

Performance evaluations of adaptive strategies in self-repairing network

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Abstract: This paper studies a relationship between an adaptive strategy and its payoff. In a recent study, the adaptive strategy has already proposed to evaluate how much the strategy obtains a payoff on average against not only itself but also other strategies. We study the adaptive strategy in two examples: a prisoner's dilemma and self-repairing network. We study the prisoner's dilemma by focusing on two parameters: benefit and cost. We reveal a condition when the strategy gets the highest adaptive measure against other strategies in the prisoner's dilemma game. Further, we apply the analysis to the self-repairing network with spatial strategies. We investigate the adaptive strategy in the self-repairing network by simulations. We revealed that the adaptive strategies get the high payoff which minimize the standard deviations in the simulations.

Keywords: self-repairing network, adaptive strategies, spatial strategies, game theory, dynamic environments

1 INTRODUCTION

Autonomous distributed systems composed of agents need to adapt changes of environments. In the autonomous distributed systems, the agents pursue their own profits to achieve assigned their own goals. The agents need to change their behavior to adapt conditions in the dynamic environments. The agents determine next actions based on their own strategies. However, decision makings of the agents will affect each other. The policy of the agent behavior is represented as a strategy in game theory. The agents need to choose the strategy that earns the high profits as much as possible and determine their next actions based on their strategies.

There are various kinds of agents in information networks. The agents, connected each other with a network structure, interact with other agents based on their own strategies. The agents will encounter other ones having not only same strategies but also different strategies. For the agents, to choose the strategies earning the high payoff to specific strategies is risky, since, the agents face with a lot kind of the other agents selecting different strategies. The neighbor agents of the agent also will affect itself as the environment. The environments dynamically change according to interactions among them. Therefore, the agents need to have the strategies that obtain the high profits on average against other strategies.

An earlier study has already proposed a concept of an adaptive strategy [1] of which how well the strategies perform against other strategies. The performance of the strategies is evaluated as adaptive measures based on interaction results of the payoffs. In the study, the authors introduced

two models as calculation examples: prisoner's dilemma [2] and self-repairing network [3, 1]. The evaluation results of the strategies from the viewpoint of the adaptive strategy showed that the strategies earning the highest payoff do not correspond to that ones getting the highest adaptive measures. The study has reported that the adaptive strategies would perform well against other strategies although they do not obtain the highest payoffs.

The related works on the adaptive strategies are fault-tolerant strategies [4] and evolutionary stable strategies [5, 6]. However, both notions aim to investigate robust strategies against other strategies. Other related studies aim to construct a good strategy (strong strategy) than other strategies. Further, they compared with the proposed and other strategies by simulations [7, 8, 9, 10]. However, the focus of the adaptive strategies is to investigate adaptiveness of the strategies that earn the high payoff on average against other strategies.

An aim of this paper is to investigate two issues on the adaptive strategies. Firstly, we analyze the iterated prisoner's dilemma game as a typical example. We reveal a certain condition in which an adaptive measure of a trigger strategy exceeds that one of always defection strategy. Secondly, we study relationships between the adaptive measures and variances of the payoffs earned by the strategies. The earlier study [1] showed the strategies earning the highest adaptive measures obtains the high payoffs on average against other strategies. For this issue, we consider a self-repairing network as another example in the autonomous distributed systems. We study the self-repairing network composed of the

agents choosing spatial strategies by numerical simulations.

In Section 2, we give a formal definition of adaptive strategies, and we consider parameter conditions on the adaptive strategies in the iterated prisoner's dilemma with three simple strategies. In Section 3, we introduce and define the self-repairing network for simulations. In Section 4, we evaluate the self-repairing network from the viewpoint of the adaptive strategies by simulations. In Section 5, we discuss the significance of the adaptive strategies on the basis of the simulation results. In Section 6, finally, we state our conclusions in this paper.

