

Model predictive control of a power-split hybrid electric vehicle system

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Abstract: This paper presents a model predictive control (MPC) approach for the energy management problem of a power-split hybrid electric vehicle (HEV) system. The MPC is suggested to optimally share the road load to the engine and the battery. By analyzing the configuration of the power-split HEV system, we developed a simplified model for better implementation of MPC. The MPC problem is solved using numerical computation method: continuation and generalized minimum residual (C/GMRES) method. The computer simulation results showed that the fuel economy was improved using the MPC approach than the ADVISOR rule based approach over three driving cycles respectively. We conclude that the MPC approach is effective for the application of power-split HEV systems energy management and has the potential for real-time implementation. The simplified modeling method of the power-split HEV system configuration can be applied to other configurations of HEV.

Keywords: energy management, hybrid electric vehicle, model predictive control

1 INTRODUCTION

In recent years, HEV has become a research hotspot due to rising cost of fossil fuels and environmental problems. HEV has an electrical power source and it can downsize the engine, optimize the engine operating point and recuperate braking energy, which helps to improve fuel economy, and reduces emissions Serrao [1].

The key technology of HEV is its energy management. A lot of works have been published on the energy management problem of HEV systems. These approaches are typical in a family of optimal control techniques. And they can be subdivided into four categories: numerical optimization, analytical optimal control theory, instantaneous optimization, heuristic control techniques Serrao [1]. The most representative of numerical optimization is dynamic programming (DP) Serrao [1] and Liu et al [2]. However DP is based on known driving cycle which is impossible to get in reality. A kind of analytical optimal control techniques is Pontryagin's minimum principle Kim et al [3]. It gives necessary conditions that the optimal solution must satisfy. It also needs to know the entire driving cycle in prior. The instantaneous optimization includes the equivalent consumption minimization strategy (ECMS) Serrao [1]. It is based on instantaneous optimization and is easy to implement in real time. However it can not guarantee the optimality over the whole driving cycle. Heuristic control techniques like rule based control strategies are robust but they are impossible to guarantee the optimality.

Although MPC Borhan et al [4] is also in numerical optimization class, its advantage is its predictive nature which can use vehicle-road-traffic information in the near future Deguchi et al [5] and Kamal et al [6] and be applicable to unknown driving cycles Kaku et al [7]. Based on a simple

and accurate model of the system, MPC can provide real time control for the system. This paper examines energy management problem of a power-split HEV system over known and unknown driving cycles. Because the power-split HEV system has functionality of both series and parallel HEV systems, it has more modes to operate the energy management system for better fuel economy. The simplified modeling method by introducing constraints to reducing the system degrees of freedom is presented.

The rest of this paper is organized as follows: In Section 2, simplified model of the power-split HEV system and MPC algorithm are presented. Section 3 gives comparative simulation results between the MPC approach and the ADVISOR Wipke et al [8] rule based approach over three different driving cycles. Section 4 provides conclusions.

2 MODELING OF THE POWER-SPLIT HEV SYSTEM

The configuration of the power-split HEV system is shown in **Fig. 1.** FD represents the final drive. The power split device (PSD) is the key component of the power-split HEV system and has both functionality of speed coupler and continuously variable transmission (CVT). There are five dynamic components: the engine, the battery, two motor/generators (M/G), and the wheels in this power-split HEV system, the only dynamic state to be considered in the optimal control problem based on known driving cycle is the battery state of charge (SOC) which can simplify the MPC algorithm for implementation. This simplification is possible because we introduce four constraints: the road load, the torque and speed relationship of the speed coupler, the

power flow relationship among the five components, and the engine optimal operating line (OOL) using CVT. We divided the optimal control problem into two levels. The high level controller determines the optimal battery power and the low level controller determines the optimal torque and speed of the engine and the motor/generators. In this paper we focus on the high level controller.

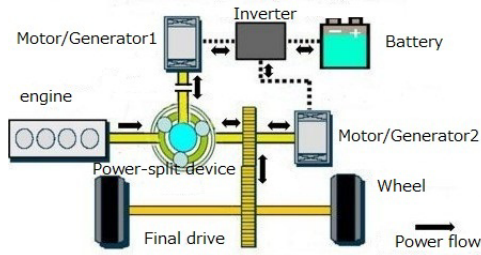


Fig. 1. Configuration of the power-split HEV system. Diagram adapted from Liu et al [2]

The torque and speed relationship of the speed coupler can be expressed as Ehsani et al [9]:

$$\begin{aligned} \tau_{eng}(t) &= -(1 + \frac{R}{S})\tau_{M/G1}(t) \\ \tau_{eng}(t) &= -(1 + \frac{S}{R})(\tau_{M/G2}(t) - \frac{\tau_{req}(t)}{g_f}) \\ S\omega_{M/G1}(t) + R\omega_{M/G2}(t) - (S + R)\omega_{eng}(t) &= 0 \end{aligned} \quad (1)$$

where S and R are the number of sun gear and ring gear teeth respectively, $\tau_{M/G1}$, $\tau_{M/G2}$, τ_{req} , and τ_{eng} are the torques of the M/G1, M/G2, the road load and the engine respectively, $\omega_{M/G1}$, $\omega_{M/G2}$ and ω_{eng} are the angular speeds of the M/G1, the M/G2, and the engine respectively.

