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Identifying Cellular Automata Rules Using Local Rule Network From Spatiotemporal Patterns

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Abstract: There are some methods of Identifying rules of cellular automata (CA) from their spatiotemporal patterns. But these methods do not consider relations of local rules. The relations include spatial ones and temporal ones. For example, a rule which satisfy the mass conservation law has spatial constraints for how to apply local rules to each cells. A local rule network represents spatial or temporal associations between local rules which are identified from spatiotemporal patterns. This paper address to construct local rule networks from spatiotemporal pattern and propose rule identification methods using the networks.

Keywords: Cellular Automata (CA), Rule Identification, Graphical Model

1 INTRODUCTION

Visualizations are used extensively for understanding behaviors of various phenomena. For example, Bayesian networks which are visualized causal associations by graphs (networks), apply to various area such as image recognition [1, 2]. Meanwhile, Wolfram classified 256 elemental rule of one dimensional cellular automata (CA) [3] into four classes by their behaviors [4]. Wolfram found differences between global behaviors of 256 rules by constructing directed graphs. The graphs represent transitions between possible configurations in certain space size. A Wolfram's graph can represent global behaviors of a rule but we cannot know local behaviors from the graph. Therefore we propose *local rule network* which consists of local rules as nodes and their transitions as arcs. A local rule network can represent associations between local rules. A rule of CA consists of local rules and each cell changes own state by applying a local rule. A local rule describes local a behavior which determine next state of a cell in a certain neighborhood pattern.

CA are known as which can generate various complex patterns from simple rules and uses for modeling of phenomena which generate spatiotemporal patterns. For example, ASEP[5] which is known as a traffic model is one of the one dimensional probabilistic cellular automata (PCA).

On the other hand, some researches deal with identifying cellular automata rules from analyzing their spatiotemporal patterns[6, 7]. Ichise proposed a method which identify a one dimensional deterministic and probabilistic cellular automaton rule from a spatiotemporal pattern generated by a computer simulation. Authors also applied the identifying method to actual phenomena and spatial prisoner's dilemma[8, 9]. The identifying method is simple which identifies local rules by scanning spatiotemporal patterns. The method can identify rules when spatiotemporal patterns are sufficiently big and simple such as patterns of Wolfram's 256 elemental rules. But the method cannot completely identify some rules which have hidden (embedded) rule such as ASEP's mass conservation law[10]. Because the method does not consider association between local rules. Therefore this paper propose local rules network which are constructed from spatiotemporal patterns and visualize local behaviors of these spatiotemporal patterns.

Section 2 explains cellular automata. Then local rule network is defined in section 3. Section 4 shows a example of patterns which the method cannot completely identify rules and discusses using local rule network.

2 CELLULAR AUTOMATA

Cellular automaton (CA) consists of cells which arranged on a lattice. Each cell has certain states and changes next own state following current neighborhood pattern. In this paper we restrict ourselves a case of one dimensional cellular automata. A time evolution of a cell is described by formula (1). Where n_i^t is a neighborhood pattern of cell i and r is neighborhood radius. When $r = 1$ current left, right and own three neighborhood determine the next state. A mapping $f : N \to S$ is a rule of CA where S is space of possible states and $N(= S^{r+1})$ is a set of possible neighborhood patterns. In a elemental CA (ECA) which is two states, three neighborhood and one dimensional CA has eight local rules because the number of possible neighborhood patterns is 2^3. Then there are $2^8 = 256$ rules in ECA.

$$\begin{aligned} s_{i+1}^t &= f(n_i^t) \\ n_i^t &= s_{i-r}^t, \cdots, s_i^t, \cdots, s_{i+r}^t \end{aligned} \qquad (1)$$

Wolfram numbered these 256 ECA rules. The numbers are defined by formula (2). Where $c_j (\in S)$ is a state after applying local rule j such as table 1. The formula (2) also can consider that below of table 1 is binary string. Table 2 shows an example of rule 90.

$$R = \sum_{j=0}^{8} c_j 2^j \quad (2)$$

Table 1. A rule of 2-states,3-neighbors,one dimensional cellular automata (ECA). The rule consists of eight local rules.

n_i^t	111	110	101	100	011	010	001	000
s_i^{t+1}	c_7	c_6	c_5	c_4	c_3	c_2	c_1	c_0

Table 2. An example of rule90. When we consider the bottom column to binary number, it corresponds to $(01011010)_2 = 90$

n_i^t	111	110	101	100	011	010	001	000
s_i^{t+1}	0	1	0	1	1	0	1	0

Fig. 1 shows a spatiotemporal pattern generated by computer simulation of rule 90. Where white cells indicate 0 and black are 1. Space size is 100 and simulation time steps are 100. Initial condition is given by random.

