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A Pattern Formation Mechanism of a Cellular Automaton Evolving on a Mutual Determination Rule of Variables and a Dynamics

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Abstract: The present study has an interest on complexity produced by removing the original boundary between different classes of concepts and it removes a boundary between a state variable and its dynamics by interacting them circularly on a two dimensional cellular automaton. It clarifies that removing the boundary creates the complexity of changing simple periodic orbits into attractive ones and proves a theorem about a period which characterizes periodic behaviors observed in simulations.

Keywords: Cellular automaton, Boundary, Complexity, Periodic attractor

1 INTRODUCTION

The cellular automaton is a computation model driven by a simple rule which describes the time evolution of a state of each cell on a lattice space. Despite its simplicity, it is much applied to simulating from life phenomena to physical ones such as crystal growth, turbulence and so on. As a typical example of cellular automaton, "life game" invented by Conway is very popular [1]. His model is thought to be able to simulate various growth patterns of living systems therefore it is called Life game. Life game's rule is quiet simple: a state of a cell at the next time is determined by the total number of active cells in its neighborhood. Namely the future states are directly determined by the past ones (the state is driven by itself); the dynamics is invariant on time. Namely there is a boundary to discriminate the state variable and its dynamics. Most of the cellular automata are modeled by the same formulation.

Meanwhile, the present study has an interest on complexity produced by removing the boundary between different class of concepts. In immunology, Jerne removed the discrimination between antibody (recognizer) and antigen (recognized entity) whereby built a new immunological theory called idiotype network theory [2]. In computer science, Lisp computer language opened a possibility of new calculation called λ calculation by removing the boundary between function (rulers) and dummy arguments (subject) [3]. In game theory, we tried to remove a boundary between player's move and its strategy by introducing a meta-rule to interact them circularly [4][5]. Masumoto and Ikegami achieved openness of a strategy space by rebuilding a framework of the game by λ calculus [6].

This study focuses on removing a boundary between a state variable and its dynamics by introducing a circular interaction between them.

This paper is composed of six sections. The second section proposes a standard two dimensional cellular automaton driven by a basic rule without interaction between state variables and its dynamics and discusses its behaviors. This standard model provides a control against a new model considered the interaction. The third section proposes new two dimensional cellular automaton, which introduces a reciprocal determination process between a state variable and its dynamics. The firth section discusses an analogous relationship between the new model and a spatial game model. In the fifth section, considering the fact that the space-temporal patterns of the new model has some 1-,3-,6- and 9-period attractors, it proves two lemma and one theorem about a period of these attractors. The last section discusses how difference introducing the reciprocal determination process makes on the dynamical behaviors.

2 ONE WAY DETERMINATION FROM A DYAMICS TO ITS VARIABLES

We consider a two dimensional square lattice with a size of its side, N. To designate a site of each cell, we prepare a horizontal and vertical axis with a suffix *i*, *j* respectively and place the origin at the top left of the lattice. The suffixes *i* and *j* take an integer value from 0 to *N*-1. We let a symbol $A^r_{i,j}$ to denote a state variable of a lattice cell (*i*, *j*) at the time *r* and the variable takes which of the two states:0 and 1. As a dynamical system of state variable $A^r_{i,j}$, we consider a linear system as the followings:

$$A^{r+1}_{i,j} = F(\vec{b}, \vec{A}^r_{i,j}) \equiv \vec{b} \cdot \vec{A}^r_{i,j} \cdot, \tag{1}$$

where $\vec{b}$ is a vector of coefficient parameters and is defined as $\vec{b} = (b_0, b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8)$ but $b_k = 1, b_i = 0 (i \neq k)$; $\vec{A}^{r+1}_{i,j}$ is defined as $(A^r_{i-1,j-1}, A^r_{i,j-1}, A^r_{i+1,j-1}, A^r_{i+1,j}, A^r_{i+1,j+1}, A^r_{i,j+1}, A^r_{i-1,j+1}, A^r_{i,j})$. As shown in Fig. 1, each component of the vector $\vec{A}^r_{i,j}$ arranges in a clockwise fashion from the upper left of the site (*i*,*j*)