The power flow relationships among the five components at the inverter and the power split device in Fig. 1. are given as:

$$\begin{aligned} P_{batt}(t) &= P_{M/G1}(t) + P_{M/G2}(t) \\ P_{req}(t) &= P_{M/G1}(t) + P_{M/G2}(t) + P_{eng}(t) \end{aligned} \quad (2)$$

where P_{batt} , $P_{M/G1}$, $P_{M/G2}$, P_{eng} , and P_{req} are the power of the battery, the M/G1, the M/G2, the engine, and the road load.

We assume that the engine always works along its OOL using CVT which can also be considered as a constraint. When the engine power is known, by looking up the table of OOL, the engine speed and torque can be obtained.

We evaluate the fuel consumption using Willans line method to reduce the complexity of the engine fuel consumption model. It was found that good approximation are obtained using the Willans line method Serrao [1]. The fuel consumption can be expressed as:

$$\dot{m}_f(t) = \dot{m}_f(P_{req}(t) - P_{batt}(t)) \approx c_f(P_{req}(t) - P_{batt}(t)) \quad (3)$$

where c_f is a constant.

The road load which are the vehicle speed and the required power at the wheels is known when the driving cycle

is known. From the configuration of the power-split HEV system, the M/G2 speed is also known as:

$$\omega_{M/G2}(t) = \frac{g_f}{r_w} v_{req}(t) \quad (4)$$

where $\omega_{M/G2}$ is the speed of the M/G2, g_f is the final drive gear ratio, r_w is the wheel radius, v_{req} is the required vehicle speed by the driving cycle.

When the driving cycle is known, the system dynamics is reduced to the battery dynamics. The optimization objective is only the fuel economy. The only state variable is the battery SOC, x_{SOC} , and the control input is the battery power. The battery model can be expressed as Kim et al [3]:

$$\dot{x}_{SOC} = -\frac{V_{OC} - \sqrt{V_{OC}^2 - 4P_{batt}R_{batt}}}{2R_{batt}Q_{batt}} \quad (5)$$

where V_{OC} , R_{batt} , and Q_{batt} are the open circuit voltage, the internal resistance, and the capacity of the battery.

When the driving cycle is unknown, the system dynamics includes the battery and the vehicle dynamics. Both the fuel economy and the driving profile are optimized. The system model is then represented by

$$\dot{x} = \begin{bmatrix} v \\ w - \frac{1}{2}\rho C_D A v^2 / m - g\mu - g \sin(\theta(p)) \\ k_p(u_1 - w) \\ -\frac{V_{OC} - \sqrt{V_{OC}^2 - 4P_{batt}R_{batt}}}{2R_{batt}Q_{batt}} \end{bmatrix} \quad (6)$$

$$x = [p \ v \ w \ x_{SOC}]^T$$

$$u = [u_1 \ P_{batt}]^T \quad (7)$$

where p , v , and w are the vehicle position, speed, and acceleration or deceleration converted from the traction force or brake force. ρ , C_D , A , m , g , μ , and $\theta(p)$ are the air density, the air drag coefficient, the frontal area of the vehicle, the vehicle mass, the gravity acceleration, the rolling resistance coefficient, and the road grade. u_1 and k_p are the vehicle acceleration or deceleration control input and the delay constant.

Due to the simplified modeling method derived from the power relationship among the engine, the battery and the road load which is general in the HEV configurations, it can be applied to other HEV configurations.

3 MODEL PREDICTIVE CONTROL

The optimal control problem based on known driving cycle is defined as:

$$\text{Min. } J_{known} = \int_t^{t+T} L_{known}(x_{SOC}(\tau|t), P_{batt}(\tau|t)) d\tau \quad (8)$$

$$\text{Subject to: } SOC_{min} \leq x_{SOC}(\tau|t) \leq SOC_{max}$$

$$P_{battmin} \leq P_{batt}(\tau|t) \leq P_{battmax} \quad (9)$$

where T is the prediction horizon, min and max denote the minimum and maximum bounds of battery SOC and power.

