

# Flight guidance and control of a winged rocket

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**Abstract:** Since Reusable Launch Vehicles (RLVs) operate in a wide range of flight conditions, the values of the parameters of the RLVs' dynamic equations are not constant. Therefore, some adaptive control methods for RLVs have been proposed. We have proposed a digital adaptive feedback linearization control method with time-scale separation to a winged rocket. In this paper, we propose a digital adaptive feedback linearization control method using unscented Kalman filter. The simulation results with a guidance using genetic algorithm show the effectiveness of the proposed control systems.

**Keywords:** Adaptive Control, Flight Control

## 1 Introduction

In late years space development is performed lively all over the world, and the research of space transportation systems to enable them is performed. Especially, Reusable Launch Vehicles (RLVs) are expected for the systems because the RLVs are the low-cost and highly reliable transport system instead of the conventional disposable rockets.

Since the RLVs have wide range flight conditions, the values of parameters of the dynamic model of the RLVs are not constant. Therefore, gain scheduling control method has been applied for the RLVs [1]. However, when the air traffic window expands, the number of required gains to be designed becomes very large and the method can only correspondence to the known change; the control performance becomes worse for the unpredictable change in the flight condition such as the abort flight.

For the change of the flight conditions, adaptive control methods using approximated linear dynamic equation of the RLVs have been researched [2–5]. However, the control performance decreases when the nonlinearity strengthens though a linear adaptive control has an excellent performance when the nonlinearity of the controlled system can be disregarded. Then, a feedback linearization method for deleting the nonlinear term by the state feedback for the nonlinear equation of motion of the RLV have been researched [6]. In addition, the method for dividing time-scale by a fast motion and a slow motion is researched for the simplification of the structure of the control system [7]. We have also proposed a digital adaptive feedback linearization control method with time-scale separation to a winged rocket [8] which is one of the RLVs.

Here, the air data obtained from a pitot tube etc. contains many noises. Therefore, in order to guarantee the good control performance of RLVs, it is



Fig. 1 Winged rocket

necessary to estimate the state variables. For estimation of the parameters and state variables of the nonlinear systems, Unscented Kalman Filter (UKF) is researched [9]. The UKF was developed as approximating a Gaussian distribution instead of linear approximating for a nonlinear system such as Extended Kalman Filter (EKF). Therefore, it is said that UKF is more effective than EKF for the estimation of a nonlinear system [10].

In this paper, we propose a digital adaptive feedback linearization control method using UKF to a winged rocket, and to validate the effectiveness of the control systems in wide range flight conditions, guidance and control simulations are done. The simulation results show that the control system has a good control performance.

## 2 Model of Winged Rocket

Fig. 1 shows an outline of a winged rocket [11] which is one of the RLVs. The body length is assumed to be 2.5[m] and the mass is 241[kg]. The rocket has two elevons and two rudders as aerodynamic control surfaces. Here, the equation of motion of the winged rocket divided into fast motion and slow motion is shown. Since the configuration of the winged rocket shown in Fig. 1 is similar for general airplanes [12]. Roll, pitch and yaw rotational rates with a fast response are assumed to be fast time scale variables  $\mathbf{x} = [p, q, r]^T$ , and angle of attack, sideslip angle

and bank angle with a slow response are assumed to be slow time scale variables  $\mathbf{y} = [\alpha, \beta, \phi]^T$ . Therefore, equations of motion are expressed as follows:

$$\dot{\mathbf{x}}(t) = \mathbf{A}(\mathbf{x})\mathbf{x}(t) + \mathbf{B}(\mathbf{x})\mathbf{u}(t) + \mathbf{C}(\mathbf{x})\mathbf{w}(t), \quad (1)$$

$$\dot{\mathbf{y}}(t) = \mathbf{D}(\mathbf{y})\mathbf{x}(t) + \mathbf{E}(\mathbf{y})\mathbf{h}(t) + \mathbf{g}(t) \quad (2)$$

where

$$\mathbf{u}(t) = [\delta_{ac}, \delta_{ec}, \delta_{rc}]^T,$$

$$\mathbf{w}(t) = [pq, qr, rp, r^2 - p^2, \alpha, \beta]^T,$$

$$\mathbf{h}(t) = \begin{bmatrix} 1 \\ \frac{1}{V_{TAS} C_\beta}, \frac{1}{V_{TAS}} \end{bmatrix}^T,$$

