The Seventeenth International Symposium on Artificial Life and Robotics 2012 (AROB 17th '12),
B-Con Plaza, Beppu, Oita, Japan, January 19-21, 2012

An improved adaptive controller in the presence of input saturation - In case of systems with available output derivatives up to the order of relative degree -

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Abstract: We have proposed a model reference adaptive control scheme (conventional control scheme) for continuous time single-input single-output linear systems with an input saturation in which i-th derivatives of the output signal ($i = 1, \cdots$, relative degree) are available. In the conventional scheme, a condition for the initial states of the controlled object has to be satisfied. In this paper, the main attention is focused on the relaxation of the condition. To achieve the objective, we propose an improve adaptive control scheme. As a result of analyzing stability of the closed loop system using the improved adaptive control scheme, a new condition for the initial states is derived. It should be emphasized that we can apply the new control scheme in a larger region of initial states compared with the conventional one.

Keywords: adaptive control, input saturation, order of systems, improvement of control performance, relaxation of restrict condition

1 INTRODUCTION

In practice, the values of the system parameters, that are coefficients of dynamic equations describing behavior of physical systems, vary due to aged deterioration or environmental changes. For the systems, adaptive control schemes [1]-[3] were useful. However, input saturation was ignored in the early works. To overcome the problem, various adaptive control schemes have been developed for single-input single-output (SISO) linear systems [4]-[16]. These schemes guarantee boundedness of all signals in the closed loop systems. But most of them could not assure asymptotical stability of a tracking error between the output of the controlled object and a desired trajectory. Although there exist some schemes that can assure asymptotical stability of the tracking error, they have a problem that controlled objects are restricted to asymptotically stable systems. Moreover, in all proposed schemes, there exists a problem that a method to improve tracking performance have not been provided.

To struggle against these problems, for the first step, the authors proposed an adaptive output tracking control scheme for SISO linear systems with an input saturation in which full states can be measured [17]. The closed loop system using the proposed controller has the following properties: 1) Stability of all signals in the closed loop systems can be guaranteed; 2) Convergence of the tracking error to zero is assured even for unstable controlled objects; 3) Setting only one design parameter, the control performance can be easily improved. For the next step, to decrease measurement signals, for n-th degree systems with an input saturation and the relative degree n_r, we proposed an extended scheme satisfying 1)-3) in which i-th derivatives of output signal ($i = 1, \cdots, n_r$) are only required [18]. However, to achieve the good properties 1)-3), the condition for the initial states of the controlled object has to be satisfied.

In this paper, the main attention is to relax the condition for the initial states derived in [18]. We propose an improved adaptive controller for SISO linear systems with an input saturation in which the output derivatives up to the order of relative degree n_r are available. In the proposed adaptive controller, it is shown theoretically that the same good properties 1)-3) can be assured under a new condition for the initial states. Moreover, it can be shown theoretically that the improved adaptive controller can be utilized in a larger region of initial states compared with the adaptive control scheme proposed in [18].

2 PROBLEM STATEMENT

In this paper, we consider the controlled objects described by

$$\left.\begin{aligned} Y(s) &= \frac{b_m B_p(s)}{A_p(s)} F(s) + \frac{C_p(s)}{A_p(s)} \\ A_p(s) &= s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0 \\ B_p(s) &= s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0 \\ C_p(s) &= c_{n-1}s^{n-1} + \cdots + c_1 s + c_0 \end{aligned}\right\} \quad (1)$$

where $A_p(s)$ and $B_p(s)$ are coprime. $f(u) = \mathcal{L}[F(s)]^{-1} \in \mathbb{R}$ is a saturation function given by

$$f(u) = \begin{cases} \sigma & \text{for} \quad u(t) > \sigma \\ u(t) & \text{for} \quad |u(t)| \le \sigma \\ -\sigma & \text{for} \quad u(t) < -\sigma \end{cases} \quad (2)$$

where $u(t) \in \mathbb{R}$ is the control input and the positive constant σ is an amplitude saturation level of the actuator.

