

Adaptive Oscillation Control Scheme for a Wheeled Mobile Robot

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Abstract: In the case when wheeled mobile robots run fast on rough terrain, due to the robot body acceleration and oscillation, sensors mounted on the robot body may be destroyed. In this paper, we propose a scheme to reduce the body acceleration at any specified location on the robot body. In the proposed scheme, a combined ideal model is designed so that the location where acceleration performance becomes best can be moved easily by setting only two design parameters. Then, an adaptive controller is developed so that the behavior of the actual mobile robot tracks that of the combined ideal model. It is ascertained by numerical simulations that the body acceleration at any specified location can be improved easily.

Key Words: Wheeled mobile robot, Oscillation, Adaptive control.

1. INTRODUCTION

In extreme environment, for example, disaster areas and minefields, there are many dangerous tasks for human. Recently, in the environment, mobile robots working instead of human have been developed^{[1]-[4]}. In the case when robots go over steps, due to the vertical acceleration of the robot body, sensors mounted on the robot body may be destroyed or the measurement accuracy of sensors becomes worse. Nevertheless, the problem had not been considered yet. If the acceleration at each location on the robot body is controlled so as to be zero, that problem can be resolved. However, in such a case, the mobile robot cannot go over a slope. Therefore, the controllers are designed by using trial and error method so that the body acceleration can be suppressed at only one specified location on the robot body. In this case, well oscillation suppression of the acceleration can be realized for a sensor mounted on the robot body. If measurement accuracy is required for the other sensor, we have to redesign the controller. However, it will cost a lot of time.

To overcome the problem stated above, we proposed a robust oscillation control scheme^[5] that can suppress the vertical acceleration at a specified location on the robot body without any redesigning of controllers. For uncertainties of the system parameters, the excellent robust performance can be also achieved. However, there is a problem that the scheme still requires accurate information of some parts of system parameters.

In this paper, we propose an adaptive oscillation control scheme. The controller does not require any information of system parameters. At first, a combined ideal model is designed based on the state space description containing the body acceleration. In the combined ideal robot model, the location where the acceleration becomes small can be moved easily by setting only two design parameters. Next, an adaptive tracking controller is developed so that the actual robot can track the behavior of the designed combined ideal

robot model. In the proposed adaptive tracking control system, for any uncertainties of system parameters, it is shown that the tracking performance can be improved by setting design parameters. At last, the usefulness of the proposed controller is shown by carrying out numerical simulations.

2. WHEELED MOBILE ROBOT

The wheeled mobile robot is shown in Fig. 1, and Fig. 2 shows the simplified model of the wheeled mobile robot. To simplify the explanation below, each large wheel is labeled as 1, 2, and 3. Each large wheel has three small wheels. In the large wheels labeled 1 and 2, the small wheels are driving wheels. The wheeled mobile robot has the mechanism that each large wheel can be rotated by the motors set at the center axis of the large wheels 1, 2 and 3. The explanations of parameters are shown in Table 1.

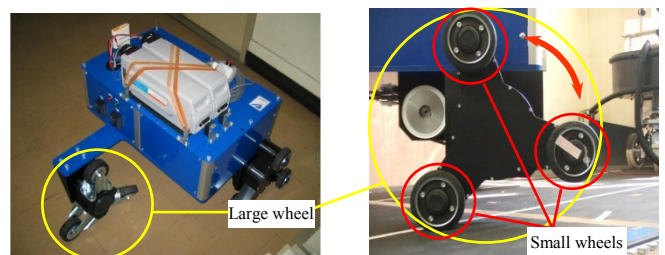


Fig. 1 Mobile robot and wheels.

