

# A centralized control system for ecological vehicle platooning using linear quadratic regulator theory

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**Abstract:** This paper presents an ecological vehicle platooning control system that aims in reducing overall fuel consumption of the vehicles in a platoon. A centralized linear quadratic regulator system for controlling the vehicles in the platoon has been developed considering the aerodynamic characteristics of the vehicle and the resistance due to the road slope. The proposed control system is simulated on a highway with up-down slopes for high speed driving. Its fuel saving performance is compared with a conventional decentralized vehicle platooning control system. Computer simulation results reveal the significant improvement in fuel economy by the proposed control system.

**Keywords:** Aerodynamic characteristics, centralized control, ecological driving, linear quadratic regulator theory, platooning, road shape information.

## 1 INTRODUCTION

In recent years, global warming and ozone depletion have been gotten widespread international attention. Since the energy consumption of vehicles accounts for a substantial amount of all the energy consumption, it is considered that so-called “eco-driving” by vehicles is important to solve such environmental issues. Eco-driving can be characterized by a fuel-efficient driving style to match various road-traffic situations. The fuel consumption caused by the aerodynamic drag during high speed driving can be reduced by platooning. As shown in Fig.1, a number of vehicles run with short spacing in a line. The aerodynamic drag of each vehicle is reduced owing to the improved airflow profile around the platoon [1]. Therefore, fuel consumption of each vehicle caused by the aerodynamic drag can be reduced. Automated control of the vehicles has been introduced for such platooning. Various conventional methods mainly focused on the platoon formation and achievement of the string stability [2] [3] [4]. On the other hand, automated driving control system for energy saving of two vehicles has been proposed [5]. However, fuel saving by synchronized driving behavior of more vehicles was not considered. To build an ecological vehicle platooning (EVP) control system, it is considered that centralized controller is efficient in synchronizing motion of the vehicles by using driving information of all the vehicles in the platoon.

In this paper, a centralized linear quadratic regulator (LQR) system for the ecological vehicle platooning is proposed to reduce the overall fuel consumption of the vehicles in the platoon. The dynamical model of the platoon including the varying aerodynamic drag coefficient depending on

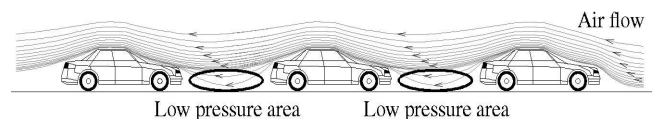


Fig. 1. Air flow profile around a platoon

vehicles' spacings and the road shape has been developed. This model is linearized around the target driving situation. The fuel consumption model based on the engine efficiency characteristics is constructed. Computer simulations considering high speed driving on a typical highway with up-down slopes were conducted. The fuel-saving performance of the proposed control system was compared with a conventional decentralized vehicle platooning control system (CVP). Significant improvement in fuel economy by the proposed control system was verified.

This paper is structured as follows. In Section 2, control for an ecological vehicle platooning is proposed. In Section 3, computer simulation results demonstrate the effectiveness of the proposed control system. In Section 4, conclusions are discussed.

## 2 CONTROL METHOD FOR ECOLOGICAL VEHICLE PLATOONING

### 2.1 Vehicle model

A platoon consisting of three vehicles is shown in Fig.2. Longitudinal movement of a vehicle in the platoon can be

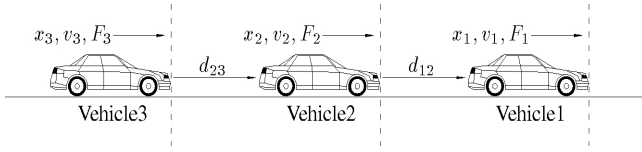


Fig. 2. A platoon consists of three vehicles

represented by the following equations:

$$\begin{aligned}
 \dot{x}_i(t) &= v_i(t), \\
 m_i \dot{v}_i(t) &= F_i(t) - f_{ai}(t) - f_{gi}(t) - f_{\mu i}, \\
 f_{ai}(t) &= \frac{1}{2} \rho A_i C_i(d_{12}(t), d_{23}(t)) v_i^2(t), \\
 f_{gi}(t) &= m_i g \sin \theta(x_i(t)), \\
 f_{\mu i} &= \mu_i m_i g, \\
 d_{12}(t) &= x_1(t) - x_2(t) - l_1, \\
 d_{23}(t) &= x_2(t) - x_3(t) - l_2,
 \end{aligned} \tag{1}$$

