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Propose of the use to Alternative Gramian for the Controller Order Reduction

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Abstract: Robust controllers have feature of corresponding to the model error and the disturbance. However, obtained controller is generally high-order. The controller order is desirable in small from a standpoint of cost and reliability. The purpose of our work is to propose of effective controller order reduction method. In this paper, we propose a controller order reduction method using alternative Gramian. We were applied this method to conventional methods. Further, we confirmed effect using the numerical example. As a result, our proposed method found some efficacy in stability and performance degradation of a closed loop system. However, when applied to other controlled objects, these was relatively-ineffective. Therefore, this method admit of improvement. As one of these improvements, we are thinking that to use Genetic Algorithm (GA) for optimal solution derivation of the riccati inequality.

Keywords: controller order reduction, gramian, H_∞ controller

1 INTRODUCTION

Traditional modern control theory had a gap by between theory and reality, because it was not considering the model error. For resolve this problem, the robust control theory was corresponding to the model error and disturbance. However, the robust controller such as H_∞ controller is generally high-order. Generally, the controller order is desirable in small from a standpoint of cost and reliability. Therefore, it is necessary obtaining low-order controller that could be suppress to performance degradation. In this paper, we focus attention on the balanced realization truncation method.

The balanced realization truncation method is one of the controller reduction. In addition, the blocked balanced realization truncation method is also one of the controller reduction method, this method is considering the characteristics of the closed-loop system input-output. In these methods, we obtain controllability Gramian and observability Gramian from Lyapunov equations. When the Gramian replaced by a solution of stringent condition, we think that could effective coordinate transformation. We replaced the Gramian by the solution of stringent condition, it will be referred to as "Alternative Gramian".

In this paper, we propose a controller order reduction method using Alternative Gramian. Alternative Gramians were used a solution of three kinds, Lyapunov inequations, Riccati equations and Riccati inequations. Moreover, controller order reduction methods were applied to the balanced realization truncation method and the block balanced realization truncation method. We confirmed the effectiveness of the proposed method by numerical examples.

2 METHOD

2.1 Controllability and Observability Gramian

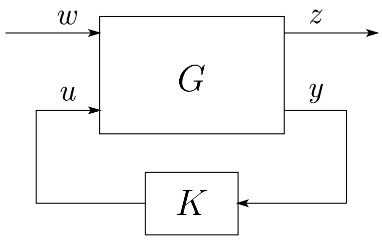

Fig. 1. The closed-loop system

Consider the closed-loop system shown in Fig.1. In the figure, w has shown the external input, z has shown the controlled output, u has shown the control input, and y has shown the measured input. Let, system G is the generalized plant, and K is the controller. G and K are represented as following.

$$G = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] = C(sI - A)^{-1}B + D \quad (1)$$

$$K = \left[\begin{array}{c|c} A_k & B_k \\ \hline C_k & D_k \end{array}\right] \quad (2)$$

where, the matrix A_k assumed to be stable. Solution Q of the following Lyapunov equation is called the "observability Gramian".

$$A_k^T Q + Q A_k + C_k^T C_k = 0 \quad (3)$$

In the same way, solution P of the following Lyapunov equation is called the "controllability Gramian".

$$A_kP + PA_k^T + B_kB_k^T = 0 \tag{4}$$

The eigenvalues size of the controllability Gramian and observability Gramian shows ease of the control and observation for corresponding states. However, the Gramian of alone cannot accurately assess effect of input-output relation. In order to solve this problem, we introduce a special realization of the controllability Gramian and observability Gramian.

2.2 Balanced realization truncation method

When controller K, controllability Gramian P and observability Gramian Q are shown in equation (2)-(4), we assume $P \geq 0$ and $Q \geq 0$. Here, we convert to $\hat{y} = Ty$ using the nonsingular matrix T. At this time, K is

$$K = \left[\begin{array}{c|c} \hat{A}_k & \hat{B}_k \\ \hline \hat{C}_k & \hat{D}_k \end{array}\right] = \left[\begin{array}{c|c} TA_kT^{-1} & TB_k \\ \hline C_kT^{-1} & D_k \end{array}\right] \tag{5}$$

Furthermore, Gramians will change as follows:

$$\hat{P} = TPT^T \tag{6}$$

$$\hat{Q} = (T^{-1})^TQT^{-1} \tag{7}$$

In particular, when these are the minimal realization, can be the following:

$$TPT^T = (T^{-1})^TQT^{-1} = \Sigma \tag{8}$$

$$\Sigma = \mathrm{diag}(\sigma_1, \sigma_2, \ldots, \sigma_n) \tag{9}$$

This is called a balanced realization. And, $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_n \geq 0$ are called Hankel singular values. Corresponding state to the zero eigenvalue of P and Q can eliminate from the transfer function matrix. When the new realization of K is split as follows

$$K = \left[\begin{array}{c|c} TA_kT^{-1} & TB_k \\ \hline C_kT^{-1} & D_k \end{array}\right] = \left[\begin{array}{ccc|c} A_{11} & \cdots & A_{1j} & B_1 \\ \vdots & & \vdots & \vdots \\ A_{i1} & \cdots & A_{ij} & B_i \\ \hline C_1 & \cdots & C_j & D \end{array}\right] \tag{10}$$