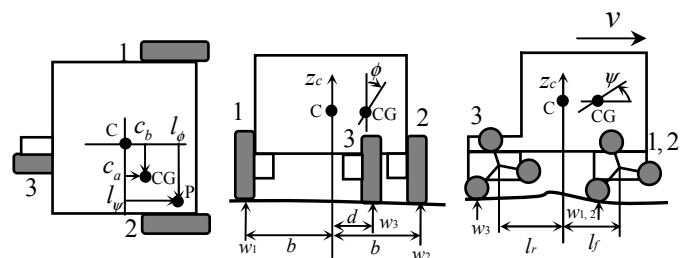


Fig. 2 Mobile robot model

Table. 1 Notation of wheeled mobile robot

| | |
|---------------------|--|
| C,CG | center and center of gravity of mobile robot body |
| z_c | vertical displacement at C |
| P | location on the robot body |
| ψ, ϕ | pitch and roll angle |
| x_1, x_2, x_3 | vertical displacement at the center of large wheels |
| w_1, w_2, w_3 | vertical displacement of road disturbance added to large wheels |
| v | longitudinal velocity of mobile robot |
| g | acceleration of gravity |
| m, i_ψ, i_ϕ | robot mass, pitch and roll moment of inertia of robot body |
| l_f, l_r, b, d | distance from C to large wheels |
| c_a, c_b | distance from C to CG |
| l_ψ, l_ϕ | distance from C to P |
| l_w | distance from the center of large wheel to the center of small wheel |
| k_a | voltage and torque conversion constant |
| r, k_G | reciprocal of armature resistance and gear ratio |
| j_{mL}, d_{mL} | moment of inertia and viscous damping constant between a motor and wheel center axis |

It is assumed that the pitch angle $\psi(t)$ and the roll angle $\phi(t)$ are small. Then, the dynamic equation of the wheeled mobile robot is given as follows.

$$\left. \begin{aligned} M_p(t)\ddot{\mathbf{x}}(t) + K_d(t)\dot{\mathbf{x}}(t) &= -K_b(t)\mathbf{u}(t) + D_p(t)\dot{\mathbf{w}}(t) - \mathbf{g}_a \\ \mathbf{x}(t) &= [x_1(t), x_2(t), x_3(t)]^T \\ \mathbf{w}(t) &= [w_1(t), w_2(t), w_3(t)]^T \\ M_c &= \text{diag}[m, i_\psi, i_\phi] \\ M_p(t) &= \left((HT_h)^T \right)^{-1} M_c (HT_h)^{-1} + K_j(t) \\ D_p(t) &= K_j(t) - M_p(t) \\ H &= \begin{bmatrix} 1 & l_f & -b \\ 1 & l_f & b \\ 1 & -l_r & d \end{bmatrix}, T_h = \begin{bmatrix} 1 & -c_a & -c_b \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ K_d(t) &= \theta_{kj} K_{j\theta}(t) \\ K_j(t) &= \theta_{kd1} K_{d\theta1}(t) + \theta_{kd2} K_{d\theta2}(t) \\ K_b(t) &= \theta_{kb} K_{d\theta}(t) \\ \theta_{kj} &= \frac{j_{mL}}{l_w^2}, \theta_{kd1} = \frac{j_{mL}}{l_w^3}, \theta_{kd2} = \frac{k_G^2 d_{mL} + r k_a^2}{k_G^2 l_w^2}, \theta_{kb} = \frac{r k_a}{k_G l_w} \\ K_{j\theta}(t) &= K_{d\theta2}(t) = \text{diag}[S_1^{-2}, S_2^{-2}, S_3^{-2}] \\ K_{d\theta1}(t) &= \text{diag}[C_1 S_1^{-4} \dot{x}_1(t), C_2 S_2^{-4} \dot{x}_2(t), C_3 S_3^{-4} \dot{x}_3(t)] \\ K_{d\theta}(t) &= \text{diag}[S_1^{-1}, S_2^{-1}, S_3^{-1}] \\ S_i &= \sin(\theta_{wi}(t)) + \cos^{-1} \frac{3}{4} \\ C_i &= \cos(\theta_{wi}(t)) + \cos^{-1} \frac{3}{4} \\ (i &= 1, 2, 3) \end{aligned} \right\} (1)$$

Where \mathbf{g}_a is the constant vector including the gravity \mathbf{g} and $\mathbf{u}(t)$ denotes an input voltage added to the motors set at the

Table. 2 Nominal values of parameters

| | | | | | |
|----------|-------|------------------|----------|-----------------------|------------------|
| m | 84.2 | kg | i_ψ | 1.72 | kgm ² |
| i_ϕ | 1.39 | kgm ² | l_w | 0.15 | m |
| c_a | 0.079 | m | c_b | 0.011 | m |
| l_f | 0.125 | m | l_r | 0.445 | m |
| b | 0.35 | m | d | 0.011 | m |
| k_a | 0.126 | N/V | k_G | 1/160 | |
| j_m | 0.21 | kgm ² | d_m | 1.72×10^{-4} | Nms/rad |
| r | 0.226 | 1/Ω | | | |