where, subscript  $i$  is an integer which indicates the  $i$ -th vehicle ( $i=1,2,3$ ) counted from the leading vehicle in the platoon. Parameters  $\rho$ ,  $g$ ,  $m_i$ ,  $A_i$ ,  $\mu_i$ , and  $l_i$  are the air density, the gravity acceleration, the mass, the projected frontal area, the coefficient of all the other friction resistances, and the length of the  $i$ -th vehicle, respectively. Variables  $x_i$ ,  $v_i$ ,  $F_i$ ,  $d_{12}$ , and  $d_{23}$  are the position, the velocity, the traction force or the brake force of the  $i$ -th vehicle, and the spacing between the neighbor vehicles, respectively. Functions  $f_{ai}$ ,  $f_{gi}$ , and  $f_{\mu i}$  denote the aerodynamic drag, the grade resistance, and all the other friction resistances of the  $i$ -th vehicle, respectively.  $C_i(d_{12}, d_{23})$  is the aerodynamic drag coefficient that depends on the spacing.  $\theta(x_i)$  is the road gradient that depends on the position of each vehicle. The time delay from input command to the traction force or the brake force is assumed to be represented by the first order lag:

$$\dot{F}_i(t) = -\frac{1}{\tau_i} F_i(t) + \frac{1}{\tau_i} u_i(t), \tag{2}$$

where,  $\tau_i$  and  $u_i$  are the time delay constant and the control input command of the traction force or the brake force of the  $i$ -th vehicle, respectively.

Around the target driving situation, the model of the vehicle' longitudinal equations are linearized. Each deviation of the velocity  $\delta v_i$ , the traction force or the brake force  $\delta F_i$ , the control input  $\delta u_i$ , and the spacings are represented by

$$\begin{aligned}
 \delta v_i(t) &= v_i(t) - v^*, \\
 \delta F_i(t) &= F_i(t) - f_{ai}^* - f_{gi}(t) - f_{\mu i}, \\
 \delta u_i(t) &= u_i(t) - f_{ai}^* - f_{gi}(t) - f_{\mu i}, \\
 \delta d_{12}(t) &= d_{12}(t) - d^*, \\
 \delta d_{23}(t) &= d_{23}(t) - d^*, \\
 f_{ai}^* &= \frac{1}{2} \rho A_i C_i(d^*, d^*) v^{*2},
 \end{aligned} \tag{3}$$

where,  $v^*$  and  $d^*$  are the target velocity and the target spacing. A state equation used in the centralized LQR control system is represented as

$$\begin{aligned}
 \dot{x}(t) &= Ax(t) + Bu(t), \\
 x(t) &= [\delta v_1 \ \delta F_1 \ \delta d_{12} \ \delta v_2 \ \delta F_2 \ \delta d_{23} \ \delta v_3 \ \delta F_3]^T, \\
 u(t) &= [\delta u_1 \ \delta u_2 \ \delta u_3]^T,
 \end{aligned} \tag{4}$$

$$A = \begin{bmatrix} -\frac{\alpha_1}{m_1} & \frac{1}{m_1} & -\frac{\beta_1}{m_1} & 0 & 0 & -\frac{\gamma_1}{m_1} & 0 & 0 \\ 0 & -\frac{1}{\tau_1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\beta_2}{m_2} & -\frac{\alpha_2}{m_2} & \frac{1}{m_2} & -\frac{\gamma_2}{m_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{\tau_2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & -\frac{\beta_3}{m_3} & 0 & 0 & -\frac{\gamma_3}{m_3} & -\frac{\alpha_3}{m_3} & \frac{1}{m_3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\tau_3} \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & 0 \\ \frac{1}{\tau_1} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \frac{1}{\tau_2} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\tau_3} \end{bmatrix}$$

$$\begin{aligned}
 \alpha_i &= \left. \frac{\partial f_{ai}}{\partial v_i} \right|_{v_i=v^*, d_{12}=d^*, d_{23}=d^*} \\
 \beta_i &= \left. \frac{\partial f_{ai}}{\partial d_{12}} \right|_{v_i=v^*, d_{12}=d^*, d_{23}=d^*} \\
 \gamma_i &= \left. \frac{\partial f_{ai}}{\partial d_{23}} \right|_{v_i=v^*, d_{12}=d^*, d_{23}=d^*},
 \end{aligned}$$

where,  $\alpha_i$ ,  $\beta_i$ , and  $\gamma_i$  are parameters of Taylor expansion of  $f_{ai}$  that ignore the terms over second-order.

