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An analysis of spatial patterns in a spatial prisoner's dilemma

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Abstract: In the natural world, cooperative behavior emerges and assumes the crucial roles. Cooperative behavior means altruistic behavior and non-cooperative behavior (defection) means selfish behavior. Although cooperators emerge in the society, cooperation has not any advantage as compared to defection in rational terms. Earlier studies proposed many mechanisms to fill a gap between theoretical prediction and experimental evidence. As previous works, the authors studied the SPD that is spatial-temporal version of the Prisoner's Dilemma to investigate the maintenance mechanisms of cooperators. Then we observed a membrane formation as a mechanism that protects cooperation from invasion of defectors. The authors consider the effects of the interaction distance on the game payoff. In the present model, interacting with distant individuals pays a higher cost than interacting with adjacent individuals. In the SPD simulation, this paper shows that cooperators emerge easier by considering the effects of the interaction distance.

Keywords: cooperation, membrane formation, spatial generosity, spatial prisoner's dilemma

1 INTRODUCTION

In the natural world, cooperative behavior emerges and assumes the crucial roles. Cooperative behavior means altruistic behavior and non-cooperative behavior (defection) means selfish behavior. A cooperator pays a cost for someone's benefit. On the other hand, a defector does not pay a cost. Although cooperators emerge in the society, cooperation has not any advantage as compared to defection in rational terms. Instead of cooperating, each individual or player should defect pursing their benefit. Natural selection obviously favors defection over cooperation and prevents evolution of cooperation unless it has mechanisms. Earlier researchers proposed many mechanisms to fill a gap between theoretical prediction and experimental evidence. Some of them are based on the game theory, especially the Prisoner's Dilemma (PD) game.

The PD has been widely studied not only in international politics but also in the evolutionary biology [1]. Nowak and May invented a spatiotemporal version of the PD named Spatial Prisoner's Dilemma (SPD) [2, 3]. The SPD also is the model as to discuss cooperative behavior. In the SPD, the effects of a spatial structure that protects cooperators and sustains are well-known results. After the invention of the SPD, many types of the SPD that are more realistic than the conventional model have engendered. They have considered the realistic factors such as Tit-for-tat (direct reciprocity), indirect reciprocity, random graph networks, scale-free networks, heredity effects and social generosity effects [4 - 6].

As previous works, we studied the SPD that treats the spatial generosity, to investigate the effects of the social generosity on cooperation. Then the authors observed a membrane formation as a mechanism that protects cooperation from invasion of defectors and maintains cooperators in the SPD model with the spatial generosity.

The membrane works as a mechanism for protects cooperators from invasion from defectors. Moreover the membrane emerges even if the lattice structure (e.g., square lattice, hexagonal lattice) and neighborhood radius change.

The robustness of the membrane for the lattice topology and the neighborhood is already shown [7]. This investigation, however, was conducted on the SPD under the ideal conditions in which all of the neighbors have equivalent value. In the realistic interaction between individuals, to interact with distant individuals has a higher cost than adjacent individuals.

In the present work, we use the SPD model that takes into account the interaction distance and the spatial generosity to investigate the effects of the distance between individuals. In the previous studies, the interaction distance has not considered. Thus, the player profits whether it is playing with the adjacent players or the distant players. In the real situation, however, playing with the distant players requires the high costs than the adjacent players. We assume that the payoffs which the players receive by playing mutual interaction are affected and discounted as the interaction distance increases.

This paper shows that cooperators emerge easier by considering the effects of the interaction distance.

This paper is organized as follows. In the next section, we explain the games models. In the section 3, we describe the membrane formation. Subsequently, we show the main results of our simulations. In the last section, we conclude and summarize this paper.

2 MODEL

2.1 Prisoner’s Dilemma

The Prisoner’s Dilemma (PD) is a one of fundamental models of the game theory. It is played just once by two players with two actions (Cooperation, C, or Defection, D). The players decide action simultaneously whether to cooperate or to defect. If both players cooperate, players receive payoff R (Reward), whereas if both players defect, players receive payoff P (Punishment). If one player cooperates and one player defects, the cooperator receives payoff S (Sucker) and the defector receives payoff T (Temptation). In the PD game, the payoffs must satisfy the equation $T > R > P > S$. In this model, whatever the opponent player chooses, the defection is the optimal choice for individuals clearly.

Therefore, both players always choose defection and receive payoff P, which is lower than that received when both choose cooperation.

The Iterated Prisoner’s Dilemma (IPD) is temporal extension of the PD. In IPD, the PD is carried out repeatedly where $2R > T + S$. For IPD, some earlier researches proposed the temporal strategy. Axelrod reported a tit-for-tat (TFT) strategy is the best strategy among other temporal strategies [1]. The TFT strategy consists of playing C in the first round and from then on copying the opponent’s action of the previous round. The TFT strategy contains temporal generosity as an element.