We obtain the low-order system K_r by truncation of low-impact parts from the high-order system K. The obtained low-order system K_r is following.

$$K_r = \left[\begin{array}{c|c} A_{11} & B_1 \\ \hline C_1 & D \end{array}\right] \tag{11}$$

This method called the balanced realization truncation method. This is one of the famous model reduction method. The controller order reduction methods had been proposed various methods such as proposed of the frequency weight based on the balanced realization method. When using the balanced realization method to the controller order reduction, it is considering the input-output characteristics. However, it was not consider stability of the closed-loop system. Wherein, the blocked balanced realization was proposed for considering the input-output characteristics of the closed-loop system. The blocked balanced realization has been considered by the input-output characteristics of the closed-loop system by to think about overall the closed-loop system.

2.3 Proposed Method 1

As previously explained, controllability and observability Gramian are solution of Lyapunov equations (2) and (3). The balanced realization using these gramians have been considered for the mutual similarity with the original system. However, it has not considered for the performance degradation of low-order system. In addition, the traditional balanced realization was not consider stability of the closed-loop system. Here, in order to consider the H_∞ characteristics, we propose to use Riccati equations as alternative of controllability and observability Gramian. For example, when this method is applied by the block balanced realization, used Riccati equations are as following.

$$\begin{aligned}(A + BR^{-1}D^TC)^TQ + Q(A + BR^{-1}D^TC) \\ + QBR^{-1}B^TQ + C^T(I + DR^{-1}D^T)C = 0\end{aligned} \tag{12}$$

$$\begin{aligned}(A + BR^{-1}D^TC)P + P(A + BR^{-1}D^TC)^T \\ + PC^TR^{-1}CP + B(I + D^TR^{-1}D)B^T = 0\end{aligned} \tag{13}$$

Where, R is

$$R = \gamma^2I - D^TD$$

or

$$R = \gamma^2I - DD^T$$

To exist solutions of equation (12) ,(13) and to be less than γ to the H_∞ norm of the system are equivalent. Therefore, when the block balanced realization is applied by these solution,the H_∞ norm of the closed-loop system will can be expected to less than γ.

2.4 Proposed Method 2

Secondly, to further reduce the performance degradation, we propose to use solution of the Lyapunov inequality on alternative of gramian. For example, when this method is applied by the block balanced realization, used Lyapunov inequality are as following.

$$AP + PA^T + BB^T \leq 0 \tag{14}$$

$$A^TQ + QA + C^TC \leq 0 \tag{15}$$

Lyapunov equation has a unique solution exists, but there is no unique solution in the Lyapunov inequality. We give the guidepost for minimization of solution. When make the Trace(P) and Trace(Q) smaller, it is expected that truncation part of the Hankel singular values can be smaller. Therefore, when we find P and Q to minimize Trace(P) and Trace(Q) , the closed-loop system performance degradation will can reduce.

2.5 Proposed Method 3

Section 2.3 proposed that we use solution of the Riccati equation as alternative for the controllability and observability Gramian for to consider of the H_∞ characteristic. Moreover, Section 2.4 proposed also that we use solution of the Lyapunov inequality for to reduce the performance degradation of the closed-loop system used the low-order controller. In this section, we propose that we use solution of Riccati inequality for to reduce the performance degradation and to consider of the H_∞ characteristic. For example, when this method is applied by the block balanced realization, used Riccati inequality are as following.

$$\begin{bmatrix} AP + PA^T & PC^T & B \\ CP & -\gamma I & D \\ B^T & D^T & -\gamma I \end{bmatrix} < 0 \tag{16}$$

$$\begin{bmatrix} A^TQ + QA & QB & C^T \\ B^TQ & -\gamma I & D^T \\ C & D & -\gamma I \end{bmatrix} < 0 \tag{17}$$

To exist solutions of equation (16) ,(17) and to be less than γ to the H_∞ norm of the system are equivalent. Therefore, when this solutions are applied to the block balanced realization, the H_∞ norm of the closed-loop system will can be expected to less than γ. Also, the closed-loop system performance degradation will can reduce.