The objectives of this optimal control problem is to minimize the fuel consumption, meanwhile, the battery SOC is maintained between the thresholds. This is achieved by minimizing the cost function L_{known} including three terms: the

fuel use, the engine use and the mechanical brake use, and the deviation of battery SOC from the reference value. The cost function L_{known} is defined as follows:

$$L_{known} = w_1 c_f (P_{req} - P_{batt}) / (1 + e^{-\beta(P_{req} - P_{batt})}) + w_2 (P_{req} - P_{batt})^2 + w_3 (x_{SOC} - SOC_d)^2 + w_4 (-\ln(x_{SOC} - SOC_{min}) - \ln(SOC_{max} - x_{SOC})) \quad (10)$$

where SOC_d is the desired SOC value. w_1, w_2, w_3 and w_4 are the weights. The sigmoid function is chosen to evaluate the vehical brake fuel consumption. The log barrier function is used as a penalizing term for violations of state constraints.

The optimal control problem based on unknown driving cycle is defined as:

$$\text{Min. } J_{unknown} = \int_t^{t+T} L_{unknown}(x(\tau|t), u(\tau|t)) d\tau \quad (11)$$

$$\text{Subject to: } SOC_{min} \leq x_{SOC}(\tau|t) \leq SOC_{max} \\ P_{battmin} \leq P_{batt}(\tau|t) \leq P_{battmax} \\ u_{1min} \leq u_1(\tau|t) \leq u_{1max} \quad (12)$$

The cost function $L_{unknown}$ is defined as follows:

$$L_{unknown} = w_x L_x + w_y L_y + w_z L_z + w_d L_d + w_e L_e + w_f L_f \\ L_x = (w - \frac{1}{2} \rho C_D A v^2 / m - g \mu)^2 \\ L_y = (v - v_d)^2 \\ L_z = c_f (m w v - P_{batt}) / (1 + e^{-\beta(m w v - P_{batt})}) \\ L_d = (x_{SOC} - SOC_d)^2 \\ L_e = (m w v - P_{batt})^2 \\ L_f = -\ln(x_{SOC} - SOC_{min}) - \ln(SOC_{max} - x_{SOC}) \quad (13)$$

where $w_x, w_y, w_z, w_d, w_e,$ and w_f are the weights, v_d is the desired vehicle speed. The first term of $L_{unknown}$ indicates the acceleration and braking cost.

At each time t , the optimal control input is computed by solving the above optimal control problems during the prediction horizon T . Only the first element of the optimal control sequence is applied. At the next time step, the prediction horizon moves forward, and the process is repeated.

4 COMPUTER SIMULATION

4.1 Simulation conditons

In this simulation, vehicle parameters are obtained from ADVISOR 2002. Fig. 2. gives the engine OOL of the power-split HEV system. The vehicle parameters are $m=1368$ [kg], $\rho=1.23$ [kg/m³], $C_D=0.3$, $A=1.746$ [m²], $g=9.8$ [m/s²], $\mu=0.015$, $V_{OC}=307.85$ [V], $R_{batt}=1.004$ [Ω] and $Q_{batt}=6$ [Ah], $c_f=0.0874$. The control parameters are $\beta=0.5$, $SOC_d=0.7$, $SOC_{min}=0.6$, $SOC_{max}=0.8$, $k_p=10$, $P_{battmin}=-20$ [kW], $P_{battmax}=20$ [kW], $u_{1min}=-3$ [m/s²], $u_{1max}=3$ [m/s²], $v_d=50$ [km/h], $w_1=4000$, $w_2=5000$, $w_3 = 3.5 \times 10^7$ and $w_4 = 10^5$, $w_x=10$, $w_y=20000$, $w_z=40000$, $w_d = 9 \times 10^7$, $w_e=2200$ and $w_f=100$. The MPC problem is solved using a numerical computation method: continuation and generalized minimum residual (C/GMRES) method Ohtsuka et al [10].

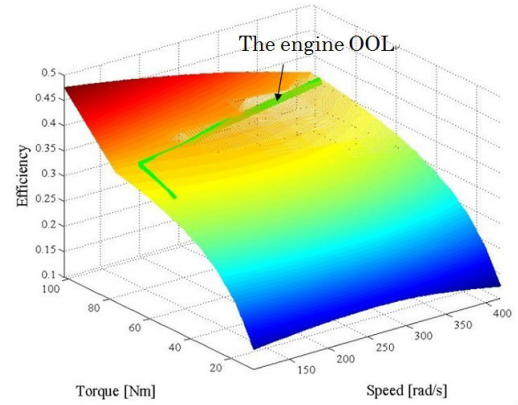


Fig. 2. The engine OOL of the power-split HEV system

Driving cycle 1 included acceleration, deceleration, and cruise scenario (see the first row of Fig. 3.). Driving cycle 2 was the standard driving cycle: Japan 10-15 (see the first row of Fig. 4.). Driving cycle 3 was an unknown driving cycle with road slopes (see the first and second row of Fig. 5.).

