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Fuzzy Servo Control of an Inverted Pendulum System

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Abstract: The studies of model based fuzzy control system are concentrating on regulator problem. But in real system, the output of the system needs to be regulated to the desired reference output which is not only zero. Servo system can regulate the output to desired reference output without steady state error against unknown disturbance. In this paper, application of fuzzy servo control for stabilizing inverted pendulum system will be discussed. The inverted pendulum system is a simple system consists of pendulum and cart but has strong nonlinearity and inherent instability. The simulations are done and the result shows that the proposed method can stabilize the system.

Keywords: Davison-Smith method, Fuzzy control, Inverted pendulum system, Nonlinear servo system

1 INTRODUCTION

The studies of model based fuzzy control system are concentrating on regulator problem, which is involved with the design of the controller that can drive all the initial condition to zero as required by the design specification. In real system, the output of the system needs to be regulated to the desired reference output which is not only zero. Servo system can regulate the output to desired reference output without steady state error against unknown disturbance. In this paper, application of fuzzy servo control for stabilizing inverted pendulum system will be discussed. The inverted pendulum system is a simple system consists of pendulum and cart but has strong nonlinearity and inherent instability. It is used as benchmark problem for study the performance and effectiveness of new control method. The idea behind of this control method is to divide the operating region of nonlinear system into small area, and to treat as a collection of local linear servo systems by using Davison-Smith method. The control rule of each local linear servo systems is calculated using pole assignment method proposed by Hikita. Fuzzy method is applied to each local linear servo system and combines it as new control rule. The simulations have been done and the results shown that proposed can stabilize the system. As shown in the result, the output of the system follows the reference given and converges to the reference value as desired.

2 SYSTEM DESCRIPTION

Let the original system S be a nonlinear system one as

$$\dot{x} = f(x,u) + d \quad (1)$$
$$y = g(x) + d_o \quad (2)$$

Where, $x \in R^n$ is state vector, $u \in R^m$ is control input, $y \in R^l$ is control output, $d \in R^n$ is state disturbance and $d_o \in R^l (m \geq l)$ is output disturbance. n, m and l are dimensions of state, input and output of the system. Assume the trajectory of nonlinear system around operating point of the system(x_i, u_i) as

$$\delta x = x - x_i \quad (3)$$
$$\delta u = u - u_i. \quad (4)$$

Where δx and δu represent the small quantity of state and input variable respectively.The system can be linearized by applying Taylor expansion to (1) and (2) around operating point (x_i, u_i), where

$$\delta \dot{x} = \frac{\partial}{\partial x^T} f(x_i, u_i)\, \delta x + \frac{\partial}{\partial u^T} f(x_i, u_i)\, \delta u + f(x_i, u_i) + d \quad (5)$$
$$y = \frac{\partial}{\partial x^T} g(x_i)\, \delta x + g(x_i) + d_o. \quad (6)$$

The linear approximated system S_i can be represented as,

$$\dot{x} = A_i x + B_i u + d_{xi} \quad (7)$$
$$y = C_i x + d_{oi} \quad (8)$$

where,
$A_i = \frac{\partial}{\partial x^T} f(x_i, u_i), B_i = \frac{\partial}{\partial u^T} f(x_i, u_i), C_i = \frac{\partial}{\partial x^T} g(x_i), d_{xi} = f(x_i, u_i) - A_i x_i - B_i u_i + d, d_{oi} = g(x_i) - C_i x_i + d_o$.
Let error of the system be as following

$$\dot{v} = y - r. \quad (9)$$

Derive the augmented system from (7),(8) and (9).

$$\frac{d}{dt}\begin{bmatrix} x \\ v \end{bmatrix} = \begin{bmatrix} A_i & 0 \\ C_i & 0 \end{bmatrix}\begin{bmatrix} x \\ v \end{bmatrix} + \begin{bmatrix} B_i \\ 0 \end{bmatrix} u + \begin{bmatrix} d_{xi} \\ d_{oi} - r \end{bmatrix} \quad (10)$$

