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Two-dimensional merging path generation using model predictive control

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Abstract: In this research, the merging problem is considered in the two-dimensional space instead of the one-dimensional space. In this paper, we set up the mathematic model of the system, formulate the two-dimensional merging problem as an optimization problem and solve it by model predictive control (MPC). To compare the simulation results with the practical situation, three typical cases were researched. In order to be more practical, the initial conditions of the cases were set according to the data obtained through analyzing the helicopter-shot video. The results represent that the MPC-controlled merging maneuver carried out safely and smoothly, and the relative positions after merging is also the same with the practical results in all the three representative conditions considered in this research. The absolute values of the accelerations of the vehicles are all below 3m/s^2, which are quite practical as well. The simulation results also represented the importance of the adjustment in driving during merging. By adjusting these vehicles, this control algorithm would generate the merging path that could avoid merging accident even in the very severe condition.

Keywords: automotive control, merging, model predictive control, optimal control, path generation.

1 INTRODUCTION

As motorization grows, it is becoming more and more important to realize a safe and ecological driving. From Ohashi et al [1] and Takayama et al [2] we could know that one of the riskiest maneuvers that a driver has to perform is to merge into the traffic, especially for the beginners and the aged people. It would take a lot courage and attention for them to conduct the maneuver. Iguchi et al [3] tells that merging is also one of the reasons for traffic congestion at the same time. The traffic throughput is often reduced at the merging part.

As a result, many researchers are now working on this subject. Jula et al [4] worked on collision avoidance merging through calculating the safe region. However, this paper does not represent the exact merging way for the vehicles. It will still be a difficult problem for the driver to merge into the traffic. Lu et al [5] worked on automated vehicle merging, and provided the 'adaptive solution', but the ecological requirement was not considered.

This paper presents an algorithm for merging path generation, in which the two-dimensional merging problem was formulated as a nonlinear optimization problem. The initial conditions were set according to the helicopter-shot video data. The input constraints, and the performance function, which realizes ecology and safety, were also set realistically. With all these elements chosen, the optimization problem was solved by the C/GMRES method proposed in Otsuka [6]. To investigate the effectiveness of the control algorithm, three representative cases were researched. The computer simulation results show that in all the three conditions the algorithm could generate smooth merging and the relative positions of the two vehicles are also practical. The results of the case, which have a quite severe initial condition, also represent the function of trajectory adjustment of this algorithm, what we think is also the key point during merging. If a vehicle merges automatically under the control of this algorithm, the driver would not feel any pressure during the merging maneuver, and the ecological requirement could also be satisfied at the same time.

The rest part of this paper is arranged as follows. In the section 2 we describe the formulation of the merging problem. The simulation results and analysis are shown in the section 3. The section 4 gives some conclusions of the whole paper.

2 MERGING PROBLEM FORMULATION

Due to the difficulty of merging and the time varying nature of the traffic situation, it is essential to predict the future situation of the traffic. Taking these aspects into account, we choose nonlinear model predictive control to solve the merging problem. As shown in **Fig. 1.**, in this research, only one vehicle on the merging lane, and one vehicle on the main lane, are considered. The vehicle on the merging lane is denoted as Vehicle1, and the vehicle on the main lane is denoted as Vehicle2 in the following parts of this paper. The merging problem is formulated as the following optimization problem.

$$\min_a J = \int_t^{t+T} L(x(\tau), a(\tau), \tau) d\tau \quad (1)$$

subject to the input constraints and the vehicle dynamics.

Here, x denotes the state variables, and a denotes the input of the system, respectively. The expression of function L is described in Subsection 2.3.

The optimal control input is updated at each time step by solving the above optimal control problems during the prediction horizon T. Only the first element of the optimal control sequence is applied to the system as the actual input. The process is repeated at each time step.