2 ADAPTIVE STRATEGY

2.1 Definition

The concept of adaptive strategies and its formal definition has already introduced in the former paper [1]. The concept of the adaptive strategies is proposed in order to construct the autonomous distributed systems which work well on average in uncertain environments. The concept of the adaptive strategies incorporates fundamental two factors: (a) cooperative and (b) self-tolerance. Cooperative means that the strategies need to behave cooperative to maintain their performance against the opponents because the agents would encounter various kinds of strategies in information networks. Self-tolerance means that the agents need to cooperate with their neighboring agents because if they defect from others who have the same strategies, they would lose future opportunities to get a higher payoff. These two fundamental factors require agents to cooperate with not only themselves but also other agents.

For designing the strategies, performance of the strategies how much they behave well against other strategies is evaluated as an adaptive measure. This measurement is defined by payoffs and strategies. Let S denote a set of strategies. Let N denote the cardinality of the strategy set S . Let i, j , and k denote natural numbers used for numbering the strategies in the strategy set. Strategy s_i is expressed as one strategy in the set S numbered as i . Let $E_p[s_i|s_j]$ denote the expected payoff of strategy s_i against s_j . Let $E_m[s_i]$ be the expected payoff of the strategy s_i for all strategies.

Let denote the adaptive measure $E[s_i]$ of a strategy s_i its strength. The adaptive measure is represented as follows:

$$E[s_i] = \frac{1}{NM} \sum_{s_j \in S} E_p[s_i|s_j] E_m[s_j] \quad (1)$$

$E_m[s_j]$ is expressed as follows:

$$E_m[s_j] = \frac{1}{NM} \sum_{s_k \in S} E_p[s_j|s_k] \quad (2)$$

The symbol M represents the maximum total payoff in the payoff matrix of the game. Formula (2) represents the

averaged performance of strategy s_j for all strategies. The adaptive measure in Formula (1) is expressed as the product of $E_p[s_i|s_j]$ and $E_m[s_j]$ to evaluate whether strategy s_i achieves the higher payoff against strategy s_j even if s_j achieves a high payoff for other strategies. The adaptive measure will decrease when strategy s_i achieves the smaller payoff and even if strategy s_j achieves the higher averaged payoff. In contrast, the adaptive measure will increase when strategy s_i gets the larger payoff and strategy s_j obtains the higher averaged payoff. The range of the measure can be normalized from zero to one. The strategy is adaptive if the measure is close to one, and it is not adaptive if the measure is close to zero. The adaptiveness of the strategies can be evaluated by comparing with the measures.

2.2 Example of adaptive strategy analysis

We present a calculation example of the adaptive measures of the adaptive strategies in the iterated prisoner's dilemma (IPD). This example uses simple three strategies: All-C (always cooperate), All-D (always defect) and Trigger (it cooperates until an opponent defects in a previous round, otherwise defects). We reveal a condition in which the adaptive measure of Trigger strategy exceeds All-D strategy's one where a discount rate is a variable parameter. The IPD is a temporal extension of the prisoner's dilemma. The prisoner's dilemma [2] is a one-shot game whereas the IPD is a repeated game. We consider a two-player game in the infinite IPD. The players determine their actions of cooperation or defection simultaneously without prior consultation before the game. The payoff for each player is determined by combinations of moves among the players.

The payoff matrix is shown in Table 1. The payoff matrix is defined with two parameters b (benefit) and c (cost). Each symbolic value in the table satisfies the conditions $T > R > P > S$ and $2R > T + S$. We assume a discount rate w of the payoff for every round. The discount rate can be used for calculating the discounted payoff regarded as the future payoff.

The expected payoffs of the infinite IPD for the three strategies can be calculated theoretically [2]. The theoretical results shown in Table 2 are calculated from the payoff matrix shown in Table 1.