2.2 Spatial Prisoner’s Dilemma

The Spatial Prisoner’s Dilemma (SPD) is a spatio-temporal version of the PD. Our model generalized SPD by introducing a spatial strategy. Each player takes over the each site of a two-dimensional lattice. There have an action and a spatial strategy, and receive a score. The spatial strategy determines the next action according to the spatial pattern of action of their neighbor. A player plays the PD game with its neighbors (eight adjacent players when *Moore* neighborhood and neighborhood radius is one), and changes its strategy to the strategy that earns the highest total score among the neighbors’. Table 1 shows the payoff matrix of the PD. In our simulations, R, S, T and P are set to 1, 0, b and 0, respectively, below following the Nowak-May’s simulations.

Table 1. The Payoff Matrix of the Prisoner’s Dilemma Game. R, S, T and P are payoff for the player 1. A single parameter b is used following the Nowak-May’s simulation.

		Player 2	
		C	D
Player 1	C	$R(1)$	$S(0)$
	D	$T(b)$	$P(0)$

The SPD is conducted in the following way with N players simultaneously.

1. Initial arrangement phase: an action and a strategy of each player are determined randomly with equal probability.
2. Action renewal phase: the next action of player will be determined by its strategy based on the neighbors’ action (excluding the player itself).
3. Interaction phase: the players play the PD game with the neighbors and player itself, and then receive the payoff according to the payoff matrix in Table 1. However, the receiving payoff is discounted by an effect of the interaction distance. The score for each player is calculated by summing up all the scores and add the sum to the current score of the player.
4. Strategy renewal phase: after (2-3) is repeated q (strategy update cycle) times, the strategy will be chosen from the strategy with the highest score among the neighbors and the player itself.

2.2.1 Weight of the Interaction Distance

We weight the payoff that the player receives from the PD game to incorporate the effects of the interaction distance. The weight w ($0 < w \leqq 1$) discounts the player’s payoff according to the decreasing function of the interaction distance. Hence, as distance between players increases, the weight decreases and a discount factor increases. The net payoffs of player x is given by

$$P_x = w \times \textit{game payoffs}.$$

In this paper, we proposed the two types of the decreasing function of the interaction distance. One is,

$$w = -\frac{d}{r+1} + 1, \tag{1}$$

where d is a interaction distance and r is a neighborhood radius. The other one is,

$$w = \frac{1}{(d+1)^2}. \tag{2}$$

For both definitions, the payoff's weight is one, when only the player plays the PD game with itself.

2.2.2 Spatial Strategy and Spatial Generosity

We conduct the SPD simulation with the spatial strategy and spatial generosity. The player decides next action according to its strategy and the pattern of the neighbors' action.

The spatial strategy determines player's action only based on the pattern of the neighbors' action. However, the patterns of the neighbors' action make an enormous amount. For example, in *Moore* neighborhood and its radius is 1, the patterns of the neighbors' action amount to 2^9 patterns. Hence, the 2^9 rules are required. For simplicity, we consider only "totalistic spatial strategy" that depends on the number of defectors in the neighbors, not on their positions. To represent a strategy, let l be number of the D actions of the neighbors excluding the player itself.

As a typical totalistic spatial strategy, we define *k*-D strategy. The player chooses D if $l \geqq k$, otherwise chooses C (Fig. 1). For instance, All-C (always cooperate) corresponds to the 9-D strategy, whereas All-D (always defect) corresponds to the 0-D strategy.

The *k*-D strategy can be regarded as a spatial version of the TFT where *k* indicates the spatial version of generosity.

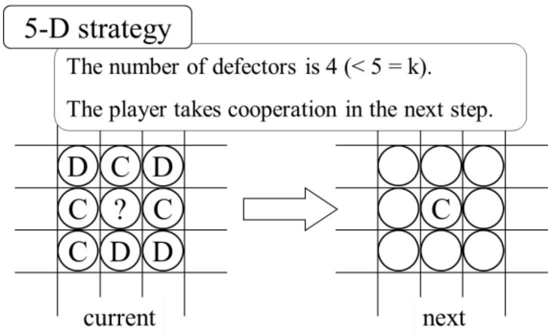

Fig. 1. An example of the spatial strategy. When *k* (spatial generosity) is five, the player tolerates to four defectors in the neighbors. Therefore, the center player will be cooperator in this situation.

3 MEMBRANE FORMATION

We already simulated the interaction between All-D (always D) vs. *k*-D instead of All-D vs. All-C (Nowak-May's simulation). In All-D vs. *k*-D interaction, we already observed the membrane as a mechanism for protects cooperation from invasion of defectors of All-D.

The membrane is composed of only the *k*-D defectors. Fig 2 shows an example of the membrane formation in *Moore* neighborhood.

In this paper, we also argue the effects of the weight of the interaction distance on the membrane formation.