The reference model is given by

$$\left.\begin{aligned} Y_M(s) &= \frac{B_M(s)}{A_M(s)} R(s) \\ A_M(s) &= s^{n_M} + a_{M(n_M-1)}s^{n_M-1} + \cdots + a_{M1}s + a_{M0} \\ B_M(s) &= b_{Mm_M}s^{m_M} + \cdots + b_{M1}s + b_{M0} \end{aligned}\right\} \quad (3)$$

where $y_M(t) = \mathcal{L}^{-1}[Y_M(s)]$ is the reference output, $r(t) = \mathcal{L}^{-1}[R(s)]$ is the reference input, $A_M(s)$ is Hurwitz polynomia, and $n_M - m_M \ge n - m$. The reference input $r(t)$ is a deterministic signal given by

$$\left.\begin{aligned} R(s) &= \frac{B_R(s)}{A_R(s)} \\ A_R(s) &= s^{n_r} + a_{R(n_r-1)}s^{n_r-1} + \cdots + a_{R1}s + a_{R0} \\ B_R(s) &= b_{Rm_r}s^{m_r} + \cdots + b_{R1}s + b_{R0} \end{aligned}\right\} \quad (4)$$

where $A_R(s)$ and $B_R(s)$ are known polynomials, and $n_r - m_r \ge 0$.

The tracking error is defined as $y_e(t) = y(t) - y_M(t)$. Then, using the polynomials

$$\left.\begin{array}{l} A_p(s) = A_{pq}(s)B_p(s) + A_{pr}(s) \\ A_{pq}(s) = s^{n-m} + a_{q(n-m-1)}s^{n-m-1} + \cdots + a_{q1}s + a_{q0} \\ A_{pr}(s) = a_{r(m-1)}s^{m-1} + \cdots + a_{r1}s + a_{r0} \end{array}\right\}, \quad (5)$$

the following state space description can be derived [18].

$$\left.\begin{array}{l} \dot{\boldsymbol{x}}_e(t) = \boldsymbol{A}\boldsymbol{x}_e(t) + b_m\boldsymbol{b}\left(f(u) - \boldsymbol{c}_z^{\mathsf{T}}\boldsymbol{z}(t) - \boldsymbol{\theta}_r^{\mathsf{T}}\boldsymbol{x}_r(t) + \phi(t)\right) \\ \boldsymbol{x}_e(t) = \left[y_e(t),\ \dot{y}_e(t),\ \cdots,\ y_e(t)^{(n-m-1)}\right]^{\mathsf{T}} \\ \boldsymbol{A} = \overline{\boldsymbol{A}} - \boldsymbol{b}\boldsymbol{a}_q^{\mathsf{T}},\ \boldsymbol{b}^{\mathsf{T}} = [0, \cdots, 0, 1] \\ \overline{\boldsymbol{A}} = \left[\begin{array}{c|c} \boldsymbol{0} & \boldsymbol{I} \\ \hline 0 & 0\cdots 0 \end{array}\right],\ \boldsymbol{a}_q^{\mathsf{T}} = [a_{q0}, \cdots, a_{q(n-m-1)}] \\ \boldsymbol{c}_z^{\mathsf{T}} = [a_{r0}, \cdots, a_{r(m-1)}] \\ \dot{\boldsymbol{x}}_r(t) = \boldsymbol{A}_r\boldsymbol{x}_r(t),\ \boldsymbol{x}_r(0) = \boldsymbol{x}_{r0} \end{array}\right\} \quad (6)$$

$$\left.\begin{array}{l} \dot{\boldsymbol{z}}(t) = \boldsymbol{A}_z\boldsymbol{z}(t) + \boldsymbol{h}\boldsymbol{c}_x^{\mathsf{T}}\boldsymbol{x}_e(t),\ \boldsymbol{z}(0) = \boldsymbol{0} \\ \boldsymbol{A}_z = \left[\begin{array}{c|c} \boldsymbol{0} & \boldsymbol{I} \\ \hline -b_0 & -b_1 \cdots - b_{m-1} \end{array}\right] \\ \boldsymbol{h}^{\mathsf{T}} = [0, \cdots, 0, 1/b_m],\ \boldsymbol{c}_x^{\mathsf{T}} = [1, 0, \cdots, 0] \end{array}\right\} \quad (7)$$

Where $\phi(t)$ is an unknown bounded exponential damping function, $\boldsymbol{\theta}_r$ is an unknown parameter, $\boldsymbol{A}_r$ is known constant matrix, and $\boldsymbol{x}_r(t) \in \mathbb{R}^{n_M+n_r}$ is a known state with respect to the reference model.