Equation(10) can be re-written as

$$\dot{z} = A_{zi}z + B_{zi}u + d_{zi} \tag{11}$$

where,
$z = \begin{bmatrix} x \\ v \end{bmatrix}, A_{zi} = \begin{bmatrix} A_i & 0 \\ C_i & 0 \end{bmatrix}, B_{zi} = \begin{bmatrix} B_i \\ 0 \end{bmatrix},$
$d_{zi} = \begin{bmatrix} d_{xi} \\ d_{oi} - r \end{bmatrix}.$
The system that has ability to follow the given reference is called as servo system.This system in (11) is controllable if controlability condition satisfies.

$$rank\left[B_{zi}\ \ A_{zi}B_{zi}\ \ A_{zi}^2B_{zi}\ \ \cdot\cdot\ \ A_{zi}^{n+l-1}B_{zi}\right] = n+l. \tag{12}$$

This condition is equivalent to the next one.

$$rank\left[\,B_i\ \ A_iB_i\ \ A_i^2B_i\ \ \cdots\ \ A_i^{n-1}B_i\,\right] = n,$$
$$rank\begin{bmatrix} A_i & B_i \\ C_i & 0 \end{bmatrix} = n+l \tag{13}$$

3 FUZZY SERVO SYSTEM

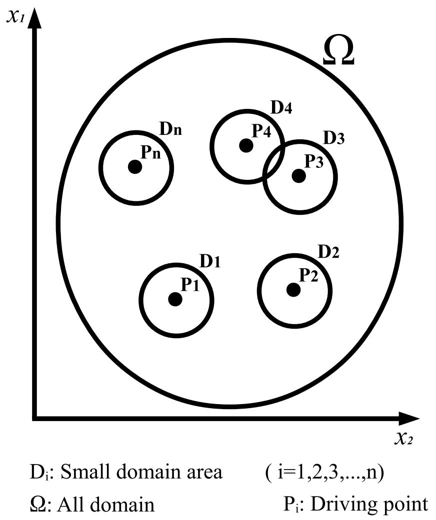

Fig. 1. Partition of driving domain Ω

In this section we apply fuzzy method to develop the control rule for the nonlinear system. First, nonlinear variable for system in (1) and (2) will be determined. Let, the element of nonlinear variable for x in funtion $f(x,u), g(x)$ as x_{ej} and can be determine as

$$\frac{\partial}{\partial x_{ej}} f\left(x,u\right) \neq const. \tag{14}$$

This definition also can be use for output equation u. The origin point $(0,0)$ are used as linear driving point. Let, the vector of the set of nonlinear variable x and u for $f(x,u), g(x)$ as $x_n \in R^{n_{xn}}$ and $u_n \in R^{n_{un}}$. The relationship between x, u and x_n, u_n can be shown as

$$x_n = C_{xn}x \tag{15}$$
$$u_n = C_{un}u \tag{16}$$

$C_{xn} \in R^{n_{xn} \times n} and C_{un} \in R^{n_{un} \times m}$ are the matrix of which its elements are 1 or 0. Define the vector for all nonlinear variable as $z_n \in R^{n_{zn}}, n_{zn} = n_{xn} + n_{un}$, the relatioship between the variable can be represent as

$$z_n = \begin{bmatrix} x_n \\ u_n \end{bmatrix} = \begin{bmatrix} C_{xn} & 0 \\ 0 & C_{un} \end{bmatrix}\begin{bmatrix} x \\ u \end{bmatrix} = C_n\begin{bmatrix} x \\ u \end{bmatrix}. \tag{17}$$