2.1 Vehicle model

To save the calculation time, a simplified vehicle dynamics is used. Instead of the actual three-dimensional movement, only the horizontal motion is considered. In this paper it is assumed that the main lane can be approximated to be straight and the width of the main lane is quite a small number compared to the length of it. Then the lateral movement of Vehicle2 is omitted. We assume that Vehicle2 always moves on the middle line of the main lane. This means that the coordinate, the velocity and the acceleration of Vehicle2 in the Y-axis direction are all 0.

Under these assumptions, the state equation of the system is as follows.

$$\dot{x} = \begin{bmatrix} \dot{x}_{hx} \\ \dot{x}_{hy} \\ \dot{x}_m \\ \dot{v}_{hx} \\ \dot{v}_{hy} \\ \dot{v}_m \end{bmatrix} = \begin{bmatrix} v_{hx} \\ v_{hy} \\ v_m \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ a_{hx} \\ a_{hy} \\ a_m \end{bmatrix} \quad (2)$$

where

$$x = [x_{hx} \ \ x_{hy} \ \ x_m \ \ v_{hx} \ \ v_{hy} \ \ v_m]^T$$
$$a = [a_{hx} \ \ a_{hy} \ \ a_m]^T$$

Here, x_{hx} and x_{hy} denote Vehicle1's coordinate in X-axis and Y-axis respectively. v_{hx} and v_{hy} denote the velocities, and a_{hx} and a_{hy} denote the accelerations of Vehicle1 in the X-axis and Y-axis respectively. The x_m, v_m and a_m denote the coordinate, the velocity and the acceleration of Vehicle2 in X-axis respectively.

During merging, the two vehicles have to adjust their velocities. Therefore the brake and the accelerator are involved. The acceleration and the deceleration have upper bounds. It is assumed that the system has the following input constraints.

$$\begin{cases} -a_{hxmax} \le a_{hx} \le a_{hxmax} \\ -a_{hymax} \le a_{hy} \le a_{hymax} \\ -a_{mmax} \le a_m \le a_{mmax} \end{cases} \quad (3)$$

Where a_{hxmax}, a_{hymax}, and a_{mmax} are the upper bounds of the absolute value of a_{hx}, a_{hy}, and a_m, respectively.

2.2 Expression of the road shape

Fig. 1. shows the figure of the merging part of the helicopter-shot roads and the initial position of the two vehicles. The ○ represents the initial position of Vehicle1 and the × represents the initial position of Vehicle2.

Fig. 1. The merging part of the roads

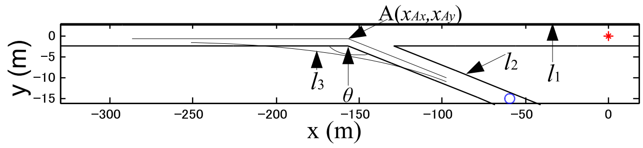

Fig. 2. The approximation results of the roads

To simplify the calculation we approximate this typical roads' figure with two straight lines and a hyperbola. The result is shown in **Fig.2.**. Choose the initial position of Vehicle2 as the origin of the coordinate system, the driving direction of Vehicle2 as the X-axis positive direction, the expressions of the lines are as follows:

$$l_1 = x_{hy} - \frac{1}{2}d = 0 \quad (4)$$
$$l_2 = kx_{hx} + x_{hy} + c = 0 \quad (5)$$
$$l_3 = 1 + \frac{(-\sin\frac{1}{2}\theta(x_{hy} - x_{Ax}) + \cos\frac{1}{2}\theta(x_{hy} - x_{Ay}))^2}{k_d(\tan\frac{1}{2}\theta)^2} - \frac{(\cos\frac{1}{2}\theta(x_{hx} - x_{Ax}) + \sin\frac{1}{2}\theta(x_{hy} - x_{Ay}))^2}{k_d} = 0 \quad (6)$$

The meanings of the variables are shown in Table 1. To keep safe, point A is set at the upper right side of the intersection point of the left side of the merging lane and the main lane.