Table 1. Payoff matrix for the proponent in the prisoner's dilemma with two parameters

		Player 2	
		C	D
Player 1	C	$R = b - c$	$S = -c$
	D	$T = b$	$P = 0$

Table 2. Expected payoff for the proponent in the infinite IPD with two parameters

		Opponent		
		All-D	Trigger	All-C
Proponent	All-D	0	b	$\frac{b}{1-w}$
	Trigger	c	$\frac{b-c}{1-w}$	$\frac{b-c}{1-w}$
	All-C	$\frac{-c}{1-w}$	$\frac{b-c}{1-w}$	$\frac{b-c}{1-w}$

From Table 2, we simply obtain the averaged payoffs for all strategies.

$$E_m[All\ D] = \frac{2b - 3c}{3b} \quad (3)$$

$$E_m[Trigger] = \frac{1-w}{3b} \left(c + \frac{2(b-c)}{1-w} \right) \quad (4)$$

$$E_m[All\ C] = \frac{2-w}{3} \quad (5)$$

We focus on the two strategies of Trigger and All-D. The adaptive measures of these two strategies can be calculated by the definitions. Therefore, we obtain adaptive measures of both strategies as follows:

$$E[All\ D] = \frac{1}{9b} \left(cw^2 + 4cw - 2bw - 6c + 4b \right) \quad (6)$$

$$E[Trigger] = \frac{1}{9b} \left(bcw^2 + (4bc - c^2)w + 6c^2 - 12bc + 4b^2 \right) \quad (7)$$

We calculate the condition of the discount rate which leads the adaptive measure of Trigger to be the highest value.

$$\begin{aligned} E[Trigger] &> E[All\ D] \\ 2b^2w + c^2w &> 6c^2 - 6bc \end{aligned} \quad (8)$$

We replace the term c/b with r . Then, we obtain

$$w > \frac{6r(r-1)}{(\sqrt{2}-r)(\sqrt{2}+r)} \quad (9)$$

According to the above result, the adaptive measure of Trigger exceeds the adaptive measure of All-D when the discount rate exceeds the threshold. Fig. 1 shows a curve of discount rate against the cost-benefit ratio calculated by Formula

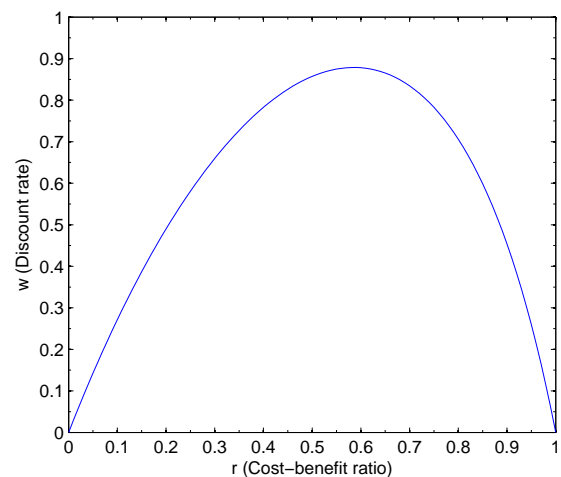


Fig. 1. Discount rate curve against cost-benefit ratio in Eq. (9).

(9). The highest discount rate is 0.88 when the cost-benefit ratio is 0.69. The discount rate increases as the cost-benefit ratio grows because the defective strategy dominates the cooperative strategies where the the discount rate is smaller than its highest value. On the other hand, the discount rate decreases as the cost-benefit ratio grows to be larger than 0.69 because the benefit is small due to the cost to be large.

3 SELF-REPAIRING NETWORK

In order to consider the relationship between the highest adaptive measures and the payoffs, we apply the adaptive strategy analysis to the self-repairing network with spatial strategies [11]. The self-repairing network is a network model in which agents repair other agents by copying their contents mutually. We consider the self-repairing network by game-theoretic approach model [3, 12]. We adopt the proposed former model defined by replicator dynamics and master equations.