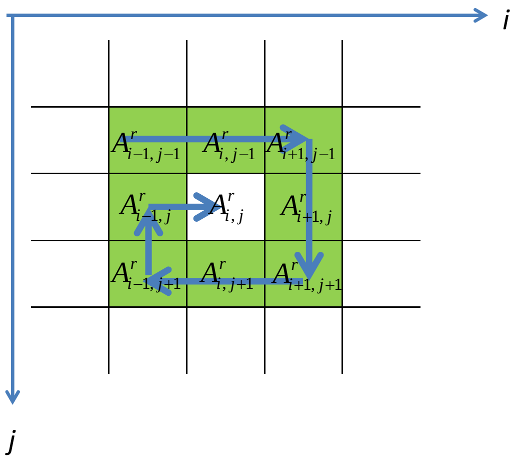

Fig.1. Arrangement of each component of the vector $\vec{A}^r_{i,j}$

2.2 Dynamic behaviors

Assuming the periodic boundary, a time evolution of spatial-temporal pattern of state variables is easily imagined from the model's definition. Only periodic motions in one direction which is determined by the vector $\vec{b}$ are observed. For example, when *k* is one, any spatial-temporal pattern moves downwards. In another case of *k* is three, any pattern moves leftwards. A period *T* is determined by *N* and is given as divisor of *N*. For example, *N* is four, *T* is one, two and four.

3 RECIPROCAL DETERMINATION BETWEEN VARIABLES AND ITS DYNAMICS

This study interests a reciprocal determination process between the state variables and its dynamics thus considers to update the parameter $\vec{b}$ through the time evolution of the state variables $\vec{A}^r_{i,j}$.We define a function $g(\vec{A}^r_{i,j})$ to calculate the sum of all cells' state in the Moore neighborhood of the cell (*i*,*j*) as

$$g(\vec{A}^r_{i,j}) = \sum_{i'=i-1}^{i+1} \sum_{j'=j-1}^{j+1} A^r_{i',j'} \quad . \tag{2}$$

Then through the value of $g(\vec{A}^r_{i,j})$ we update the parameter $\vec{b}$ as following:

When $g(\vec{A}^r_{i,j})$ is equal with *k*,

$$b^r_i = \begin{cases} 1 & (i = k) \\ 0 & (i \neq k) \end{cases} \tag{3}$$

After all, Eq.(1) results in

$$A^{r+1}_{i,j} = F(\vec{b}^r, \vec{A}^r_{i,j}) \equiv \vec{b}^r \cdot \vec{A}^r_{i,j}. \tag{4}$$

Fig.2 shows a situation that the state variable and its dynamics determine circularly.

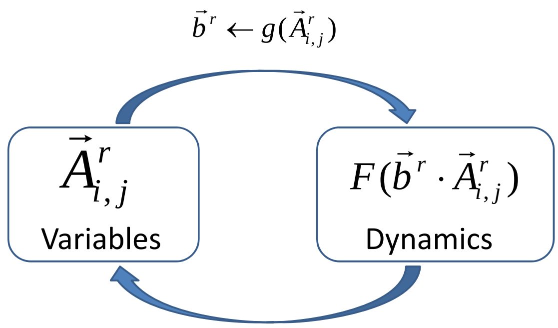

Fig.2 Reciprocal determination process between variables and its dynamics

4 A VIEW FROM SPATIAL GAME MODELS

Our proposed model is a particular kind of spatial game systems. In the common spatial games, a player has a game with each of its neighborhood players and in the next round uses the move of the highest scoring player among them.

We mention the correspondence relationship between the common spatial game model and our proposed model. We let the state variable $A^r_{i,j}$ to represent a move of a player at the site (*i*,*j*) at a certain time *r* thus a player's move is 0 or 1.