Table 1. Variables in l_1, l_2, and l_3

d	the width of the main lane
k	the slope of the merging lane
c	a constant of the road shape
θ	the angle between the main lane and the merging lane
x_{Ax}	abscissa of the point A
x_{Ay}	ordinate of the point A
k_d	a constant of the shape of the hyperbola

2.3 Performance function

Subject to the input constrains, there are also many requirements which must be considered during the merging maneuver. These requirements are realized by the performance function as equation 1. The expression of function $L(x(\tau), a(\tau), \tau)$ is as follows.

$$\begin{aligned} L &= \frac{\omega_1}{r^2} - \omega_2 \log(l_1) - \omega_3 \log(l_2) - \omega_4 \log(l_3) \\ &+ \omega_5 (v_{hx} - v_m)^2 + \omega_6 a_{hx}^2 + \omega_7 a_{hy}^2 + \omega_8 a_m^2 \\ &+ \omega_9 (v_{hx} - v_i)^2 + \omega_{10} (v_m - v_i)^2 + \omega_{11} x_{hy}^2 \quad (7) \end{aligned}$$

Where ω_1, ω_2, ω_3, ω_4, ω_5, ω_5, ω_6, ω_7, ω_8, ω_9, ω_{10}, and ω_{11}, represent the weight of every term respectively, r is the relative distance of the two vehicles, defined in equation 8, and v_i is the desired value of the velocity.

$$r = ((x_{hx} - x_m)^2 + x_{hy}^2)^{\frac{1}{2}} \quad (8)$$

The cost function L consists of 11 terms. The first term is the cost due to the relative distance r. The second term, the third term and the forth term are the barrier functions to represent the road shape. Among them, the second term represents the restriction of the line l_1, which is shown in **Fig. 2.**, the third term represents the restriction of the line l_2, and the forth term represents the restriction of the line l_3. All the barrier functions are chosen as minus logarithm functions, because the restrictions of the road shape could never be violated, or there will be an accident. The fifth term describes the cost due to relative velocity, because running at the same speed would generate smooth merging. The terms from the sixth one to the eighth one describe the requirements of ecology. They minimize the accelerations and decelerations to save unnecessary fuel consumption. The ninth term and the tenth term make the two vehicles run as closely as possible at the desired value of the velocity v_i. The eleventh term is used to reduce the overshoot of Vehicle1.

3 SIMULATION RESULTS AND ANALYSIS

To investigate the effectiveness of the control algorithm, three representative cases are researched. The computer simulation was conducted with the time step $h = 0.01$s, and the prediction horizon is $T = 1$s. The values of the parameters are chosen suitably as follows: $\omega_1 = 4.0$, $\omega_2 = 0.01$, $\omega_3 = 0.01$, $\omega_4 = 0.01$, $\omega_5 = 0.01$, $\omega_6 = 0.03$, $\omega_7 = 0.01$, $\omega_8 = 0.01$, $\omega_9 = 0.01$, $\omega_1 0 = 0.01$, $\omega_1 1 = 0.01$, $d = 5$m, $k = -0.16$, $c = 24$, $\theta = 3.0$rad, $x_{Ax} = -160$m, $x_{Ay} = -2.0$m, $k_d = 0.01$, $v_i = -17$m/s (about 60km/h), $a_{hxmax} = a_{hymax} = a_{mmax} = 3$m/s^2.

3.1 Case 1

The initial conditions of the two involved vehicles in this case were set according to the data drawn from the helicopter-shot video. The obtained initial conditions are as follows: $(x_{hx}, x_{hy}) = (-59\text{m}, -15\text{m})$, $x_m = 0$m, $v_{hx} = -9.9$m/s, $v_{hy} = 1.6$m/s, $v_m = -20$m/s, $a_{hx} = 0$m/s^2, $a_{hy} = 0.03$m/s^2, $a_m = -0.05$m/s^2. The actual merging trajectories of this situation is shown in **Fig. 3.**, while the simulation result is shown in **Fig. 4.**. In **Fig. 3.** the ◯ represents the position of Vehicle1 and the × represents the position of Vehicle2 of every time respectively. In **Fig. 4.** The ∘ represents the position of Vehicle1 and the ∗ represents the position of Vehicle2 of every time respectively. The time in these two figures is: $t_0 = 0$s, $t_1 = 4$s, $t_2 = 8$s, $t_3 = 12$s, $t_4 = 16$s respectively. **Fig. 5.** shows the variation of the variables during the merging maneuver.