Where $C_n \in R^{n_{zn} \times (n+m)}$. Next, we divide the region of nonlinear variable z_n,into small area D_i around driving point P_i. Fuzzy method has been apply to each local linear system and combines it and treated as a collection of local linear servo systems (Fig.1). The fuzzy rule R_i can be define as

$$R_i: \quad IF\ \ z_n \in D_i\ \ THEN\ \ S\ \ is\ \ S_i. \tag{18}$$

Where, the fuzzy rules are $-N \leq i \leq N$. Let ω_i define as Gaussian type fuzzy membership function (Fig.2) as following.

$$\omega_i \;=\; e^{-(z_n - z_{ni})^T Q_n (z_n - z_{ni})} \tag{19}$$
$$Q_n \;=\; {Q_n}^T > 0 \tag{20}$$

ρ_i can be define as,

$$\rho_i = \frac{\omega_i}{\sum_{i=-N}^{N} \omega_i}. \tag{21}$$

The Fuzzy Servo System can be re-write as,

$$\dot{z} = \sum_{i=-N}^{N} \rho_i \left(A_{zi}z + B_{zi}u + d_{zi}\right). \tag{22}$$

In order to stabilized the system in (22), the control input u can be defines as

$$u = -\sum_{i=-N}^{N} \rho_i K_i z. \tag{23}$$

K_i is the feedback coefficient matrix of the input for small area D_i. This matrix calculated using Hikita method [3] based on servo system proposed by Davison-Smith[2]. Let

$$f_{ij} \;=\; -(\lambda_{ij}I - A_{zi})^{-1}B_{zi}g_{ij} \tag{24}$$
$$(j \;=\; 1,2,3,...n+l),$$
$$g_{ij} \;\in R^m, f_{ij} \in R^{n+l}.$$

λ_{ij} be the eigenvalues, g_{ij} be non-zero vector and f_{ij} be eigenvector of the system. The feedback co-efficient matrix can be represented as

$$K_i \;=\; \left[\ g_{i1}\ \ \ldots\ \ g_{i(n+l)}\ \right]\left[\ f_{i1}\ \ \ldots\ \ f_{i(n+l)}\ \right]^{-1} \tag{25}$$
$$(-N \leq i \leq N)$$

The input u in (23) are applied to system in (1) and (2).

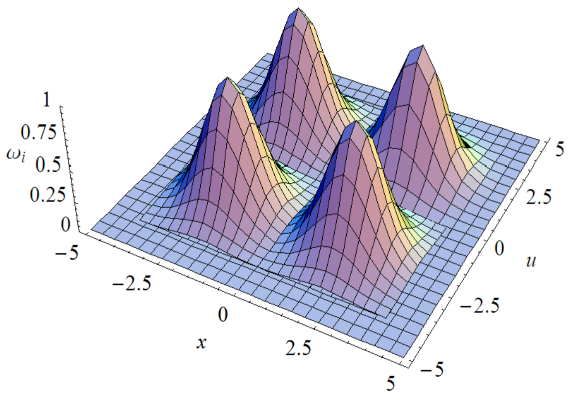

Fig. 2. Gaussian type fuzzy membership function ω_i

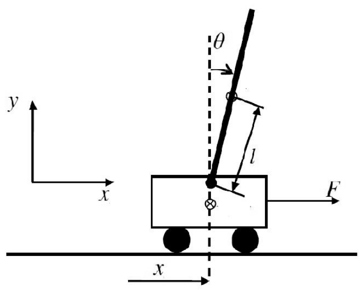

Fig. 3. Inverted Pendulum system

4 SIMULATION AND RESULTS

The inverted pendulum used in this simulation is shown in Fig.3. It consists of cart and a pendulum. The cart is free to move the horizontal direction when the force F applies to it. We assume that the mass of the pendulum and cart are homogenously distributed and concentrated in their center of the gravity and the friction of the cart is proportional only to the cart velocity and friction generating by the pivot axis is proportional to the angular velocity of the pendulum. The parameters used for simulation are shown in Table 1.The mathematical model of inverted pendulum system can be described as the following.