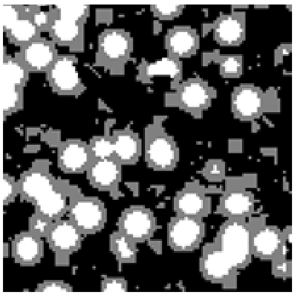

Fig. 2. The membrane formation generated by SPD simulation. *Black cells* indicate All-D players. *White and gray cells* indicate C and D players of *k*-D. In this snapshot, *k* is 6. The cooperators are covered by the membrane (*gray*).

4 SIMULATION

We carried out a SPD simulation in order to investigate the effects of the incorporating of the interaction distance with the parameter list in Table 2.

Table 2. Parameter list for the simulations

Parameter name	Description	Value		
$L \times L$	Size of lattice	700×700		
N	Number of players	490,000		
T	Number of steps	1,000		
r	Neighborhood radius	1	2	3
b	Bias of defectors of the payoff matrix in Table 1	1.8001, 2.0001	1.5625, 2.0001	1.4849, 2.0001

In a certain neighborhood radius, we carried out the two types of the simulation. In these simulations, we set a different parameter *b* (bias for defectors of the parameter matrix in Table 1).

1. The parameter *b* is set to be a minimum value that allows All-D to expand in the sea of cooperators from a single All-D player following the earlier study
2. The parameter *b* is set to be 2.0001. It does not satisfy the condition of the IPD ($2R > T + S$). This value favors a situation that defectors drive out cooperators in the IPD. Thus, it is favorable situation for the defector.

As a result, we observed the membrane formation whether the payoff is weighted (Fig. 3). Fig. 4 plots the time evolution of the cooperators when $r = 1$ and (1). In 6-D simulation, most players cooperate. This is because the All-D player could not invade and take over the cooperators

(Fig. 5). The cooperators receive higher payoff than defectors by an impact of self-interaction increasing.

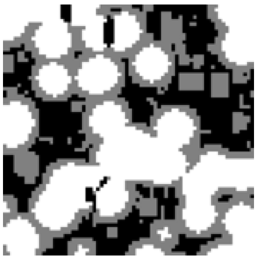

Fig. 3. The membrane formations emerge when $k = 6$, step $t = 10$, and the expression (1).

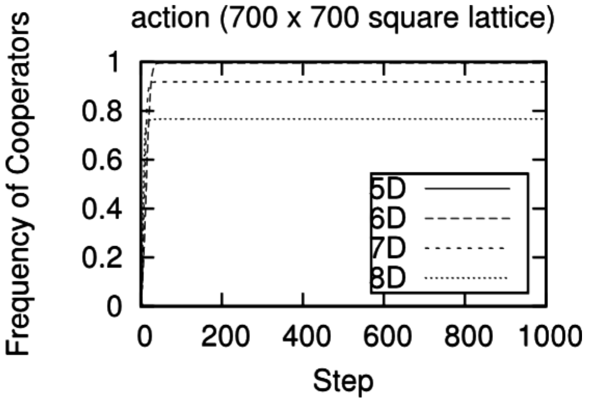

Fig. 4. The time evolution of the frequency of the cooperators. In the 6-D simulation, most players cooperate.

Fig. 5. This snapshots show step $t = 100$ of Fig. 3 simulation. The color code is same as in Fig. 3. The cooperators are not invaded by the defectors and remain.

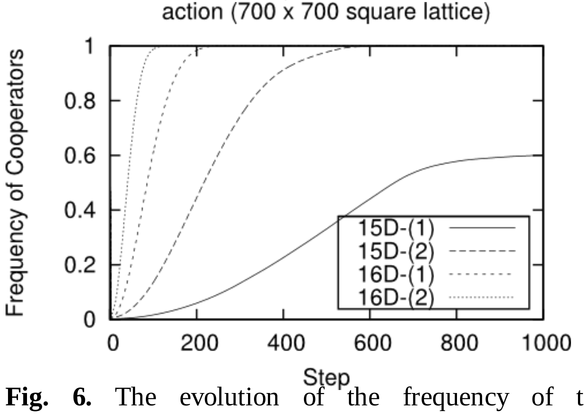

Fig. 6. The evolution of the frequency of the cooperators when $r = 2$. as the effect of the interaction distance increasing, the promotion of cooperation is preferred to defection.

Fig. 6 shows the time evolution of the cooperators when neighborhood radius $r = 2$. The cooperators easily emerge when (2). The interaction distance strongly influences a payoff weight when (2) than (1). Thus, as the effect of the interaction distance increasing, the promotion of cooperation is preferred to defection.

4 CONCLUSION

In conclusion, we have investigated the effect of the interaction distance on cooperation. In order to investigate this effect, we assumed that interacting with the distant individuals requires the high costs than the adjacent individuals. The payoff is discounted as the interaction distance increases.

In our model, cooperation easily emerges by incorporating the payoff’s weight. The impact of the self-interaction increases by the introducing the payoff weight.

Moreover, we observed the membrane formation whether the payoff weight is introduced.

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