When the player (*i*,*j*) has a game with each of its Moore neighborhood players, our model determines the highest scoring player by the function $g(\vec{A}^{r}_{i,j})$. Concretely speaking, when the value of g is k, our model let the highest scoring player to be the one in the k-th site in the clockwise fashion from the top-left of the site (i,j). Fig. 3 shows a case of k is 4. Meanwhile, we can construe determining $A^{r+1}_{i,j}$ by Eq. (4) as for the player (i,j) to copy the highest scoring player's move as its own move at the next round, which is known as the copy strategy in the common spatial games.

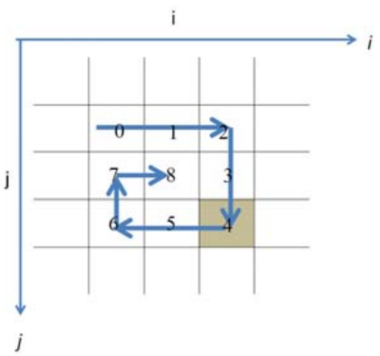

Fig. 3 A rule to assign k to each site. When k is 4, gray colored cell is selected as the highest scoring player's site.

In the present game, what sort of game situation must be established for a player to win? Only when not only the player but also all of the neighborhood players choose the same move 1, the player can be the winner.

The uniform state of all players taking the specific move 0 or 1 is unstable. Because when one player takes a opposite move in the uniform state, the system unable to return the original state.

5 THEOREMS ABOUT PERIODIC BEHAVIORS

We examined all of spatial temporal patterns observed in the case of the system size is 3 and clarified the emerged patterns are periodic and they are classified into 1, 3, 6 and 9 period. Fig. 4 is a typical example of the observed periodic behaviors. This section discusses why the observed period is limited to multiples of 3. We first mention a lemma on determination of the period. For the proof, we define two symbols: P^r and $S_{L,L'}$. Let P^r to be a configuration pattern of all players' action at the round r. Let $S_{L,L'}$ to be a translation to shift a configuration pattern P by L and L' in a vertical and horizontal direction respectively.

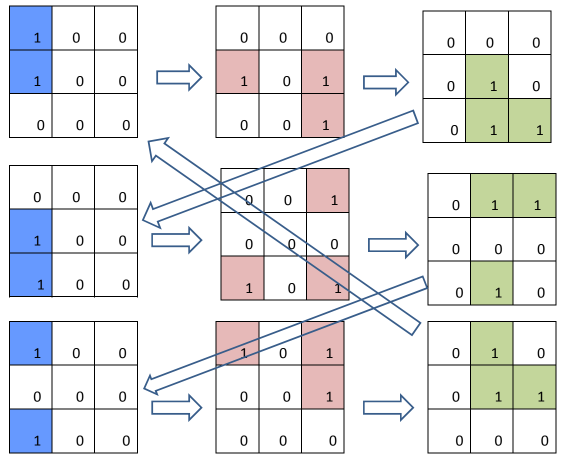

Fig. 3 Period-9 behavior

Lemma 1

Assume L, L' and n such that $S_{L,L'}P^1 = P^{n+1}$ exist, t hen $S_{L,L'}P^r = P^{n+r}$ for any r succeeds.

Proof

The assumption $S_{L,L'}P^1 = P^{n+1}$ succeeds thus for any i, j,

$$A^1_{i,j} = A^{n+1}_{i+L,j+L'}. \qquad (5)$$

Eq. (5) leads to

$$\vec{A}^1_{i,j} = \vec{A}^{n+1}_{i+L,j+L'}. \qquad (6)$$

And from Eq. (6),

$$g(\vec{A}^1_{i,j}) = g(\vec{A}^{n+1}_{i+L,j+L'}) \qquad (7)$$

Due to Eqs. (6) and (7), for any i, j

$$A^2_{i,j} = A^{n+2}_{i+L,j+L'} \qquad (8)$$

Therefore,

$$S_{L,L'}P^2 = P^{n+2} \qquad (9)$$

Through the same discussions, for any r,

$$S_{L,L'}P^r = P^{n+r} \qquad (10)$$

On the basis of lemma 1, we prove a theorem on the period of the dynamics.