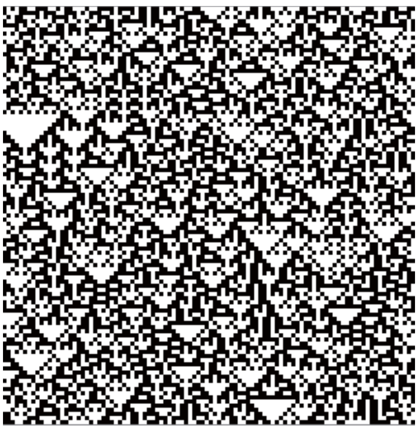

Fig. 1. A spatiotemporal pattern which is generated by rule90. White cells indicate 0 and black is 1. Space size is 100 and simulation time steps are 100. Initial condition is given by random.

3 LOCAL RULE NETWORK

Formula (3) defines a local rule network $G = (V, E)$. Fig. 2 shows a small example of local rule network. V is a set of nodes which are local rules and E is a set of arcs which are transitions (l_j, l_k) from local rule l_j to l_k. Where n_j is jth neighborhood pattern. For example, n_7 corresponds to $(1, 1, 1)$ in Table 1. When the neighborhood pattern of cell i at time step t is $(0, 0, 0)$, time step $t+1$ is $(0, 1, 0)$ and the state of cell i at time step $t+2$ is 0, there is a transition from local rule $((0, 0, 0), 1)$ to $((0, 1, 0), 0)$. Each arc has transition frequency $w(j, k)$ as weight. Then transition frequencies satisfy condition (4).

$$\begin{aligned} V &= \{l_j | l = (n_j, c_j), n_j \in N\} \\ E &= \{(l_j, l_k) | l_j, l_k \in V\} \end{aligned} \quad (3)$$

$$\sum_{l_k \in V} w(l_j, l_k) = 1 \quad \forall l_j \in V \quad (4)$$

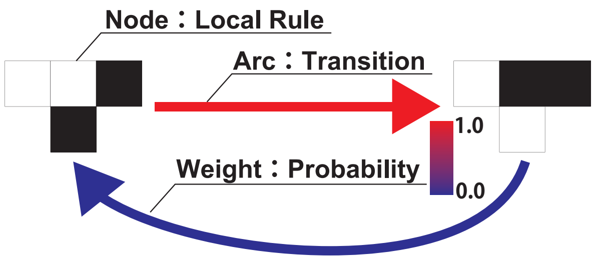

Fig. 2. An example of local rule network. Nodes correspond to local rules and arcs correspond to transitions between local rules. Each arc has transition frequency as a weight.

A local rule network is generated by scanning given a spatiotemporal pattern and finding local rules and their transitions. Then a transition frequency is obtained from the number of appearance. Fig. 3 shows a local rule network generated by a spatiotemporal pattern of rule 90 when space size is 1000, simulation time steps is 1000 and initial condition is random. Rule 90 is classified as class 3 which behaves chaos by Wolfram. Therefore rule 90 generates complex fractal patterns. Then the local rule network visualizes that there are many transitions and the rule is complex. Meanwhile, Fig. 4 shows a local rule generated by rule 5. This network has some hub local rules and it shows rule 5 is simple rule. Actually a spatiotemporal pattern of rule 5 is monotonous such as Fig. 5.

4 APPLICATION TO ASEP

ASEP is a traffic model and two states, three neighbors and one dimensional probabilistic CA. Each cell has a state, existing a car on the cell $(= 1)$or not existing $(= 0)$. Each car on the cell moves to front cell in probability p when front cell is empty. The number of cars is constant. Thus cars suddenly appears or disappears. It means that ASEP satisfies the mass

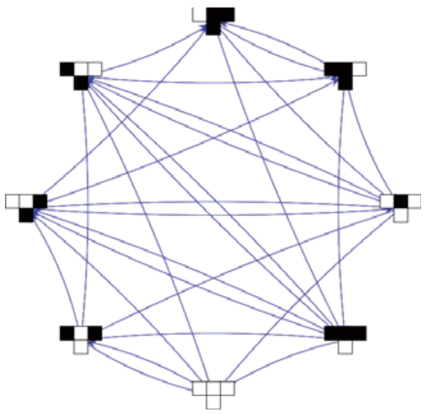

Fig. 3. A local rule generated from a spatiotemporal pattern of rule90. Each node indicates a two states and three neighbors local rule. White cells indicate 0 and black is 1.