Using a known constant vector $\boldsymbol{d} \in \mathbb{R}^{n-m}$, the Hurwitz matrix $\boldsymbol{A}_e$ is defined as $\boldsymbol{A}_e = \overline{\boldsymbol{A}} - \boldsymbol{b}\boldsymbol{d}^{\mathsf{T}}$. Then, defining the signal $\ell(t) = \boldsymbol{c}_z^{\mathsf{T}}\boldsymbol{z}(t)$, the tracking error system (6) can be rewritten as

$$\left.\begin{array}{l} \dot{\boldsymbol{x}}_e(t) = \boldsymbol{A}_e\boldsymbol{x}_e(t) + b_m\boldsymbol{b}\left(f(u) - q(t)\right) \\ q(t) = \boldsymbol{\theta}(t)^{\mathsf{T}}\boldsymbol{\omega}(t) + \ell(t),\ \ell(t) = \boldsymbol{c}_z^{\mathsf{T}}\boldsymbol{z}(t) \\ \boldsymbol{\theta}(t)^{\mathsf{T}} = \dfrac{1}{\varepsilon}\left[\phi(t),\ \boldsymbol{\theta}_e^{\mathsf{T}},\ \boldsymbol{\theta}_r\right],\ \varepsilon \geq 1 \\ \boldsymbol{\omega}(t)^{\mathsf{T}} = \varepsilon\left[-1,\ \boldsymbol{x}_e(t),\ \boldsymbol{x}_r(t)\right] \\ \boldsymbol{\theta}_e = \dfrac{1}{b_m}(\boldsymbol{a}_q - \boldsymbol{d}) \end{array}\right\} \quad (8)$$

where $\boldsymbol{\theta}(t)$ and $\ell(t)$ are unknown signals. The design parameter ε is a positive constant. The design parameter ε is introduce to reduce the initial value $\|\boldsymbol{\theta}(0)\|$.

The controlled objective is that the tracking error becomes asymptotically stable even if a controlled object is not asymptotically stable. To achieve the controlled objective, the following assumptions are made:

A1 Coefficients of polynomials $A_p(s)$, $B_p(s)$ and $C_p(s)$ are unknown constants.
A2 The plant is minimal phase.
A3 Sign of b_m is known. It is assumed that the sign is positive hereafter.
A4 Relative degree $n - m$ is known.
A5 $y(t)^{(i)},\ i = 0, \cdots, n - m - 1$ are available.
A6 There exists a known positive constant δ_z such that $\overline{\boldsymbol{A}}_z = \boldsymbol{A}_z + \delta_z\boldsymbol{I}$ is Hurwitz matrix.
A7 There exists bounded positive constant $\overline{\rho}_{M1}$ and $\overline{\rho}_{M2}$ such that $|\boldsymbol{\theta}_r\boldsymbol{x}_r(t) - \phi(t)| \leq \overline{\rho}_{M1}$ and $\|\boldsymbol{x}_r(t)\| \leq \overline{\rho}_{M2}$.
A8 The relation $\sigma - \overline{\rho}_{M1} > 0$ holds.
A9 The saturation level σ is known.

The assumptions A1 ~ A4 are the same assumptions made in conventional adaptive control scheme. From the assumption A2, $\boldsymbol{A}_z$ becomes Hurwitz. Since $\phi(t)$ is bounded exponential damping function, it is also seen that there exist positive constants ρ_ϕ, δ_ϕ such that $\left\|\dot{\phi}(t)\right\| \leq \rho_\phi\exp(-\delta_\phi t)$. The assumptions A5 ~ A9 are introduced to develop the adaptive controller mentioned later. From the assumption A5, the signal $\boldsymbol{x}_e(t)$ is available.

3 DEVELOPMENT OF ADAPTIVE CONTROLLER

3.1 Conventional adaptive controller

In [18], the controller developed under the assumptions A1~A9 is given by

$$\left.\begin{array}{l} u(t) = \hat{q}(t) = \hat{\boldsymbol{\theta}}(t)^{\mathsf{T}}\boldsymbol{\omega}(t) + \hat{\ell}(t) \\ \hat{\boldsymbol{\theta}}(t) = \left[\hat{\theta}_\phi(t),\ \hat{\boldsymbol{\theta}}_e(t)^{\mathsf{T}},\ \hat{\boldsymbol{\theta}}_r(t)\right]^{\mathsf{T}} \end{array}\right\}, \quad (9)$$