Theorem

Assume $S_{L,L'}P^k = P^{n+k}$ and $S^r_{L,L'}P^1 = P^1$ then the period T is given as $n \times r$.

Proof

By the first assumption,

$$P^{2n+1} = S_{L,L'}P^{n+1} = S^2_{L,L'}P^1 \qquad (11)$$

Applying S to Eq. (11) for r-2 times,

$$P^{r\times n+1} = S^r_{L,L'}P^1 \qquad (12)$$

By the second assamption and Eq. (1),

$$P^{r\times n+1} = P^1 \qquad (13)$$

So that

$$T = r \times n. \tag{14}$$

The following lemma answers how r is determined.

Lemma 2
Assume $S^r_{L,L'}P^1 = P^1$, $L \neq 0$ and $L' \neq 0$ then
$r = L.C.M\left(L.C.M(N,|L|)/|L|, L.C.M(N,|L'|)/|L'|\right).$
But if $L(L') = 0$ then
$r = L.C.M(N,|L|)/|L|$ $(L.C.M(N,|L'|)/|L'|)$
where $L.C.M(x,y)$ represents least common multiple of x and y.

Referring to lemma 2, we try to answer why the period is the multiple of 3 when the system size N=3.
All possibilities of (L, L') on the parallel translation $S^r_{L,L'}$ is (1,0),(0,1),(1,1),(2,0),(0,2),(2,1),(1,2) and (2,2).

1. $(L, L') = (1,0), (01)$
   $r = L.C.M(3,1) = 3.$
2. $(L, L') = (1,1)$
   $r = L.C.M\left(L.C.M(3,1), L.C.M(3,1)\right)$
   $= L.C.M(3,3) = 3$
3. $(L, L') = (2,0), (0,2)$
   $r = L.C.M(3,2)/2 = 6/2 = 3.$
4. $(L, L') = (2,1), (1,2)$
   $r = L.C.M\left(L.C.M(3,2)/2, L.C.M(3,1)/1\right)$
   $= L.C.M(3,3) = 3$
5. $(L, L') = (2,2)$
   $r = L.C.M\left(L.C.M(3,2)/2, L.C.M(3,2)/2\right)$
   $= L.C.M(3,3) = 3$

In all the cases, r is 3 thus based on the theorem the period is limited to the multiple of 3. This result is consistent with the fact that the observed period is 3, 6 and 9.

6 DISCUSSIONS

The main interest of the present study is what is the characteristic behaviors created by the reciprocal determination process between the variable and its dynamics. To clarify this question, we prepared two models: the former is not taken the reciprocal determination process between the variable and its dynamics into consideration and the latter is done, and examined the difference of behaviors between the former and the latter. The both models have periodic orbits but only the latter's periodic orbit is "attractive". Fig. 5 displays 3-period attractor and some part of its basin structure and shows the process of 6 different spatial patterns suctioned into the 3-periodic orbit.

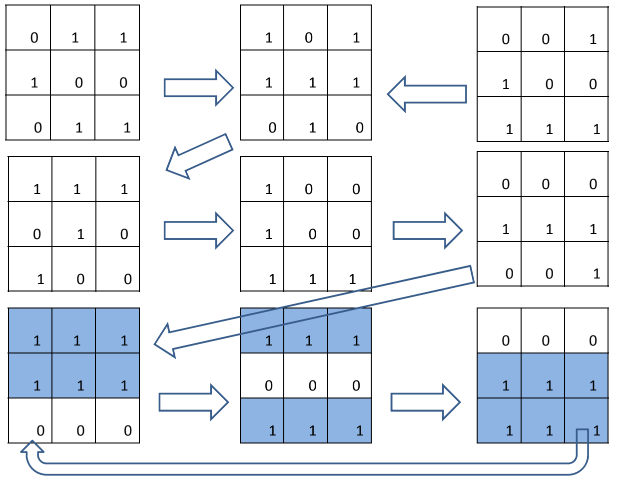

Fig. 5 3-periodic attractor

As a future work, we will clarify why non attractive periodic orbits transmutes into attractive ones by introducing the reciprocal determination process between the variable and its dynamics.

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