center axis of the large wheels 1, 2 and 3. In order to add a suspension characteristic to the wheeled mobile robot (1), the input voltage is given by

$$\left. \begin{aligned} \mathbf{u}(t) &= -\bar{K}_b^{-1}(t)(\mathbf{u}_a(t) - k_p \mathbf{x}(t) - \bar{K}_j(t)\dot{\mathbf{w}}(t) + \bar{\mathbf{g}}_a) \\ \bar{K}_b(t) &= \bar{\theta}_{kb} K_{b\theta}(t), \bar{K}_j(t) = \bar{\theta}_{kj} K_{j\theta}(t) \end{aligned} \right\} (3)$$

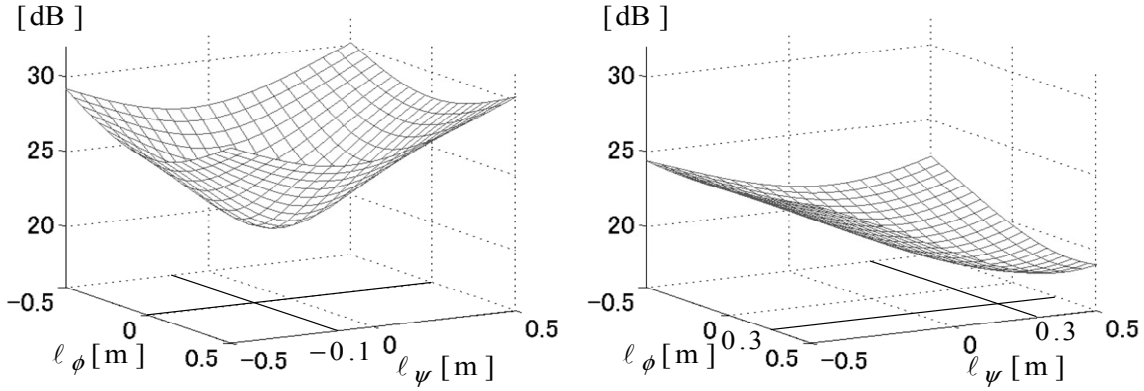
where $\mathbf{u}_a(t)$ is the new input signal introduced to control the acceleration of the robot body. The parameter $\bar{\bullet}$ denotes the parameters \bullet in which the values of element are nominal values shown in table 2.

To develop a controller achieving a small acceleration of the robot body, the state space description including the acceleration of the robot body is introduced as follows. The symbols O_n, I_n used below denote $n \times n$ zero matrix and $n \times n$ unit matrix.

$$\left. \begin{aligned} \dot{\xi}(t) &= (\Gamma - B_\xi M_p^{-1}(t) F(t)^T) \xi(t) + B_\xi M_p^{-1}(t) \frac{\theta_{kb}}{\theta_{kb}} \boldsymbol{\mu}(t) \\ &\quad + D_1(t)\dot{\mathbf{w}}(t) + D_2(t)\dot{\mathbf{w}}(t) - B_\xi M_p^{-1}(t) \left(\mathbf{g}_a - \frac{\theta_{kb}}{\theta_{kb}} \bar{\mathbf{g}}_a \right) \\ \boldsymbol{\mu}(t) &= \dot{\mathbf{u}}_a(t) + \mathbf{u}_a(t), \\ \xi(t) &= [\mathbf{x}(t)^T \quad \dot{\mathbf{x}}(t)^T \quad (H\dot{\mathbf{q}}(t))^T]^T, \dot{\mathbf{q}}(t) = [\ddot{z}_c(t) \quad \ddot{\psi}(t) \quad \ddot{\phi}(t)]^T \\ \Gamma &= \begin{bmatrix} O_3 & I_3 & O_3 \\ O_3 & O_3 & I_3 \\ O_3 & O_3 & O_3 \end{bmatrix}, F(t) = \begin{bmatrix} k_p \frac{\theta_{kb}}{\theta_{kb}} I_3 \\ \dot{K}_d(t) + K_d(t) + k_p \frac{\theta_{kb}}{\theta_{kb}} I_3 \\ M_p(t) + \dot{K}_j(t) + K_d(t) \end{bmatrix} \\ D_1(t) &= \begin{bmatrix} O_3 & & \\ & -I_3 & \\ & & M_p^{-1}(t)(\dot{K}_j(t) + K_j(t) + K_d(t) - \frac{\theta_{kb}}{\theta_{kb}} (\dot{K}_j(t) - \bar{K}_j(t))) \end{bmatrix} \\ D_2(t) &= \begin{bmatrix} O_3 & O_3 & M_p^{-1}(t) \left(K_j(t) - \frac{\theta_{kb}}{\theta_{kb}} \bar{K}_j(t) \right) \end{bmatrix}^T \\ B_\xi &= [O_3 \quad O_3 \quad I_3]^T \end{aligned} \right\} (4)$$