$$\left.\begin{array}{l} \dot{\hat{\boldsymbol{\theta}}}(t) = -\alpha^3 g(t)\tilde{\boldsymbol{x}}_e(t)^{\mathsf{T}}\boldsymbol{P}\boldsymbol{b}\boldsymbol{\Gamma}\boldsymbol{\omega}(t),\ \boldsymbol{\Gamma} > 0 \\ \dot{\hat{\ell}}(t) = -\delta_z\hat{\ell}(t) - \dfrac{1}{\rho_\ell}\alpha^3 g(t)\tilde{\boldsymbol{x}}_e(t)^{\mathsf{T}}\boldsymbol{P}\boldsymbol{b},\ \hat{\ell}(0) = 0 \\ \tilde{\boldsymbol{x}}_e(t) = \boldsymbol{x}_e(t) - \hat{\boldsymbol{x}}_e(t) \\ \dot{\hat{\boldsymbol{x}}}_e(t) = \boldsymbol{A}_e\hat{\boldsymbol{x}}_e(t) + \alpha^2\tilde{\boldsymbol{x}}_e(t),\ \hat{\boldsymbol{x}}_e(0) = \boldsymbol{x}_e(0) \\ g(t) = 1 - (1-\beta)(1 - \delta_{\tilde{e}f}(t)),\ 1 \geq \beta > 0 \\ \delta_{\tilde{e}f}(t) = \left\{\begin{array}{ll} 1 & \text{for}\quad \tilde{\boldsymbol{x}}_e(t)^{\mathsf{T}}\boldsymbol{P}\boldsymbol{b}\tilde{f}(u) \leq 0 \\ 0 & \text{for}\quad \tilde{\boldsymbol{x}}_e(t)^{\mathsf{T}}\boldsymbol{P}\boldsymbol{b}\tilde{f}(u) > 0 \end{array}\right. \\ \tilde{f}(u) = f(u) - u(t) \end{array}\right\}, \quad (10)$$

where $\hat{\theta}_\phi(t)$, $\hat{\boldsymbol{\theta}}_e(t)$ and $\hat{\boldsymbol{\theta}}_r(t)$ are estimate values corresponding to the unknown signal $\phi(t)/\varepsilon$ and constants $\boldsymbol{\theta}_e/\varepsilon$ and $\boldsymbol{\theta}_r/\varepsilon$, respectively. $\hat{\boldsymbol{x}}_e(t)$ and $\hat{\ell}(t)$ are estimates of $\boldsymbol{x}_e(t)$ and $\ell(t)$, respectively. The design parameters α and β are positive constants, $\boldsymbol{\Gamma}$ is a positive definite matrix. The design parameter ρ_ℓ is a positive constant satisfying

$$\left.\begin{array}{l} \rho_\ell \leq \dfrac{\delta_z}{b_m}\left\{\left(\dfrac{4}{\underline{\alpha}} + \dfrac{6}{\rho_{\hat{e}}}\right)\dfrac{\|\boldsymbol{c}_z^{\mathsf{T}}\boldsymbol{h}\boldsymbol{c}_x^{\mathsf{T}}\|^2}{\lambda_{\min}[\boldsymbol{Q}]} + \dfrac{4\|\boldsymbol{c}_z^{\mathsf{T}}\overline{\boldsymbol{A}}_z\|^2}{\rho_z\lambda_{\min}[\boldsymbol{Q}_z]}\right\}^{-1} \\ \rho_z = \left(\dfrac{4}{\underline{\alpha}} + \dfrac{6}{\rho_{\hat{e}}}\right)^{-1}\dfrac{\lambda_{\min}[\boldsymbol{Q}_z]}{4}\dfrac{\lambda_{\min}[\boldsymbol{Q}]}{\|\boldsymbol{P}_z\boldsymbol{h}\boldsymbol{c}_x^{\mathsf{T}}\|^2} \\ \rho_{\hat{e}} = \dfrac{\lambda_{\min}[\boldsymbol{Q}]}{6}\left(\dfrac{\|\boldsymbol{P}\|}{\underline{\alpha}}\right)^{-1} \end{array}\right\}, \quad (11)$$

where $\underline{\alpha}$ denotes a lower bound of the design parameter α such that $\alpha > \underline{\alpha} > 0$, design parameters $\boldsymbol{Q}$ and $\boldsymbol{Q}_z$ are positive definite matrices. $\boldsymbol{P}$ and $\boldsymbol{P}_z$ are the solutions of the following Lyapunov equations.