For simplicity, we assume the network large, well-mixed population. Each agent has a state either normal or abnormal. We denote the frequency of normal agents (abnormal agents) by ρ_N (ρ_A). Each agent determines the next action: *repair* (C, cooperation) or *not repair* (D, defection). We denote the frequency of repair agents (not repair agents) in the network by ρ_C (ρ_D). The agents determine their actions based on their strategies.

The repairing is done between the two agents. The two agents are randomly chosen from the network, then they repair each other based on their decisions. The repair success rate is different by the state of the agent. We denote the repair success rate of the normal and abnormal agents by α and β respectively. We assume to simplify the model that the repair by the normal agents is always successful ($\alpha = 1$). The

repaired agent becomes normal if the repairing is successful, otherwise the repaired agent becomes abnormal. We assume that the normal agents become abnormal by spontaneous failure. We denote the failure rate by λ .

From above definitions, the repairing dynamics is expressed as follows:

$$\frac{d\rho_N}{dt} = \alpha\rho_N\rho_A\rho_C + \beta\rho_A^2\rho_C - (1 - \alpha)\rho_N^2\rho_C - (1 - \beta)\rho_A\rho_N\rho_C - \lambda\rho_N \quad (10)$$

Each term represents the repair interactions between the two agents. The first and second terms express the increase of normal agents while the third and fourth terms represent the decrease of the abnormal agents.

The agents have their own resources consumed by repairing. The normal agents (abnormal agents) have the maximum resources b_N (b_A). If the normal agents (abnormal agents) do the repairing, then the agents consume their resources only c_N (c_A). The agents deal with the remained resources as the available resources. The resources of the agents are filled in the beginning of the step.

The agents determine their next actions based on the strategies. We introduce spatial strategies [11] to the agents. The spatial strategies determine the actions involving the effects of neighborhood decision making. Especially, kC strategy of the spatial strategies is introduced into the agents. In an original definition of kC strategy, it determines the next action *repair* if the number of defeaters exceed the threshold k . We modify the strategy from the original one to the strategy one that determine the next actions based on the frequency of silent agents ρ_D in the previous step. The range of the threshold k is from 0 to 1. We denote the frequency of the kC strategy by ρ_k .

For calculating the payoff of each strategy, we define a function $\rho_C^k(\rho_C^p)$ that returns a binary value 0 or 1 according to the frequency ρ_C^p of the repair agents in the previous step. The function $\rho_C^k(\rho_C^p)$ is defined as follows:

$$\rho_C^k(\rho_C^p) = \begin{cases} 0 & (\rho_C^p > k) \\ 1 & (\rho_C^p \leq k) \end{cases} \quad (11)$$

Let denote $W(kC)$ the expected payoff of the strategy kC . The $W(kC)$ is expressed as follows:

$$W(kC) = \rho_k(\rho_N((b_n - c_n)\rho_C^k(\rho_C^p) + b_n(1 - \rho_C^k(\rho_C^p))) + \rho_A((b_a - c_a)\rho_C^k(\rho_C^p) + b_a(1 - \rho_C^k(\rho_C^p)))) \quad (12)$$

Let denote \overline{W} expected payoff of the whole network. The expected payoff \overline{W} is expressed as follows:

$$\overline{W} = \rho_{k_1}W(k_1C) + \rho_{k_2}W(k_2C) \quad (13)$$

We consider the self-repairing network with the agents implementing two different strategies. The strategies are denoted as k_1C and k_2C . Therefore, the dynamics of the competition among the strategies is expressed as follows:

$$\frac{d\rho_{k_1}}{dt} = (W(k_1C) - \overline{W})\rho_{k_1} \quad (14)$$

Therefore, the dynamics of the self-repairing network with the spatial strategies are expressed by the two equations (10) and (14) are expressed.

4 SIMULATIONS

We make a round-robin tournament of the self-repairing network with spatial strategies. We evaluate the spatial strategies in the self-repairing network from the viewpoint of the adaptive strategy. We choose two strategies from the strategy set and run the simulations with the two strategies. Parameters for simulations are shown in Table 3. After simulations, we calculate the adaptive measure for each strategy.