$$\mathbf{g}(t) = \begin{bmatrix} \frac{g(C_\phi C_\alpha C_\theta + S_\alpha S_\theta)}{V_{TAS} C_\beta} \\ \frac{g(S_\theta C_\alpha S_\beta + C_\theta S_\phi C_\beta - C_\theta C_\phi S_\alpha S_\beta)}{V_{TAS}} \end{bmatrix},$$

$$\mathbf{A}(\mathbf{x}) = \mathbf{I}_d^{-1} \begin{bmatrix} L_p & 0 & L_r \\ 0 & M_q & 0 \\ N_p & 0 & N_r \end{bmatrix}, \quad \mathbf{B}(\mathbf{x}) = \mathbf{I}_d^{-1} \begin{bmatrix} L_{\delta_a} & 0 & L_{\delta_r} \\ 0 & M_{\delta_e} & 0 \\ N_{\delta_a} & 0 & N_{\delta_r} \end{bmatrix},$$

$$\mathbf{C}(\mathbf{x}) = \mathbf{I}_d^{-1} \begin{bmatrix} I_{xz} & I_{yy} - I_{zz} & 0 & 0 & 0 & L_\beta \\ 0 & 0 & I_{zz} - I_{xx} & I_{xz} & M_\alpha & 0 \\ I_{xx} - I_{yy} & -I_{zz} & 0 & 0 & 0 & N_\beta \end{bmatrix},$$

$$\mathbf{D}(\mathbf{y}) = \begin{bmatrix} -C_\alpha \tan \beta & 1 & -S_\alpha \tan \beta \\ S_\alpha & 0 & -C_\alpha \\ 1 & S_\phi \tan \theta & C_\phi \tan \theta \end{bmatrix},$$

$$\mathbf{E}(\mathbf{x}) = \begin{bmatrix} -\frac{L}{m} & 0 \\ 0 & \frac{Y}{m} \\ 0 & 0 \end{bmatrix}, \quad \mathbf{I}_d = \begin{bmatrix} I_{xx} & 0 & I_{xz} \\ 0 & I_{yy} & 0 \\ I_{xz} & 0 & I_{zz} \end{bmatrix}$$

and  $\theta$  is pitch angle,  $m$  is mass,  $I_{ij}$  is moment/product of inertia,  $g$  is gravitational acceleration,  $V_{TAS}$  is true airspeed,  $L_*$ ,  $M_*$  and  $N_*$  are aerodynamic rolling, pitching and yawing moments,  $\delta_a$ ,  $\delta_e$  and  $\delta_r$  are aileron, elevator and rudder deflection angles, and  $S_* = \sin *$ ,  $C_* = \cos *$ .

When  $\dot{\mathbf{x}}$  and  $\dot{\mathbf{y}}$  are discretized for forward difference by a sampling period  $T$ , Eqs. (1) and (2) are expressed as follows:

$$\mathbf{x}(k+1) = \mathbf{A}_T(\mathbf{x})\mathbf{x}(k) + \mathbf{B}_T(\mathbf{x})\mathbf{u}(k) + \mathbf{C}_T(\mathbf{x})\mathbf{w}(k) \quad (3)$$

$$\mathbf{y}(k+1) = \mathbf{y}(k) + \mathbf{D}_T(\mathbf{y})\mathbf{x}(k) + \mathbf{E}_T\mathbf{h}(k) + T\mathbf{g}(k) \quad (4)$$

where  $\mathbf{A}_T(\mathbf{x}) = \mathbf{I} + T\mathbf{A}(\mathbf{x})$ ,  $\mathbf{B}_T(\mathbf{x}) = T\mathbf{B}(\mathbf{x})$ ,  $\mathbf{C}_T(\mathbf{x}) = T\mathbf{C}(\mathbf{x})$ ,  $\mathbf{D}_T(\mathbf{y}) = T\mathbf{D}(\mathbf{y})$ ,  $\mathbf{E}_T = T\mathbf{E}$  and  $\mathbf{I}$  is identity matrix.

### 3 Control System

In this section, the structure of a control system for Eqs. (3) and (4) in case the value of parameters are known is shown. Moreover, adaptive estimation using UKF which inhibits the suppress of a sensor noise and can respond to wide range flight environment flexibly is described.