$$\left.\begin{array}{l} \boldsymbol{A}_e^{\mathsf{T}}\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A}_e = -\boldsymbol{Q},\ \boldsymbol{Q} > 0 \\ \boldsymbol{A}_z^{\mathsf{T}}\boldsymbol{P}_z + \boldsymbol{P}_z\boldsymbol{A}_z = -\boldsymbol{Q}_z,\ \boldsymbol{Q}_z > 0 \end{array}\right\} \quad (12)$$

The estimated signal $\hat{\boldsymbol{x}}_e(t)$ and the design parameter α is introduced to improve performance of the estimators for $\hat{\boldsymbol{\theta}}(t)$ and $\hat{\ell}(t)$. The design parameter β is introduced to guarantee tracking performance between the control input signal $\hat{q}(t)$ and the ideal input signal $q_d(t) = \boldsymbol{\theta}(t)^{\mathsf{T}}\boldsymbol{\omega}(t) + \ell(t)$. The switch function $\delta_{\tilde{e}f}(t)$ is introduced to assure stability of the closed loop system with the input saturation.

To show the property of the closed loop system, we define the following Lyapunov function candidate

$$\left.\begin{array}{l} V(t) = V_1(t) + \rho_{v2}V_2(t) \\ V_1(t) = \alpha^3\tilde{\boldsymbol{x}}_e(t)^{\mathsf{T}}\boldsymbol{P}\tilde{\boldsymbol{x}}_e(t) + V_e(t) \\ \qquad + \rho_z\boldsymbol{z}(t)^{\mathsf{T}}\boldsymbol{P}_z\boldsymbol{z}(t) + b_m\rho_\ell\tilde{\ell}(t)^2 \\ \qquad + b_m\tilde{\boldsymbol{\theta}}(t)^{\mathsf{T}}\boldsymbol{\Gamma}^{-1}\tilde{\boldsymbol{\theta}}(t) \\ V_2(t) = \dfrac{2\rho_\phi\sqrt{b_m}\left\|\boldsymbol{\Gamma}^{-\frac{1}{2}}\right\|}{\varepsilon\delta_\phi}\exp(-\delta_\phi t) \\ \rho_{v2} = \dfrac{V_2(0) + \sqrt{V_2(0)^2 + 4V_1(0)}}{2} + \delta_v \\ \tilde{\ell}(t) = \ell(t) - \hat{\ell}(t),\ \tilde{\boldsymbol{\theta}}(t) = \boldsymbol{\theta}(t) - \hat{\boldsymbol{\theta}}(t) \end{array}\right\}, \quad (13)$$

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$$V_e(t) = \rho_{\hat{e}}\hat{\boldsymbol{x}}_e(t)^{\mathsf{T}}\boldsymbol{P}\hat{\boldsymbol{x}}_e(t) \tag{14}$$

where $\tilde{\boldsymbol{\theta}}(t)$ and $\tilde{\ell}(t)$ are the parameter estimation errors, and δ_v is a design parameter of a positive constant.

Define a input estimation error $\tilde{q}(t)$ as $\tilde{q}(t) = q_d(t) - \hat{q}(t) = \tilde{\boldsymbol{\theta}}(t)^{\mathsf{T}}\boldsymbol{\omega}(t) + \tilde{\ell}(t)$. In the closed loop system using the conventional adaptive controller, the following theorem holds [18].