Table 4 shows the round-robin tournament results for the failure rate $\lambda = 0.01$. According to the result, the $0.6C$ strategy obtains the highest averaged payoff among the strategies, while the $1C$ strategy gets the highest adaptive measure and smallest standard deviation. The $0.6C$ strategy gets the high payoff against other strategies except the $1C$ strategy. The $1C$ strategy obtains the high payoff against not only itself but also other strategies although $1C$ strategy agents are repaired by other strategy agents. This difference makes the $1C$ strategy get the highest adaptive measure.

Table 5 shows the round-robin tournament result for the failure rate $\lambda = 0.04$. According to the result, the $0.6C$ strategy obtains the highest averaged payoff among the strategies, while the $0.6C$ strategy gets the highest adaptive measure and smallest standard deviation. This result is contrary to the round-robin tournament one where the failure rate $\lambda = 0.01$. For both cases, 0.6 strategy obtains the higher payoff than $0.6C$. Unfortunately, the $0.6C$ strategy causes the decrease of the payoff when it repairs itself. The $1C$ strategy makes its standard deviation the smallest. In other words, the strategy obtaining the highest adaptive measure also gets the smallest standard deviation.

According to two cases, the standard deviations of the adaptive strategies become the smallest values. The adaptive strategies earn the high payoff on average and make the standard deviations the smallest.

5 DISCUSSION

In the previous section, we compared the two results where the different failure rates are used. The round-robin

Table 3. Parameters for numerical simulations

Parameter	Name	Value
T	Step	500
α	Repair success rate by normal agents	1.0
β	Repair success rate by abnormal agents	0.1
b_N	Available resources of normal agents	1.0
c_N	Repair cost of normal agents	0.25
b_A	Available resources of abnormal agents	0.5
c_A	Repair cost of abnormal agents	0.25
λ	Failure rate	0.01, 0.04
k	k value for kC strategy	0.0-1.0 (0.2 step)
$\rho_k(0)$	Initial frequency of kC strategy	0.5
$\rho_C(0)$	Initial frequency of cooperators	0.5

Table 4. Adaptive measure and statistical values (The failure rate $\lambda = 0.01$)

	0.0	0.2	0.4	0.6	0.8	1.0	Averaged payoff	Standard deviation	Adaptive measure
0.0	0.424	0.424	0.424	0.002	0.002	0.002	0.213	0.211	0.046
0.2	0.424	0.424	0.424	0.002	0.002	0.002	0.213	0.211	0.046
0.4	0.424	0.424	0.424	0.002	0.002	0.002	0.213	0.211	0.046
0.6	0.845	0.845	0.845	0.358	0.358	0.003	0.542	0.325	0.151
0.8	0.714	0.714	0.714	0.358	0.358	0.003	0.477	0.265	0.137
1.0	0.534	0.534	0.534	0.531	0.528	0.260	0.487	0.102	0.168

Table 5. Adaptive measure and statistical values (The failure rate $\lambda = 0.04$)

	0.0	0.2	0.4	0.6	0.8	1.0	Averaged payoff	Standard deviation	Adaptive measure
0.0	0.357	0.357	0.357	0.002	0.002	0.002	0.180	0.178	0.033
0.2	0.357	0.357	0.357	0.002	0.002	0.002	0.180	0.178	0.033
0.4	0.357	0.357	0.357	0.002	0.002	0.002	0.180	0.178	0.033
0.6	0.712	0.712	0.712	0.292	0.292	0.003	0.454	0.276	0.105
0.8	0.582	0.582	0.582	0.292	0.292	0.003	0.389	0.216	0.093
1.0	0.507	0.507	0.507	0.505	0.504	0.252	0.464	0.094	0.136

tournaments demonstrated the considerable results. The standard deviations of the strategy obtaining the highest adaptive measure are the smallest values among the strategies. In one of the two cases, the strategy does not correspond to that one obtaining the highest averaged payoff. On the other hand, the strategy obtaining the highest adaptive measure correspond to that one obtaining the highest averaged payoff. However, the adaptive strategies could get the high payoff on average against given strategies set and make its standard deviation the smallest in them.

