et al (1995), Novel Type of Phase Transition in a System of Self-driven Particles, Physical Review Letters, 75(6):1226-1229

[2] Jadbabaie A, Lin J, and Morse AS (2001), Coordination of Groups of Mobile Autonomous Agents Using Nearest Neighbor Rules, IEEE Trans. Automatic Control, 48(6):988-1001

[3] Olfati-Saber R and Murray RM (2004), Consensus Problems in Networks of Agents with Switching Topology and Time-delays, IEEE Trans. Automatic Control, 49(9):1520-1533

[4] Ren W and Atkins E (2005), Consensus Seeking in Multi-agent Systems under Dynamically Changing Interaction Topologies, *IEEE Trans. Automatic Control*, 50(5):655-661

[5] Ren W, Moore K and Chen Y (2006), High-order Consensus Algorithms in Cooperative Vehicle Systems, Int.Conf. on Networking, Sensing and Control, 457-462

[6] Jiang F, Wang L and Xie G (2010), Consensus of High-order Dynamic Multi-agent Systems with Switching Topology and Time-varying Delays, J Control Theory Appl, 8(1):52-60

[7] Lin P and Jia Y (2009), Further Results on Decentralised Coordination in Networks of Agents with Second-order Dynamics. IET Control Theory and Applications, 3(7):957-970

[8] Lin P, Jia Y and Li L (2008), Distributed Robust H_∞ Consensus Control in Directed Networks of Agents with Time-delay. System & control letters, 57(8):643-653

[9] Lin P and Jia Y (2010), Robust H_∞ Consensus Analysis of a Class of Second-order Multi-agent Systems with Uncertainty. IET Control Theory and Applications, 4(3):487-498

[10] Godsil C and Royle G (2001), Algebraic Graph Theory. New York: Springer-Verlag