#### 3.1 Known parameter case

In the case that the value of parameters of Eqs. (3) and (4) are known the control commands for Eqs. (3) and (4) are defined as follows [8]. First, the rotational rate command for the slow state, i.e. control input to Eq. (4) is defined as

$$\mathbf{x}_c(k) = \mathbf{D}_T^{-1} \{ \mathbf{y}_d(k+1) - \mathbf{y}(k) - \mathbf{E}_T(\mathbf{y})\mathbf{h}(k) - T\mathbf{g}(k) - \mathbf{P}[\mathbf{y}_d(k) - \mathbf{y}(k)] \} \quad (5)$$

where  $\mathbf{P} = \text{diag}\{p_i\}$  ( $i = \alpha, \beta, \phi$ ) is the gain matrix and  $\mathbf{y}_d$  is the value which filtered  $\mathbf{y}_c$  by a low pass filter. From Eqs. (4) and (5), the equation of the output error  $\mathbf{e}_s(k) = \mathbf{y}_d(k) - \mathbf{y}(k)$  is

$$\mathbf{e}_s(k+1) = \mathbf{P}\mathbf{e}_s(k). \quad (6)$$

And if  $p_i$  is selected to satisfy  $0 < p_i < 1$ , the output error  $\mathbf{e}_s(k)$  tends to zero as  $k$  tends to infinity.

Second, the control input for the fast state (Eq. (3)) is defined as

$$\mathbf{u}(k) = \mathbf{B}_T^{-1} \{ \mathbf{x}_d(k+1) - \mathbf{A}_T(\mathbf{x})\mathbf{x}(k) - \mathbf{C}_T(\mathbf{x})\mathbf{w}(k) - \mathbf{A}[\mathbf{x}_d(k) - \mathbf{x}(k)] \} \quad (7)$$

where  $\mathbf{A} = \text{diag}\{\lambda_i\}$  ( $i = p, q, r$ ) is the gain matrix and  $\mathbf{x}_d$  is the value which filtered  $\mathbf{x}_c$  by a low pass filter. From Eqs. (3) and (7), the equation of the output error  $\mathbf{e}_f(k) = \mathbf{x}_d(k) - \mathbf{x}(k)$  is

$$\mathbf{e}_f(k+1) = \mathbf{A}\mathbf{e}_f(k). \quad (8)$$

And if  $\lambda_i$  is selected to satisfy  $0 < \lambda_i < 1$ , the output error  $\mathbf{e}_f(k)$  tends to zero as  $k$  tends to infinity.

#### 3.2 Estimation by unscented Kalman filter

In the case of unknown parameters, we introduce the matrices  $\hat{\mathbf{A}}_T(\mathbf{x})$ ,  $\hat{\mathbf{B}}_T(\mathbf{x})$ ,  $\hat{\mathbf{C}}_T(\mathbf{x})$ , etc. those coefficients are estimated values of the coefficients in  $\mathbf{A}_T(\mathbf{x})$ ,  $\mathbf{B}_T(\mathbf{x})$ ,  $\mathbf{C}_T(\mathbf{x})$ , etc., respectively. Moreover, these matrices include the state variables. Therefore, states and parameters are estimated for the improvement of the control performance. For estimation of the parameters and state variables of nonlinear systems, UKF is used [9]. The UKF calculates the covariance of estimation by using the finite sample point that is called a sigma point around the mean. Then, sigma points are propagated through the nonlinear functions, and statistic is approximated, nonlinear system is estimated. Because the correction by the sample point is done, error margins are smaller than EKF. A detailed explanation of the UKF is omitted for want of space.

For Eqs. (5) and (7), the augmented state vector  $\bar{\mathbf{x}}$  to estimate the parameters at the same time by