Where $\boldsymbol{\mu}(t) = \dot{\mathbf{u}}_a(t) + \mathbf{u}_a(t)$ is the virtual input. The wheeled mobile robot with $\boldsymbol{\mu}(t) = 0$ is called the passive robot.

The control objective is to develop an adaptive controller so that the vertical acceleration at any specified location on the robot body can be reduced to a small value easily. To meet the objective, the following assumptions are made for the actual mobile robot (4).



(a) Maximum gain surface of the passive robot (b) Maximum gain surface of the combined ideal robot model
Fig. 3 Properties of the passive robot and the combined ideal robot model

- A1 The body acceleration $\ddot{q}(t)$ can be measured.
- A2 The vertical displacements $x(t)$ at the center of large wheels and its velocity $\dot{x}(t)$ can be measured.
- A3 The acceleration $\ddot{w}(t)$ of road disturbance is bounded and can be measured

3. COMBINED IDEAL ROBOT MODEL

A combined ideal robot model that achieves the control objective is given by^[5]

$$\left. \begin{aligned} \dot{\xi}_m(t) &= \Gamma \xi_m(t) - B \bar{M}_p^{-1} k_b f_m(t) + \bar{D} \ddot{w}(t) \\ \xi_m(t) &= \left[x_m(t)^T, \dot{x}_m(t)^T, (H \ddot{q}_m(t))^T \right]^T \end{aligned} \right\} (6)$$

$$\left. \begin{aligned} \bar{D}(t) &= \left[O_3, -I_3, \bar{M}_p^{-1} k_d \right]^T \\ f_m(t) &= \sum_{i=1}^2 \gamma_{\phi i} \sum_{j=1}^2 \gamma_{\psi j} F_{m(2i+j-2)}(t) \xi_{m(2i+j-2)}(t) \\ \gamma_{\phi i} &= \frac{(-1)^i l_{\phi p} + l_{\phi m}}{2l_{\phi m}}, \gamma_{\psi i} = \frac{(-1)^{i-1} l_{\psi p} + l_{\psi m}}{2l_{\psi m}}, i=1,2 \\ \xi_{mi}(t) &= \Omega_i \xi_{di}(t), i=1,2,3,4 \\ F_{mi}(t) &= \bar{M}_p H T_i H^{-1} \bar{M}_p^{-1} (\bar{F} - G_i) \Omega_i^{-1}, i=1,2,3,4 \\ \Omega_i &= \text{diag} \left[H T_i H^{-1}, H T_i H^{-1}, H T_i H^{-1} \right], i=1,2,3,4 \\ T_i &= I_3 - \mathbf{b} \begin{bmatrix} 0, & l_{\psi p} - l_{\psi m}, & l_{\phi p} + (-1)^{i-1} l_{\phi m} \end{bmatrix}, i=1,2 \\ T_i &= I_3 - \mathbf{b} \begin{bmatrix} 0, & l_{\psi p} + l_{\psi m}, & l_{\phi p} + (-1)^{i-1} l_{\phi m} \end{bmatrix}, i=3,4 \\ \dot{\xi}_{di}(t) &= (\Gamma + \bar{B}(\bar{F} - G_i)) \xi_{di}(t) + \bar{D}_w \ddot{w}(t) \\ \xi_{di}(t) &= \left[x_{di}(t)^T, \dot{x}_{di}(t)^T, (H \ddot{q}_{di}(t))^T \right]^T, i=1,2,3,4 \end{aligned} \right\} (7)$$

where the design parameters $l_{\psi} = l_{\psi p}, l_{\phi} = l_{\phi p}$ are introduced to the specified location where the vertical acceleration becomes minimum. Hereafter, the location where the vertical acceleration becomes minimum is called the best location. The feedback gains $G_i, i=1 \sim 4$ are designed by using the optimal control theory so that the vertical acceleration of

ideal robot models $\xi_{di}(t), i=1 \sim 4$ become small at the specified locations $l_{\psi} = -l_{\psi m}, l_{\phi} = (-1)^i l_{\phi m}, i=3,4$ and $l_{\psi} = l_{\psi m}, l_{\phi} = (-1)^i l_{\phi m}, i=1,2$, respectively. $\Omega_i, i=1 \sim 4$ are transformation matrixes that are introduced to specified the best location.