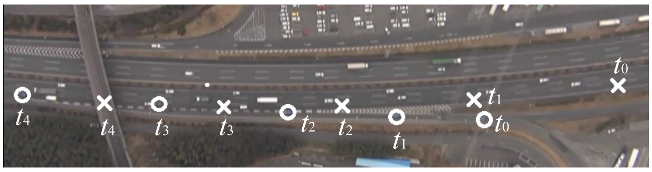

Fig. 3. The actual merging result of the case 1

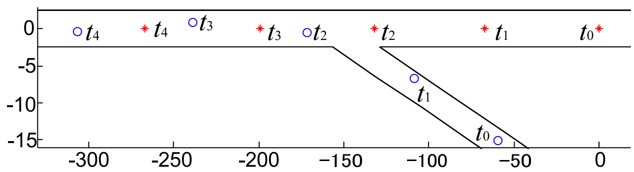

Fig. 4. Merging trajectories of the two vehicles of the case 1

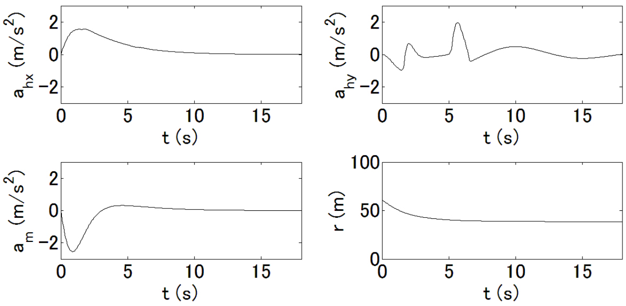

Fig. 5. Variation of the variables in the case1

We could see from **Fig. 4.** that Vehicle1 merged into the main lane without hitting any side of the main lane and the merging lane, and ran in front of Vehicle2 after merging. The time-history of r is shown in the second diagram in the second row of **Fig. 5.**. From it we could see that during the merging maneuver Vehicle1 kept an appropriate distance with Vehicle2. We could also see from **Fig. 5.** that all accelerations were kept below 3m/s^2.

3.2 Case 2

The initial condition of the two vehicles were set as follows: $(x_{hx}, x_{hy}) = (-59\text{m}, -15\text{m})$, $x_m = -59$m, $v_{hx} = -9.9$m/s, $v_{hy} = 1.6$m/s, $v_m = -20$m/s. Accelerations are all the same with the case 1. Since $x_{hx} = x_m$, $|v_{hx}| < |v_m|$, and accelerations are very small, without control, Vehicle1 would go to the behind of Vehicle2 and follow it. The simulation results, shown in **Fig. 6.** and **Fig. 7.**, represented that Vehicle1 merged to the behind of Vehicle2 successfully just as it was supposed to be.