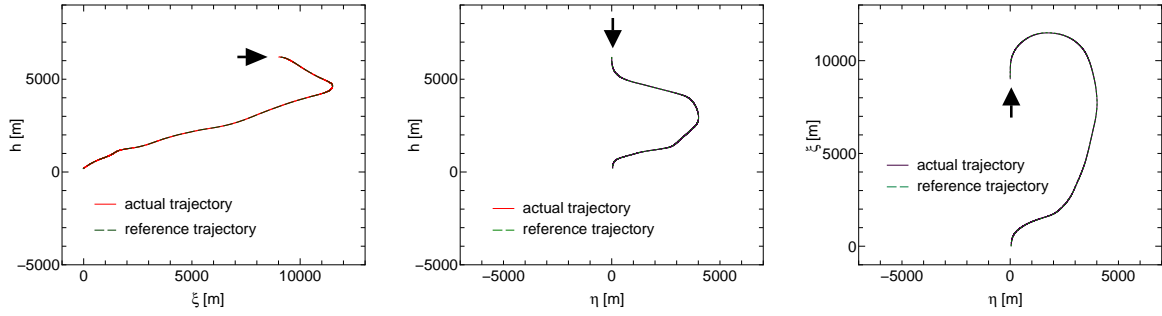


Fig. 2 Flight trajectory

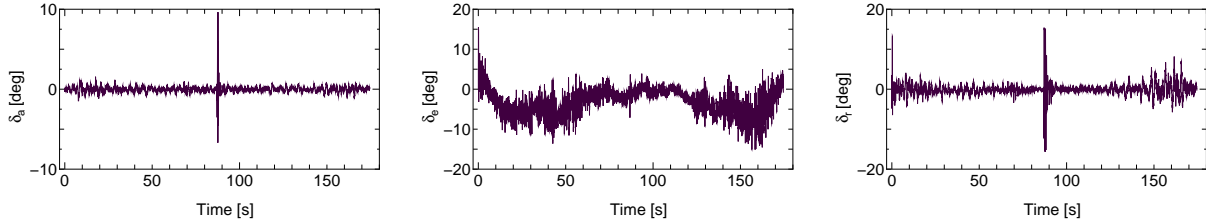


Fig. 3 Input

the UKF is introduced as follows:

$$\bar{\mathbf{x}}(k) = [p \ q \ r \ \alpha \ \beta \ \phi \ \theta \ V_{TAS} \ I_{xx} \ I_{yy} \ I_{zz} \ I_{xz} \ L_p \ L_r \ M_q \ N_p \ N_r \ L_{\delta_\alpha} \ L_{\delta_r} \ M_{\delta_e} \ N_{\delta_\alpha} \ N_{\delta_r} \ L_\beta \ M_\alpha \ N_\beta \ \frac{X}{m} \ \frac{Y}{m} \ \frac{Z}{m}]^T \quad (9)$$

And the augmented measurement vector is  $\bar{\mathbf{y}}(k) = [p \ q \ r \ \alpha \ \beta \ \phi \ \theta \ V_{TAS}]^T$ .

#### 4 Simulation

To validate the adaptive control system described above, computer simulations for a 6-DOF nonlinear winged rocket model [13] considering the atmospheric fluctuation are performed.

The simulation condition is follows. When the winged rocket is in level flight those velocity is 93[m/s], the guidance is started at  $h = 6200$ [m],  $\xi = 9000$ [m] and  $\eta = 0$ [m] in altitude, downrange and crossrange, respectively. Moreover, the wind shear that the wind velocity in a downrange direction rapidly changes from  $-10$ [m/s] to  $10$ [m/s] while flying was generated by about the 2500[m] in altitude. The guidance to the target point uses optimal trajectory generation using a genetic algorithm (GA) [14] which is a real-time guidance method. This method performs orbital optimization using GA, in order to suit flight environment without a preplanned reference trajectory and to generate a flexible trajectory, and it outputs an angle of attack command and a bank angle command required in order to realize the trajectory. The guidance is updated every 1[s]. The aerodynamic coefficient of the winged rocket uses the value obtained from the wind tunnel examination result. The actuators of elevons and rudders are used

that the attenuation coefficient  $\zeta = 0.7$  and the natural frequency  $\omega_n = 72$ [rad/s]. The sampling period of the control system is  $T = 0.01$ [s]. The gains are  $\lambda_p = 0.94$ ,  $\lambda_q = 0.92$ ,  $\lambda_r = 0.94$ ,  $p_\alpha = 0.95$ ,  $p_\beta = 0.96$  and  $p_\phi = 0.97$ . The time constants of the low-pass filters for  $p$ ,  $q$ ,  $r$ ,  $\alpha$  and  $\beta$  are 0.2, 0.3, 0.5, 0.5 and 0.5[s], respectively. The UKF [9] is utilized for state and parameter estimation.

Fig. 2 shows the flight trajectory. From Fig. 2, we can see that the winged rocket can follows the reference trajectory. Fig. 3 shows the time history of the control input commands to the actuators. And Fig. 4 shows the time history of the output  $\mathbf{y}$  and the estimated error  $e_* = * - \bar{*}$ . In Fig. 4 a dashed lines show the desired value obtained from the guidance. Since the wind shear is considered in this simulation, from Fig. 3 we can see that the input commands have vibrations with small amplitude. However, from Fig. 4, it can be seen that the estimation by the UKF is performed good and the output  $\mathbf{y}$  follows the desired value. From the simulation result, it can be confirmed that the applied control method is effective for the flight control of the winged rocket.