4.2 Simulation results

Driving cycle 1:

The simulation results of driving cycle 1 were presented in Fig. 3. using the ADVISOR simulation results as comparison. We observed from these results that the MPC algorithm performed better than the ADVISOR rule based algorithm. In the case of MPC, the battery assisted vehicle driving and recuperated vehicle braking power properly.

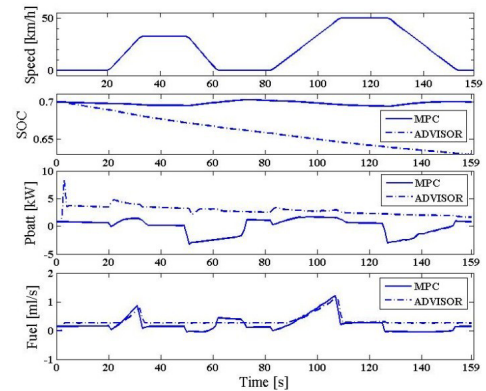


Fig. 3. Performance of the MPC algorithm and the ADVISOR rule based algorithm over driving cycle 1

Driving cycle 2:

Japan 10-15 simulation results (see Fig. 4.) showed that the MPC algorithm performed similarly as driving cycle 1.

Driving cycle 3:

The unknown driving cycle simulation results (see Fig. 5.) showed that the MPC algorithm could also use the road slope information well to reduce the fuel consumption. The vehicle accelerated before the upslope to make use of the kinetic energy. The battery recuperated vehicle braking power during the vehicle downslope driving. Since over this low road load

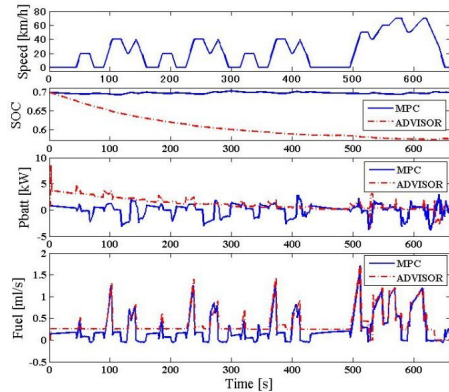


Fig. 4. Performance of the MPC algorithm and the ADVISOR rule based algorithm over driving cycle 2

driving cycle the ADVISOR rule-based algorithm used the motor assist driving mode, the fuel consumption reduction by MPC was not evident.

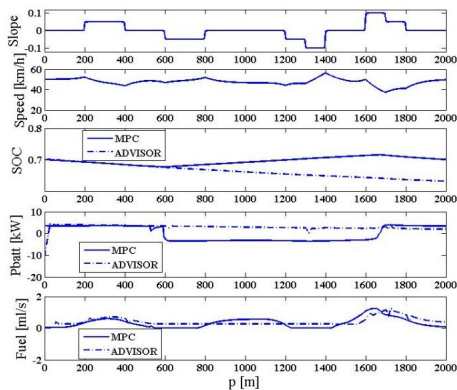


Fig. 5. Performance of the MPC algorithm and the ADVISOR rule based algorithm over driving cycle 3

The overall fuel economy results over the three driving cycles were presented in **Table 1**. We can see that the MPC approach can improve fuel economy significantly and keep the final SOC near the initial SOC compared to the ADVISOR approach over the three driving cycles. Although the ADVISOR approach considers the idling, accessories losses, battery efficiency and motor/generator efficiency, it performs poorly according to the road load, which results in the large deviation of the battery SOC. Therefore the MPC approach results are optimistic because of not considering those losses, but the results are still reasonably better compared with the ADVISOR approach.

5 CONCLUSION

MPC of a power-split HEV system was presented. The simplified system model was developed. The simulation results of the MPC algorithm using known and unknown driving cycle revealed a significant improvement of the fuel econ-

Table 1. Fuel economy comparison results

Method	Cycle	Initial SOC	Final SOC	Fuel (km/l)
MPC(known)	Cycle1	0.7	0.6997	26.40(+33.64%)
ADVISOR	Cycle1	0.7	0.6302	17.52
MPC(known)	Cycle2	0.7	0.6995	23.97(+32.87%)
ADVISOR	Cycle2	0.7	0.5745	16.09
MPC(unknown)	Cycle3	0.7	0.7000	38.49(+16.19%)
ADVISOR	Cycle3	0.7	0.6312	32.26

omy compared to the ADVISOR rule based algorithm. Because the MPC algorithm uses simplified system model and can be applied to unknown driving cycle, it has the potential for the real-time implementation. The simplified modeling method of the power-split HEV system configuration can also be applied to other configurations of HEV.

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