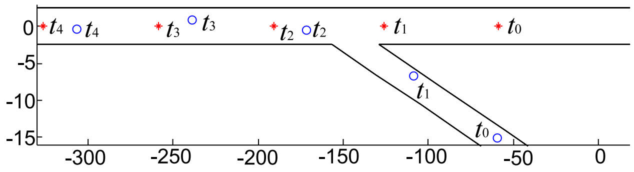

Fig. 6. Merging trajectories of the two vehicles of the case 2

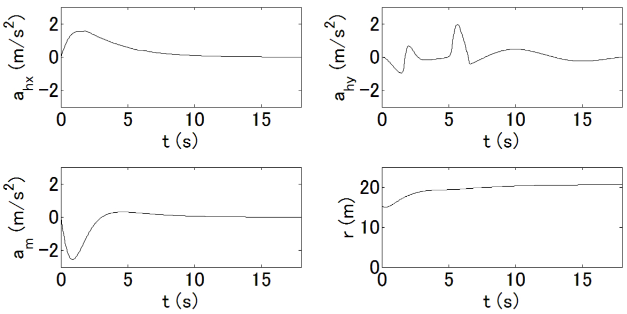

Fig. 7. Variation of the variables in the case 2

We could see from **Fig. 7.** that Vehicle1 kept an appropriate distance with Vehicle2 too. Accelerations are always below 3m/s^2 as well.

3.3 Case 3

As it was considered in the two dimensional space, x_{hy} could be involved in the calculation of r. This enables an extreme condition, that is: $(x_{hx}, x_{hy}) = (-59\text{m}, -15\text{m})$, $x_m = -59\text{m}$, $v_{hx} = -16\text{m/s}$, $v_{hy} = -1.6\text{m/s}$, $v_m = -16\text{m/s}$, $a_{hx} = 0\text{m/s}^2$, $a_{hy} = -0.03\text{m/s}^2$, $a_m = 0\text{m/s}^2$. Since $x_{hx} = x_m$, $v_{hx} = v_m$, $a_{hx} = a_m$, if no adjustment of trajectories is employed, the two vehicles will collide with each other. However the simulation results shown in **Fig. 8.** and **Fig. 9.** represented that Vehicle1 merged successfully to the behind of Vehicle2, and kept an appropriate distance. The accelerations were kept below 3m/s^2. The relative distance was more than 2m, hence the collision was avoided successfully.

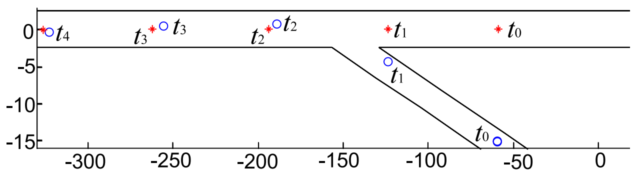

Fig. 8. Merging trajectories of the two vehicles of the case 3

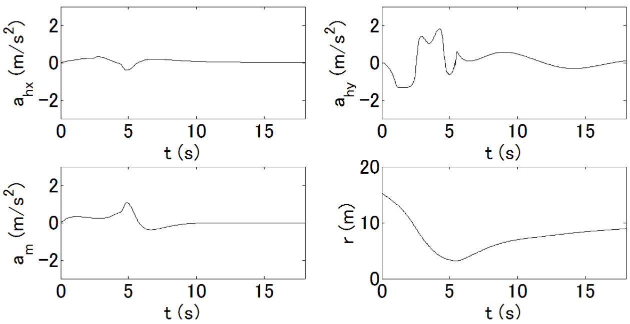

Fig. 9. Variation of the variables in the case 3

4 CONCLUSION

With the algorithm proposed in this paper, in all the three representative conditions the trajectories of the two vehicles showed that the merging vehicle merged successfully. The distance between the merging vehicle and the main lane vehicle is kept long enough during the merging maneuver as well. So the requirement of safety is satisfied. Besides, both the absolute values of the accelerations of the two vehicles can be kept below the specified value. Furthermore, the relative positions of the two vehicles in the simulation results are the same with the actual situations. That is, in the case 1 Vehicle1 became the preceding vehicle, which is the same with the helicopter-shot result, and in the case 2 Vehicle1 became the following vehicle as it should be. Remarkably, in the case 3 Vehicle1 slowed down a little to keep the safe distance between Vehicle2 and itself, instead of colliding with Vehicle2. This clearly shows the function of trajectory adjustment of this algorithm, which we think is also the key point of merging. The simulation results of the three cases show that the proposed algorithm enables smooth merging. Under the control of this algorithm, the two vehicles could merging smoothly by adjusting with each other.

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