3 RESULT

As a numerical example, we treat the four-disk controller system. The state-space realization of generalized plant is at the left. In this paper, we design the H_∞ controller set at design specification γ=1.2. The obtained controller is the 8th-order, and this controller meets the specifications as follows:

$$\begin{aligned} \|F_l(G,K)\|_\infty &= \|G_{11} + G_{12}K(I - G_{22}K)^{-1}G_{21}\|_\infty \\ &= 1.1963 < 1.2 \end{aligned}$$

We reduce order of this controller from 7th-order to 1st-order. We showing in Table 1-3 the H_∞ norm of the closed-loop system that is including by reduced-order controller. Where, in Table 1-3, "BT" has shown the balanced realization truncation method, "WBT" has shown the balanced realization truncation method with frequency weight($W = G(I + KG)^{-1}$), "BBT" has shown the block balanced realization truncation method. Moreover, "no mark" has shown a method using Lyapunov equations (conventionally method), "Ric(eq)" has shown a method using Riccati equations (Proposed method 1), "Lyap (ineq)" has shown a method using Lyapunov inequalities(Proposed method 2), "Ric(ineq)" has shown a method using Riccati inequalities(Proposed method 3). "U" has shown unstable system.

$$A = \left[\begin{array}{cccccccc} -0.161 & -6.004 & -0.58215 & -9.9835 & -0.40727 & -3.982 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right],$$

$$B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$C_1 = \left[\begin{array}{cccccccc} 0 & 0 & 0 & 0 & 0.00055 & 0.011 & 0.00132 & 0.018 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$C_2 = [\ 0 \quad 0 \quad 6.4432 \times 10^{-3} \quad 2.3196 \times 10^{-3} \quad 7.1252 \times 10^{-2} \quad 1.0002 \quad 0.10455 \quad 0.99551\ ]$$

$$D_{11} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$D_{21} = \begin{bmatrix} 0 & 1 \end{bmatrix}, \quad D_{22} = 0$$

Table 1. H_∞ norm of $F_l(G, K_r)$

Order of K_r	BT	BT+Lyap(ineq)	BT+Ric(eq)	BT+Ric(ineq)
7	1.3211	U	1.3183	1.3240
6	1.2072	U	1.2055	1.2089
5	U	U	U	U
4	U	U	U	U
3	U	U	U	U
2	U	20.305	U	U
1	U	U	U	U

Table 2. H_∞ norm of $F_l(G, K_r)$

Order of K_r	WBT	WBT+Lyap(ineq)	WBT+Ric(eq)	WBT+Ric(ineq)
7	1.2482	7.9282	1.2548	1.2415
6	1.2015	2.4321	1.2029	1.1979
5	1.3018	13.613	1.3576	1.5902
4	1.3205	15.825	1.3354	1.2444
3	U	U	U	4.0781
2	U	U	U	2.3907
1	U	U	U	U

Table 3. H_∞ norm of $F_l(G, K_r)$

Order of K_r	BBT	BBT+Lyap(ineq)	BBT+Ric(eq)	BBT+Ric(ineq)
7	1.2462	12.684	1.2623	1.2038
6	1.2018	2.3397	1.2029	1.1959
5	U	145.07	U	1.4605
4	1.2336	7.7959	1.2412	1.1989
3	U	U	U	U
2	U	U	U	2.3056
1	U	U	U	U

From the Table 1, the method using Lyapunov inequality can be the most reduced order. However, the H_∞ norm of 2nd order by Lyapunov inequality is over 20, system performance degradation can not reduced in this method. In addition, when it used the balanced realization truncation method by alternative gramian that is solution of Riccati equations or Ricacati inequality, those are almost no variation by original balanced realization. Therefore, an alternative gramian unfitted for the balanced realization truncation method.

Next, from the Tabale 2, the method using Riccati inequality can be the most reduced order. Especially, when it was used Riccati inequality in 6th order, the H_∞ norm of the closed-loop system including the low-order controller is under γ. From this, we find that performance degradation is suppressed.

Finally, from the Table 3, the method using Riccati inequality can be the most reduced order. When the controller order is 7th, 6th and 4th, the H_∞ norm is under γ. In addition, In the 2nd order, closed-loop is stable, and performance degradation is most small. Therefore, we found significant effect by applying the solution of Riccati inequality to the block balanced realization from these results.

4 CONCLUSION

In this paper, we have proposed the controller order reduction method which used solution of the Riccati inequality etc. as substitute for the controllability grammian and the observability grammian. Moreover, this paper applied a proposed method to the block balanced realization etc, and looked at an effect of a proposed method by numerical examples. As a result, We were found that our proposed method obtained some efficacy in stability and performance degradation of a closed loop system. However, when applied to other controlled objects, these was relatively-ineffective. Therefore, this method admit of improvement. As one of these improvements, we are thinking that to use Genetic Algorithm (GA) for optimal solution derivation of the Riccati inequality.

REFERENCES

[1] G.Wang,V.Sreeram and W.Q.Liu,"Balanced performance preserving controller reduction",System and Control Letters 46,pp99-110 (2002).

[2] T.Mita,"H_∞ Seigyo-[H_∞ Control]",Tokyo,Japan :Shokodo (1994).

[3] K.Z.Liu,"Senkei Robust Seigyo-[Linear Robust Control]",Tokyo,Japan :Corona Publishing (2002).

[4] K.Zhou,J.C.Doyle and K.Glover,"Robust and optimal control, Prentice-Hall",Englewood Cliffs, NJ (1996).