#### 5 Conclusion

In this paper, we propose a digital adaptive feedback linearization control method using UKF to a winged rocket. From the numerical simulation, we showed that the control system of the winged rocket has a good control performance.

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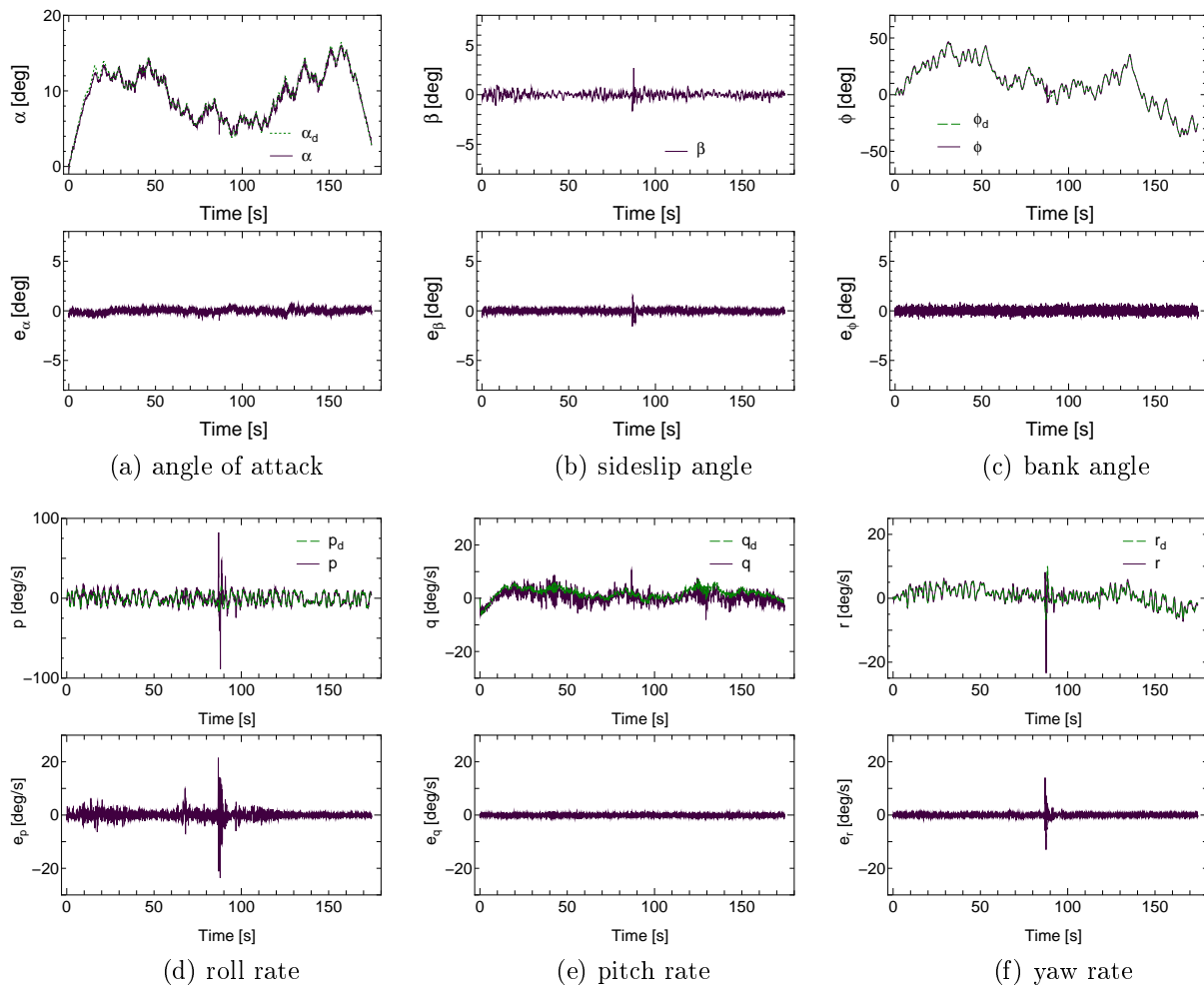


Fig. 4 Output and estimated error

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