Theorem 1 If the initial states satisfy

$$\left.\begin{aligned}
&\rho_v^2 \geq V(0)\\
&\quad = \rho_{\hat{e}}\boldsymbol{x}_e(0)^{\mathsf{T}}\boldsymbol{P}\boldsymbol{x}_e(0) + b_m\tilde{\boldsymbol{\theta}}(0)^{\mathsf{T}}\boldsymbol{\Gamma}^{-1}\tilde{\boldsymbol{\theta}}(0)\\
&\qquad + \rho_{v2}V_2(0)\\
&\rho_v = \frac{1}{2\beta\rho_{u3}}\Big\{-(\rho_{u1} + \beta\rho_{u2})\\
&\qquad + \sqrt{(\rho_{u1} + \beta\rho_{u2})^2 + 4(\sigma - \bar{\rho}_{M1})\beta\rho_{u3}}\Big\}\\
&\rho_{u1} = \sqrt{2\max\left\{\frac{\|\boldsymbol{\theta}_e\|^2}{\rho_{\hat{e}}\lambda_{\min}[\boldsymbol{P}]},\ \frac{\|\boldsymbol{c}_z\|^2}{\rho_z\lambda_{\min}[\boldsymbol{P}_z]}\right\}}\\
&\rho_{u2} = \sqrt{\frac{2}{b_m}}\max\left\{\frac{\varepsilon\bar{\rho}_{M2}}{\lambda_{\min}\left[\boldsymbol{\Gamma}^{-\frac{1}{2}}\right]},\ \frac{1}{\sqrt{\rho_\ell}}\right\}\\
&\qquad + \frac{\varepsilon}{\sqrt{b_m}\lambda_{\min}\left[\boldsymbol{\Gamma}^{-\frac{1}{2}}\right]}\\
&\rho_{u3} = \sqrt{\frac{2}{b_m\lambda_{\min}\left[\boldsymbol{\Gamma}^{-1}\right]\lambda_{\min}[\boldsymbol{P}]}\max\left\{\frac{1}{\underline{\alpha}^3},\ \frac{1}{\rho_{\hat{e}}}\right\}}
\end{aligned}\right\}, \tag{15}$$

in addition, the lower bound of the design parameter $\underline{\alpha}$ is given by

$$\underline{\alpha}^2 \geq 2b_m\|\boldsymbol{\theta}_e\|\sqrt{\boldsymbol{b}^{\mathsf{T}}\boldsymbol{P}\boldsymbol{b}\lambda_{\min}[\boldsymbol{P}^{-1}]}, \tag{16}$$

the closed loop system using the controller (9) – (12) becomes stable, and the error signals $\boldsymbol{x}_e(t)$, $\tilde{\boldsymbol{x}}_e(t)$, $\boldsymbol{z}(t)$, $\tilde{\ell}(t)$ and $\tilde{q}(t)$ converge to zero. When the design parameters are fixed except for the design parameter α, the following relation holds.

$$\tilde{q}(t)^2 \leq \exp\left(-\alpha\bar{\rho}_{q1}t\right)\tilde{q}(0)^2 + \alpha^{-1}\bar{\rho}_{q2} \tag{17}$$

Where $\bar{\rho}_{qi},\ i = 1, 2$ are bounded positive constants independent of the design parameter α. ■

From Theorem 1, we have the following remarks.

Remark 1 It is seen from Theorem 1 that $\tilde{q}(t)$ converges to zero rapidly as the design parameter α increases. Then, the control input signal (9) becomes close to the ideal input $u(t) = q_d(t)$. Therefore, it can be expected that if the value of the design parameter α is large enough, the oscillations caused by the estimator of unknown parameters are hard to occur in the control input signal.

Remark 2 Consider the case when $\tilde{q}(t)$ converges to zero rapidly. Since $\boldsymbol{x}_e(t)$ converges to zero, from the assumption A8, it is seen that there exists $t_1 \geq 0$ such that $|q(t)| \leq \sigma,\ t_1 \geq t$. Then, for $t \geq t_1$, $\boldsymbol{x}_e(t)$ converges to zero at the convergent rate specified by the system matrix $\boldsymbol{A}_e$.

Remark 3 From (15), ρ_v becomes large as the desing parameter β decreases. The upper bound of ρ_v is

$$\lim_{\beta\to 0}\rho_v = \frac{\sigma - \bar{\rho}_{M1}}{\rho_{u1}}. \tag{18}$$

From this fact, it can be seen that if the condition

$$\left.\begin{aligned}
&\frac{(\sigma - \bar{\rho}_{M1})^2}{2}\min\left(\frac{\lambda_{\min}[\boldsymbol{P}]}{\|\boldsymbol{\theta}_e\|^2},\ K_1K_2\right) > \boldsymbol{x}_e(0)^{\mathsf{T}}\boldsymbol{P}\boldsymbol{x}_e(0)\\
&K_1 = \frac{\lambda_{\min}[\boldsymbol{Q}_z]\lambda_{\min}[\boldsymbol{Q}]\lambda_{\min}[\boldsymbol{P}_z]}{16\|\boldsymbol{P}_z\boldsymbol{h}\boldsymbol{c}_x^{\mathsf{T}}\|^2\|\boldsymbol{c}_z\|^2}\\
&K_2 = \frac{6}{9 + \dfrac{\lambda_{\min}[\boldsymbol{Q}]}{\|\boldsymbol{P}\|}}
\end{aligned}\right\} \tag{19}$$

is satisfied, for any initial condition of the parameter estimation error $\tilde{\boldsymbol{\theta}}(0)$ and $V_2(0)$, there exist design parameters ε, $\boldsymbol{\Gamma}$, and β such that the first equation of (15) holds.