Fig. 3 shows the properties of the passive robot and the designed combined ideal robot model. To show the gain characteristics relating to the vertical acceleration on the robot body, the maximum value of the norm of transfer function vector from the road disturbance $\ddot{w}(t)$ to the vertical acceleration at a location (l_{ψ}, l_{ϕ}) is plotted. Hereafter, the curve surface such as shown in Fig. 3 (a) is called the maximum gain surface. Fig.3 (a) shows the maximum gain surface of the passive robot and Fig. 3 (b) shows that of the combined ideal robot model. The two design parameters l_{ψ}, l_{ϕ} are set as $l_{\psi} = 0.3\text{m}, l_{\phi} = 0.3\text{m}$.

As shown in Fig. 3 (b), the combined ideal robot model has better performance in suppression of the vertical acceleration than the passive robot, and the best location is corresponding to the specified location $l_{\psi} = 0.3\text{m}, l_{\phi} = 0.3\text{m}$. From this fact, it can be concluded that the control objective can be achieved if the behavior of the actual mobile robot can track that of the combined ideal robot model.

4. ADAPTIVE TRACKING CONTROLLER

To develop a tracking controller, the tracking error between the actual mobile robot (4) and the combined ideal robot model (6) is defined as $\tilde{\xi}(t) = \xi(t) - \xi_m(t)$, and the new signal is defined by $\eta(t) = [2I_3 \quad 3I_3 \quad I_3] \tilde{\xi}(t) = L^T \tilde{\xi}(t)$. Then, the error equation is given by

$$\begin{aligned} \dot{\eta}(t) &= -d\dot{\eta}(t) - \frac{1}{2} M_p^{-1}(t) \dot{M}_p(t) \eta(t) \\ &+ \frac{\theta_{kb}}{\theta_{kb}} M_p^{-1}(t) \left\{ \mu(t) + \mathcal{O} \omega_{\eta}(t) + \omega_k(t) + \left(K_j(t) - \frac{\theta_{kb}}{\theta_{kb}} \bar{K}_j(t) \right) \ddot{w}(t) \right\}, (8) \end{aligned}$$

$$\left. \begin{aligned} \Theta &= \frac{1}{\theta_{kb}} \left[\left((HT_h)^T \right)^{-1} M_c (HT_h)^{-1} \theta_{kj} I_3 \quad \theta_{kd1} I_3 \quad \theta_{kd2} I_3 \quad \mathbf{g}_a \right] \\ \omega_\eta(t) &= \bar{\theta}_{kj} \left[\omega_1(t)^T \quad \omega_{j\theta}(t)^T \quad \omega_{d\theta1}(t)^T \quad \omega_{d\theta2}(t)^T \quad -1 \right]^T \\ \omega_{j\theta}(t) &= K_{j\theta} \omega_1(t) + \omega_2(t) + \dot{K}_{j\theta} \omega_3(t) \\ \omega_{d\theta1}(t) &= K_{d\theta1} \omega_4(t) + \dot{K}_{d\theta1} \omega_5(t), \quad \omega_{d\theta2}(t) = K_{d\theta2} \omega_4(t) + \dot{K}_{d\theta2} \omega_5(t) \\ \omega_1(t) &= L^T \Gamma \ddot{\xi}(t) + d\eta(t) - H\dot{q}(t) + \bar{M}_p^{-1} (\bar{K}_b f_m - \bar{K}_d \dot{w}(t)) \\ \omega_2(t) &= \ddot{w}(t), \omega_3(t) = \dot{w}(t) - H\dot{q}(t) + \frac{1}{2} \dot{M}_p(t) \eta(t) \\ \omega_4 &= -\dot{x}(t) + \dot{w}(t) - H\dot{q}(t), \omega_5 = -\dot{x}(t) \\ \omega_k(t) &= -k_p \dot{x}(t) - k_v x(t) - \left(\dot{\bar{K}}_j(t) + \bar{K}_j(t) \right) \dot{w}(t) + \bar{g}_a \end{aligned} \right\} (9)$$

where Θ is unknown constant matrix, $\omega_\eta(t)$ and $\omega_k(t)$ are known signal vectors and d is design parameter.