$$(M+m)\ddot{x}+mlcos\theta\ddot{\theta}+D\dot{x}-mlsin\theta\dot{\theta}^2=F \tag{26}$$

$$mlcos\theta\ddot{x}+\frac{4}{3}ml^2\ddot{\theta}-mglsin\theta+C\dot{\theta}=0 \tag{27}$$

With output as

$$y=x_1 \tag{28}$$

Define an error of the system as

$$\dot{v}=e=y-r. \tag{29}$$

Table 1. Parameters of Inverted Pendulum System

Parameter	Description	Value
Mass of the cart	M	0.165kg
Mass of the pendulum	m	$0.12kg$
Distance from pivot to center of mass of the pendulum	l	$0.25m$
Gravitational constant	g	$9.80m/s^2$
Co-efficient of friction for pivot	C	$0.01kgm/s$
Co-efficient of friction for cart	D	$4.0kg/s$

Re-arranged the equation in term of $x_1=x, x_2=\dot{x}, x_3=\theta, x_4=\dot{\theta}$ and $F=u$. The new equation for inverted pendulum are shown in (30), (31),(32) and (33).

$$\dot{x_1} = x_2 \tag{30}$$

$$\dot{x_2} = 3cosx_3\left(mglsinx_3-Cx_4\right) -4l\left(u-Dx_2+mlsinx_3x_4^2\right)/ \left\{4l(m+M)+3mlcosx_3^2\right\} \tag{31}$$

$$\dot{x_3} = x_4 \tag{32}$$

$$\dot{x_4} = -3\left\{(m+M)(mglsinx_3-Cx_4) - mlcosx_3(u-Dx_2+mlsinx_3x_4^2)\right\}/ \left[ml^2\left\{-4(m+M)+3mcosx_3^2\right\}\right] \tag{33}$$

The nonlinear variable for the system are calculated using (14). In this example the nonlinear variable are x_2, x_3, x_4 and u. The linearization has been done to mathematical model of the inverted pendulum system using Taylor expansion as shown in (5) and (6) around of the driving points for nonlinear variables x_2, x_3, x_4 and u. Where,

$$\delta x_2 = x_2 - x_{2i} \tag{34}$$

$$\delta x_3 = x_3 - x_{3i} \tag{35}$$

$$\delta x_4 = x_4 - x_{4i}. \tag{36}$$

$$\delta u = u - u_i. \tag{37}$$

The linearization has been done and the linear servo system can be represent as

$$\dot{z} = A_{zi}z + B_{zi}u + d_{zi}. \tag{38}$$

The controllability of the servo system of inverted pendulum system has been investigated using (12). In this case, the output of system has been setting as shown in (28) because of the controllability problem. Let,

$$Q_n=\begin{bmatrix} q_2 & 0 & 0 & 0 \\ 0 & q_3 & 0 & 0 \\ 0 & 0 & q_4 & 0 \\ 0 & 0 & 0 & q_5 \end{bmatrix} \tag{39}$$

and the nonlinear variables x_2, x_3, x_4 and u can be represent as

$$z_n = \begin{bmatrix} x_2 \\ x_3 \\ x_4 \\ u \end{bmatrix}, C_n = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (40)$$

The fuzzy membership function ω_i can be represent as

$$\begin{aligned} \omega_i &= e^{-(z_n - z_{ni})^T Q_n (z_n - z_{ni})} \\ &= e^{-q_2(x_2 - x_{2i})^2 - q_3(x_3 - x_{3i})^2 - q_4(x_4 - x_{4i})^2 - q_5(u - u_i)^2} \end{aligned} \quad (41)$$

So that,

$$\rho_i = \frac{\omega_i}{\sum_{i=-N}^{N} \omega_i}. \quad (42)$$

The fuzzy servo system for inverted pendulum system is,

$$\dot{z} = \sum_{i=-N}^{N} \rho_i \left(A_{zi} z + B_{zi} u + d_{zi} \right). \quad (43)$$