3.2 Improved adaptive controller

If the left side of the first equation in (19) can become large, the region of the initial states satisfying Theorem 1 can be expanded. To achieve this, the Lyapunov function candidate is redefined as (13) and

$$V_e(t) = \boldsymbol{x}_e(t)^{\mathsf{T}}\boldsymbol{P}\boldsymbol{x}_e(t). \tag{20}$$

Analyzing the derivative of (13) and (20), the improved adaptive controller is developed as (9) and

$$\left.\begin{aligned}
&\dot{\hat{\boldsymbol{\theta}}}(t) = -\alpha^3 g(t)\bar{\boldsymbol{x}}_e(t)^{\mathsf{T}}\boldsymbol{P}\boldsymbol{b}\boldsymbol{\Gamma}\boldsymbol{\omega}(t),\ \boldsymbol{\Gamma} > 0\\
&\dot{\hat{\ell}}(t) = -\delta_z\hat{\ell}(t) - \frac{1}{\rho_\ell}\alpha^3 g(t)\bar{\boldsymbol{x}}_e(t)^{\mathsf{T}}\boldsymbol{P}\boldsymbol{b},\ \hat{\ell}(0) = 0\\
&\bar{\boldsymbol{x}}_e(t) = \tilde{\boldsymbol{x}}_e(t) + \frac{1}{\alpha^3}\boldsymbol{x}_e(t)\\
&\tilde{\boldsymbol{x}}_e(t) = \boldsymbol{x}_e(t) - \hat{\boldsymbol{x}}_e(t)\\
&\dot{\hat{\boldsymbol{x}}}_e(t) = \boldsymbol{A}_e\hat{\boldsymbol{x}}_e(t) + \alpha^2\tilde{\boldsymbol{x}}_e(t),\ \hat{\boldsymbol{x}}_e(0) = \boldsymbol{x}_e(0)\\
&g(t) = 1 - (1 - \beta)(1 - \delta_{\tilde{e}f}(t)),\ 1 \geq \beta > 0\\
&\delta_{\tilde{e}f}(t) = \begin{cases} 1 & \text{for}\quad \bar{\boldsymbol{x}}_e(t)^{\mathsf{T}}\boldsymbol{P}\boldsymbol{b}\tilde{f}(u) \leq 0\\ 0 & \text{for}\quad \bar{\boldsymbol{x}}_e(t)^{\mathsf{T}}\boldsymbol{P}\boldsymbol{b}\tilde{f}(u) > 0\end{cases}\\
&\tilde{f}(u) = f(u) - u(t)
\end{aligned}\right\}, \tag{21}$$

where the design parameter ρ_ℓ satisfies

$$\left.\begin{aligned}
&\rho_\ell \leq \frac{\delta_z}{4b_m}\left\{\frac{\|\boldsymbol{c}_z^{\mathsf{T}}\boldsymbol{h}\boldsymbol{c}_x^{\mathsf{T}}\|^2}{\lambda_{\min}[\boldsymbol{Q}]} + \frac{\|\boldsymbol{c}_z^{\mathsf{T}}\bar{\boldsymbol{A}}_z\|^2}{\rho_z\lambda_{\min}[\boldsymbol{Q}_z]}\right\}^{-1}\\
&\rho_z = \frac{\lambda_{\min}[\boldsymbol{Q}_z]}{16}\frac{\lambda_{\min}[\boldsymbol{Q}]}{\|\boldsymbol{P}_z\boldsymbol{h}\boldsymbol{c}_x^{\mathsf{T}}\|^2}
\end{aligned}\right\}. \tag{22}$$

Then, for the closed-loop system using the improved adaptive controller, the following theorem holds.