We developed the following adaptive tracking controller based on the error equation (8).

$$\hat{\Theta}(t)^T = \Gamma_\eta \omega_\eta(t) \eta(t)^T - \delta \hat{\Theta}^T(t), \quad \delta > 0 \quad \Gamma_\eta > 0 \quad (10)$$

$$\mu(t) = -\beta \eta(t) - \hat{\Theta}(t) \omega_\eta(t) - \omega_k(t) \quad \beta > 0 \quad (11)$$

Where $\hat{\Theta}(t)$ is the estimated matrix of the unknown matrix Θ . δ and β are positive design parameters and Γ_η is a positive definite matrix.

The following theorem holds in the mobile robot using controller (10) and (11).

Theorem 1

The adaptive wheeled mobile robot using the controller (10) and (11) becomes stable. In addition, there are positive values $\bar{\rho}_1, \bar{\rho}_2$ independent of β such that

$$\|\eta(t)\|^2 \leq \bar{\rho}_1 e^{-\beta t} + \frac{\bar{\rho}_2}{\beta^2}. \quad (12)$$

5. NUMERICAL SIMULATION

We show numerical simulation results to confirm the effectiveness of the proposed controller.

The mobile robot is running at the speed 0.5[m/s], and the left front wheel runs over the road disturbance with the amplitude 0.03m and the frequency 2Hz. The design parameters are set as $\beta = 50^2$, $\Gamma_\eta = 50^3 I_3$, $d = 10$, $\delta = 0.01$. The specified location was set as $l_\psi = 0.3\text{m}$, $l_\phi = 0.3\text{m}$. The initial value of $\hat{\Theta}(t)$ is set as $\hat{\Theta}(t) = 0$.

Fig. 4 shows the responses at the specified location of

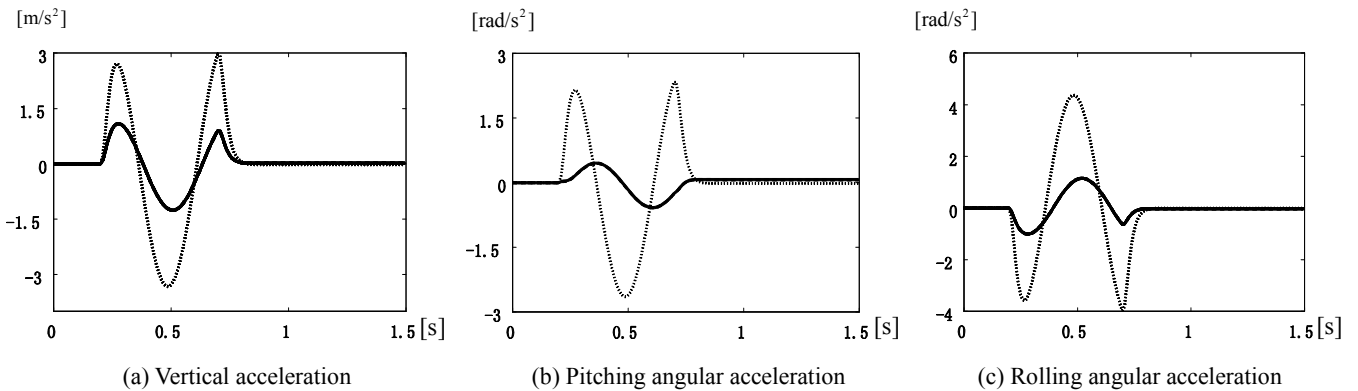


Fig. 4 Responses of the controlled mobile robot

$l_\psi = 0.3\text{m}, l_\phi = 0.3\text{m}$. Fig. 4 (a) is the vertical acceleration, (b) is the pitching angular acceleration, and (c) is the rolling angular acceleration. Dotted lines show the responses of the passive robot and thick solid lines are responses of the controlled mobile robot. The responses of the combined ideal model are shown by using thin lines. However since thin lines and thick solid lines are almost same, the thin lines disappeared.

6. CONCLUSION

We propose an adaptive oscillation controller for wheeled mobile robots. Carrying out numerical simulations, it has been shown that for any parameter uncertainties, the oscillation attenuation performance can be achieved.

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