The set of poles used is $\lambda = [-1.2 - 1.3 - 1.4 - 1.5 - 1.6]$ .The feedback coefficient matrices K_i are calculated using Hikita Method. The input of the system is

$$u = -\sum_{i=-N}^{N} \rho_i K_i z. \quad (44)$$

with $x_{2i} = h_2 i(-N_2 \leq i \leq N_2), x_{3i} = h_3 i(-N_3 \leq i \leq N_3), x_{4i} = h_4 i(-N_4 \leq i \leq N_4), u_i = h_5 i(-N_5 \leq i \leq N_5), N_2 = 5, N_3 = 5, N_4 = 5, N_5 = 5, N = (N_2) \times (N_3) \times (N_4) \times (N_5) = 625$, increment $h_2 = 0.1, h_3 = 0.01, h_4 = 0.1, h_5 = 0.1$. The initial condition is $z_0 = [0\ 0\ \theta\ 0\ 0]^T$ with value of angle, θ is 33^o.The simulation are done for three different values of Q_n. Where, in simulation 1: $q_2 = 1.0, q_3 = 1.0, q_4 = 1.0, q_5 = 1.0$, simulation 2: $q_2 = 14.0, q_3 = 14.0, q_4 = 14.0, q_5 = 14.0$ and simulation 3: $q_3 = 0.1, q_3 = 0.1, q_4 = 0.1, q_5 = 0.1$.

The graph in Fig.4 and Fig.5 are the results for the simulation. The result shows that not only the output of system $y = x_1 = x$, but the value of $x_3 = \theta$ also converges to zero. The effect of varying the value of matrix Q_n also can be observe in the result. The simulation shows that the fuzzy servo control can stabilize inverted pendulum system.

5 CONCLUSION

In this paper, we discuss about the application of the fuzzy servo control on inverted pendulum system. The theory used have been explained in detail and the simulations have been done. As shown in the result, the output of the system follows the reference given and converges to the reference value as desired. For future work, study of characteristic of the proposed method will be done by doing the simulation using other system together with the study of the stability issue.

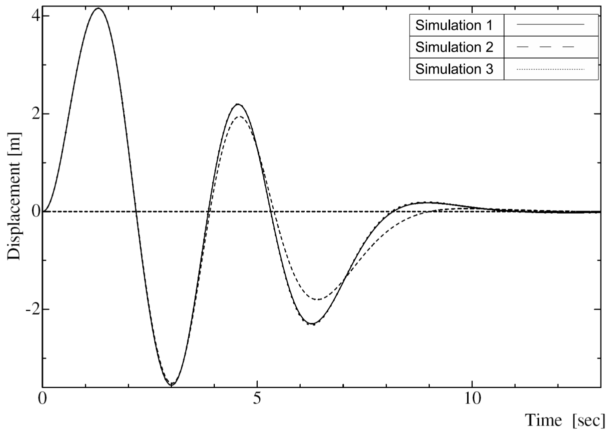

Fig. 4. Simulation result for $y = x_1 = x$

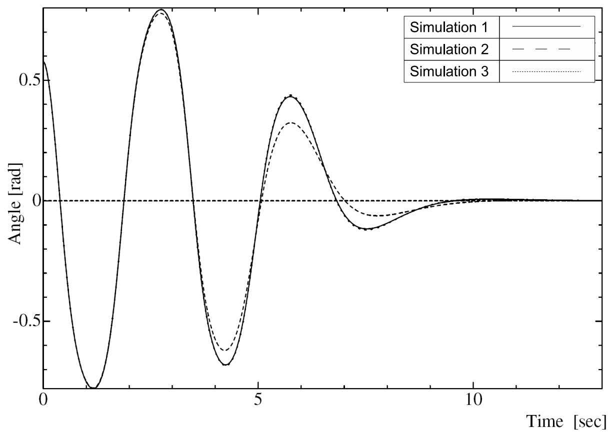

Fig. 5. Simulation result for $x_3 = \theta$

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