Theorem 2 If the initial states satisfy

$$\left.\begin{aligned}
&\rho_{v_m}^2 \geq V(0)\\
&\quad = \boldsymbol{x}_e(0)^{\mathsf{T}}\boldsymbol{P}\boldsymbol{x}_e(0) + b_m\tilde{\boldsymbol{\theta}}(0)^{\mathsf{T}}\boldsymbol{\Gamma}^{-1}\tilde{\boldsymbol{\theta}}(0)\\
&\qquad + \rho_{v2}V_2(0)\\
&\rho_{v_m} = \frac{1}{2\beta\rho_{u3}}\Big\{-(\rho_{u1} + \beta\rho_{u2})\\
&\qquad + \sqrt{(\rho_{u1} + \beta\rho_{u2})^2 + 4(\sigma - \bar{\rho}_{M1})\beta\rho_{u3}}\Big\}\\
&\rho_{m1} = \sqrt{\max\left\{\frac{\|\boldsymbol{\theta}_e\|^2}{\lambda_{\min}[\boldsymbol{P}]},\ \frac{\|\boldsymbol{c}_z\|^2}{\rho_z\lambda_{\min}[\boldsymbol{P}_z]}\right\}}\\
&\rho_{m2} = \frac{1}{\sqrt{b_m}}\max\left\{\frac{\varepsilon(\bar{\rho}_{M2} + 1)}{\lambda_{\min}\left[\boldsymbol{\Gamma}^{-\frac{1}{2}}\right]},\ \frac{1}{\sqrt{\rho_\ell}}\right\}\\
&\rho_{m3} = \frac{1}{\sqrt{b_m\lambda_{\min}\left[\boldsymbol{\Gamma}^{-1}\right]\lambda_{\min}[\boldsymbol{P}]}}
\end{aligned}\right\}, \tag{23}$$

the closed loop system using the controller (9), (12), (21), and (22) becomes stable, and the error signals $\boldsymbol{x}_e(t)$, $\tilde{\boldsymbol{x}}_e(t)$, $\boldsymbol{z}(t)$, $\tilde{\ell}(t)$ and $\tilde{q}(t)$ converge to zero. When the design parameters are fixed except for the design parameter α, there exist the bounded positive constant $\overline{\rho}_{qi}$, $i = 1, 2$ independent of the design parameter α satisfying (17). ■

For the lack of space, the proof is omitted. □

From Theorem 2, it can be ascertained that the properties stated in Remark 1 and Remark 2 hold. Moreover, we have the following remark for initial states.

Remark 4 From (23), ρ_{v_m} becomes large as the desing parameter β decreases. The upper bound of ρ_{v_m} is

$$\lim_{\beta \to 0} \rho_{v_m} = \frac{\sigma - \overline{\rho}_{M1}}{\rho_{m1}}. \tag{24}$$

From this fact, it can be seen that if the condition

$$(\sigma - \overline{\rho}_{M1})^2 \min\left(\frac{\lambda_{\min}[\boldsymbol{P}]}{||\boldsymbol{\theta}_e||^2},\ K_1\right) > \boldsymbol{x}_e(0)^{\mathsf{T}} \boldsymbol{P} \boldsymbol{x}_e(0) \tag{25}$$

is satisfied, for any initial condition of the parameter estimation error $\tilde{\boldsymbol{\theta}}(0)$ and $V_2(0)$, there exist design parameters $\varepsilon, \boldsymbol{\Gamma}$, and β such that the first equation of (23) holds.

From Remark 3 and Remark 4, we have following fact. Comparing with the value of (19) and (25), since the parameter K_2 becomes $K_2 < 1$, it can be easily ascertained that the value of the left hand side of (25) becomes at least two times larger than and equal to the value of the left side of (19). This means that the improved adaptive controller can be applied to the larger region of the initial states compared with the conventional one proposed in [18].

Therefore, it can be concluded that the improved adaptive controller can achieve good properties mentioned in Remark 1 and Remark 2 and the relaxation of the condition of the initial states.

4 CONCLUSION

In this paper, the main attention was to relax the condition for the initial states of the closed loop system using the conventional adaptive control scheme proposed in [18], we proposed the improved adaptive control scheme. Using the proposed adaptive controller, it is shown theoretically that the following properties can be achieved: 1) Stability of all signals i the closed loop systems can be guaranteed; 2) The tracking error can converges to zero even for unstable controlled objects; 3) Tracking performance can be improved by setting the only one design parameter α. Moreover, it was shown that we can apply the improved adaptive control scheme in the larger region of the initial states compared with the control scheme proposed in [18].

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