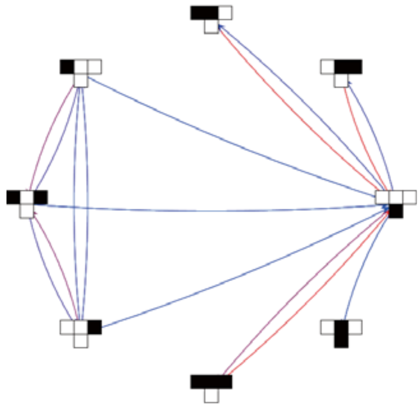

Fig. 4. A local rule generated from a spatiotemporal pattern of rule5. Each node indicates a two states and three neighbors local rule. White cells indicate 0 and black is 1. A spatiotemporal pattern is generated with 1000 space size and 1000 simulation time steps.

conservation law. But the mass conservation law cannot find a identified rule by the identifying method. While the method identify same rules from a pattern of satisfying the mass conservation law and not satisfying one [10]. Therefore we estimate whether spatiotemporal patterns satisfy the mass conservation law using local rule networks. Fig. 6 is generated by ASEP simulation and Fig. 7 is generated by a similar rule to ASEP but it does not satisfy the mass conservation law. These patterns shows major differences by comparison.

Fig. 8 is a local rule network generated from Fig. 6 and Fig. 9 is generated from Fig. 7. Fig. 9 shows that it has some transitions which do not appear in Fig. 8. These transitions break the mass conservation law. Thus local rule network can bring out differences between rules which identified as same rules by the identifying method.

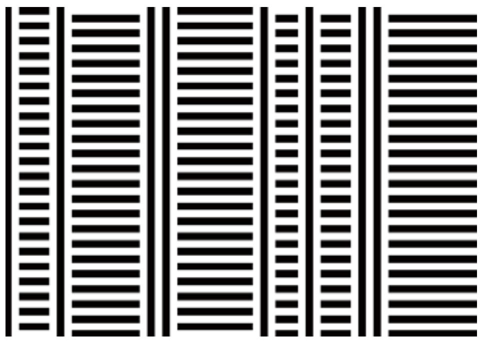

Fig. 5. A part of spatiotemporal pattern which is used for generating Fig. 4. White cells indicate 0 and black is 1.

5 CONCLUSIONS

This paper proposed a visualization of cellular automata rules from spatiotemporal patterns. A local rule network visualizes associations between local rules. Then we also shows an example of analysis in ASEP using local rule networks. Local rule network can bring out differences between rules which identified as same rules by the identifying method.

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Fig. 6. A spatiotemporal pattern of ASEP with $p = 0.8$, 100 space size and 100 simulation time steps. White cells indicate 0 and black is 1.

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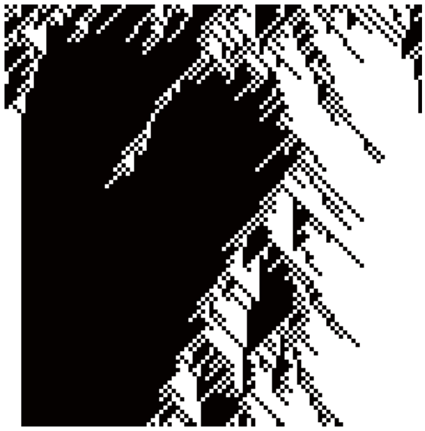

Fig. 7. A spatiotemporal pattern of general PCA with 100 space size and 100 simulation time steps. Its result of identification similar to Fig. 6 one. White cells indicate 0 and black is 1.

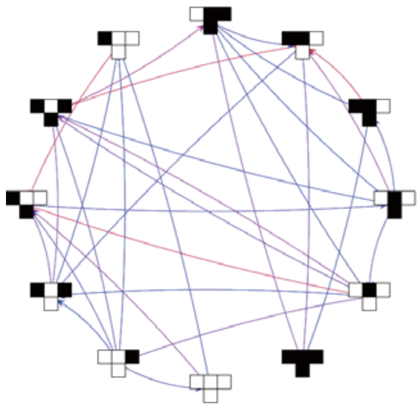

Fig. 8. A local rule network generated from Fig. 6.

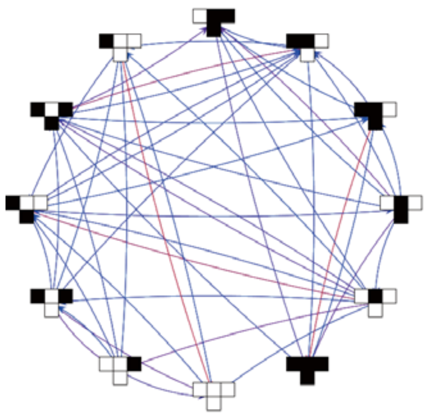

Fig. 9. A local rule network generated from Fig. 7.