In information network, the agents encounter various kinds of agents implementing different strategies. The agents need to adapt behavior of the other agents in order to pursue their goals. According to the simulation results, the adaptive strategies could obtain high payoffs on average against other strategies. Further, their lack of the payoffs compared with the averaged payoffs would be the smallest or small values. The neighborhood of the one agent can be regarded as

its environment. For the agents, to adapt for a specific environment is not reasonable because of they would face and need to follow the changes of the environments. The adaptive strategies would adapt to different environments by the simulation results.

6 CONCLUSIONS

We studied a relationship between an adaptive strategy and its payoff in both examples of a prisoner's dilemma and self-repairing network. In the prisoner's dilemma, we considered the adaptive strategies what conditions make the strategies get the highest adaptive measure. We analyze the adaptive strategies in the prisoner's dilemma game with simple three strategies and certain parameters. For the analysis, this paper focused on the two parameters: benefit and cost. We revealed the condition of which the strategy gets the highest adaptive measure against other strategies in the prisoner's dilemma.

Further, we considered the adaptive strategies in the self-repairing network with spatial strategies. The dynamics of the self-repairing network involve interactions on a mutual repairing and strategy update. We investigated the adaptive strategy in the self-repairing network by numerical simulations. In the results, we showed that the standard deviations of the strategies obtaining the highest adaptive measures are the smallest values among the strategies even the strategies do not earn the highest averaged payoffs. We revealed that the adaptive strategies would get the high payoff which minimize the standard deviations.

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REFERENCES

- [1] Tokumitsu M (2011) Adaptive strategies: Toward a design of adaptive systems for dynamic environments. In: Proceedings of the International Conference on Management of Emergent Digital EcoSystems. New York, NY, USA: ACM, MEDES '11.
- [2] Axelrod R (1984) *The Evolution of Cooperation*, Basic Books. New York.
- [3] Ishida Y, Tokumitsu M (2008) Asymmetric interactions between cooperators and defectors for controlling self-repairing. *Lecture Notes in Computer Science* 5179: 440–447.
- [4] Pelc A (2010) Fault-tolerant strategies in the iterated prisoner's dilemma. *Information Processing Letters* 110: 389–395.
- [5] Weibull JW (1997) *Evolutionary Game Theory*. The MIT. Press.
- [6] Smith JM (1982) *Evolution and the Theory of Games*. Cambridge, UK: Cambridge University Press.
- [7] Beaufils B, Delahaye J, Mathieu P (1996) Our meeting with gradual, a good strategy for the iterated prisoner's dilemma. In: *Artificial Life V: Proceedings of the Fifth International Workshop on the Synthesis and Simulation of Living Systems*.
- [8] Beaufils B, Delahaye JP, Mathieu P (1998) Complete classes of strategies for the classical iterated prisoner's dilemma. In: Porto VW, Saravanan N, Waagen D, Eiben AE, editors, *Evolutionary Programming VII*. Berlin: Springer, pp. 33–41.
- [9] Nowak M, Sigmund K (1993) A strategy of win-stay, lose-shift that outperforms tit-for-tat in the Prisoner's Dilemma game. *Nature* 364: 56–58.
- [10] Tzafestas E (2000) Spatial games with adaptive tit-for-tats. In: *Proceedings of the 6th International Conference on Parallel Problem Solving from Nature*. London, UK: Springer-Verlag, PPSN VI, pp. 507–516.
- [11] Ishida Y, Mori T (2005) Spatial strategies in a generalized spatial prisoner's dilemma. *Artificial Life and Robotics* 9: 139–143.
- [12] Tokumitsu M, Ishida Y (2011) An adaptive control technique for a connection weight of agents in a self-repairing network. *Lecture Notes in Computer Science* 6557: 44–55.