## 2.2 Centralized control system using LQR

The performance index using LQR is formulated as

$$J = \int_0^\infty (x^T Q x + u^T R u) dt, \tag{5}$$

where,  $Q$  is positive semidefinite matrix, and  $R$  is positive definite matrix. The control input is calculated by

$$u_{LQR} = -R^{-1} B^T P x(t), \tag{6}$$

where,  $P$  is the solution of the following equation:

$$A^T P + P A - P B R^{-1} B^T P + Q = 0. \tag{7}$$

## 3 COMPUTER SIMULATION

In the computer simulations, a typical vehicle equipping continuously variable transmission (CVT) was selected. The parameters of the vehicles were  $m_i = 1480$  [kg],  $A_i = 2.87$  [m<sup>2</sup>],  $l_1 = l_2 = 4.3$  [m], and  $\tau_i = 0.1$ . The driving condition were  $\rho = 1.2$  [kg/m<sup>3</sup>] and  $\mu = 0.01$ . To construct the function  $C_i$ , a set of data that represents the relationship of  $C_i$ ,  $d_{12}$ , and  $d_{23}$  obtained from wind tunnel experiments [1] was adopted. The approximation results of  $C_i$  are shown in Fig.3 and Fig.4. The fuel consumption of the vehicle was

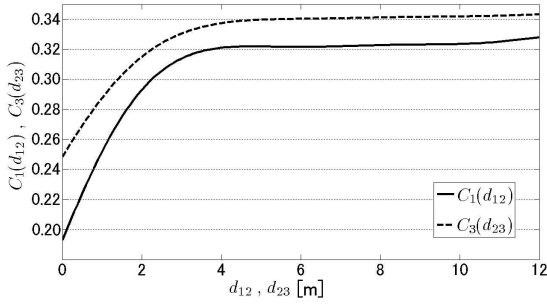


Fig. 3. Approximation results of  $C_1$  and  $C_3$

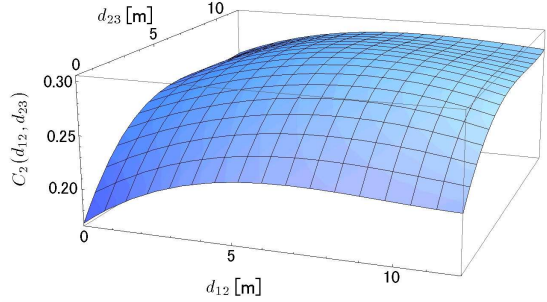


Fig. 4. Approximation result of  $C_2$

estimated as

$$f_{uel\ i}(t) = \begin{cases} \frac{P_i(t)}{\eta_i(P_i(t))Q} & \text{for } u_i \geq 0 \\ 0 & \text{for } u_i < 0 \end{cases},$$

$$P_i(t) = (m_i \dot{v}_i(t) + f_{ai}(t) + f_{gi}(t) + f_{\mu i})v_i(t) + P_c = F_i(t)v_i(t) + P_c, \quad (8)$$

where,  $f_i$  is the fuel consumption per unit time,  $P_i$  is the engine power output needed for driving a vehicle,  $\eta_i$  shown in Fig.5 is the maximum efficiency based on the engine characteristics map [6],  $Q = 34.5$  [MJ/L] is the calorific value of gasoline,  $P_c = 845.825$  [W] is the constant power required when the vehicle is idling. This equation was matched with the fuel consumption rate of the selected vehicle on the Japan 10-15 mode test.

At the initial time ( $t = 0$ ),  $v_1(0) = 95$  [km/h],  $v_2(0) = 90$  [km/h],  $v_3(0) = 85$  [km/h],  $d_{12}(0) = d_{23}(0) = 10$  [m]. The desired driving situation was set as  $v_i^* = 100$  [km/h] and  $d_{12}^* = d_{23}^* = 2$  [m]. Altitude and slope of the road shapes used here are shown in Fig.6.  $\sin \theta$  was approximated as  $\sin \theta \approx \theta$  since the road slope is very small. The matrices  $Q$  and  $R$  are defined as

$$Q = \text{diag}[ 340 \ 80 \ 12 \ 360 \ 80 \ 12 \ 380 \ 80 ],$$

$$R = \text{diag}[ 380 \ 390 \ 400 ]. \quad (9)$$

The sampling period was set as 0.01 [s]. To evaluate the performance of the proposed EVP control system, its computer simulation results were compared with a CVP control system. In the CVP control system [2], the control inputs of

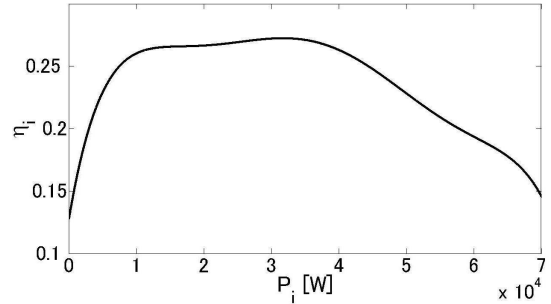


Fig. 5. Approximate engine efficiency  $\eta_i$

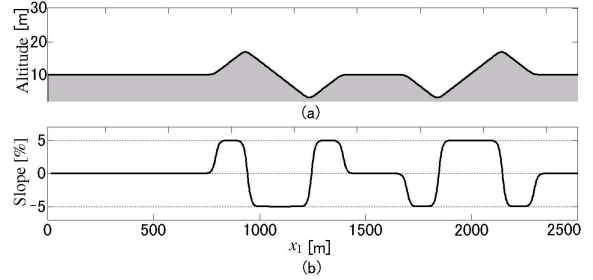


Fig. 6. Road shapes (a) altitude, (b) slope

the traction force or the brake force of the vehicles are represented as follows:

$$u_{CVP\ 1}(t) = k_a(v^* - v_1(t)) + \int_0^t k_p(v^* - v_1(t'))dt',$$

$$u_{CVP\ 2}(t) = k_1(v_1(t) - v_2(t)) - k_2(hv_2(t) + \delta_{min} - d_{12}(t)),$$

$$u_{CVP\ 3}(t) = k_1(v_2(t) - v_3(t)) - k_2(hv_3(t) + \delta_{min} - d_{23}(t)), \quad (10)$$

where,  $k_a = 1184$ ,  $k_p = 296$ ,  $k_1 = 29600$ ,  $k_2 = 2960$ ,  $\delta_{min} = 0.5$ , and  $h = 0.05$  are constant parameters. To confirm the advantage of the platoon driving, the solitude driving (SOL) of a vehicle without the platooning ( $u_{SOL} = u_{CVP\ 1}$ ) was also simulated. The initial velocity of this vehicle was the same as the velocity of the lead vehicle of the CVP.

The fuel consumption of the vehicles of the EVP, the CVP and the SOL at a distance of 2500 [m] are shown in Fig.7. Here, the improvement rates mean that the fuel consumption of the  $i$ -th vehicle of the EVP is compared with those of the CVP and the SOL. It is clear that the vehicles of the EVP further reduce the fuel than the CVP, and the platoon driving can save more fuel than the SOL.

The computer simulation results shown in Fig.8 and Fig.9 indicate that: During the early time period (0-25 [s]), the lead vehicle of the EVP accelerated slightly while the middle vehicle and the tail vehicle accelerated. This maneuver indicated that the EVP synchronously controlled the vehicles, the fast convergence in their spacings was achieved. The control inputs were small owing to the low acceleration of the vehicles. The vehicles of the CVP accelerated independently

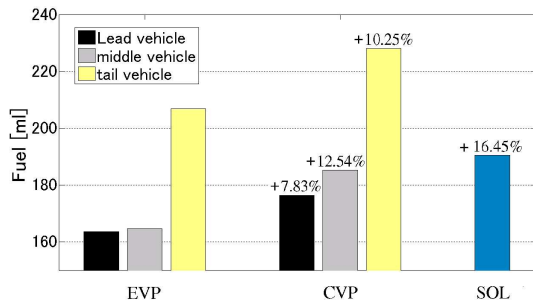


Fig. 7. Fuel consumption in 0-2500 [m]

since each vehicle of the CVP did not use the information of the other vehicles. By this maneuver, the slow convergence in their spacings was observed, and the control inputs were very large because of the high acceleration of the vehicles. During the uphill-downhill driving (25-90 [s]), the vehicles of the EVP tried to keep a low speed during uphill driving and a high speed during downhill driving. The amplitude of the control inputs was small owing to these driving policies. The vehicles of the CVP tried to keep the desired speed. This driving policy needed the large control inputs during the uphill driving, and wasted the kinetic energy of the vehicles during the downhill driving.

#### 4 CONCLUSIONS

A centralized linear quadratic regulator system for ecological vehicle platooning considering the aerodynamic drag varied by the spacing and the road shape has been presented. The performance of the proposed control system was confirmed by the computer simulations. The synchronized driving obtained by the centralized control has provided the fast convergence of the spacings. The excessive acceleration and deceleration during the uphill and downhill driving have been avoided. The results revealed the significant improvement in fuel economy by using the proposed control system in comparison with those of a conventional vehicle platooning.

#### REFERENCES

- [1] Marcu B, Browand F (1999), Aerodynamic forces experienced by a 3-vehicle platoon in a crosswind. SAE Paper 1999-01-1324
- [2] Swaroop D, Hedrick J K, Chien C C et al (1994), Comparison of spacing and headway control laws for automatically controlled vehicles. *Vehicle System Dynamics* 23:597-625
- [3] Zhang Y, Elias B, Kosmatopoulos et al (1999), Autonomous intelligent cruise control using front and back information for tight vehicle following maneuvers. *IEEE Transactions on Vehicular Technology* 48(1): 319-328
- [4] Levine S W, Athans M (1966), On the optimal error regulation of a string of moving vehicles. *IEEE Transactions on Automatic Control* 11(3): 355-361
- [5] Kaku A, Kamal M A S, Mukai M et al (2009), Model predictive control of a platoon for optimal fuel consumption based on vehicle aerodynamics (in Japanese). *Proceedings of the 11th Symposium on Motion and*

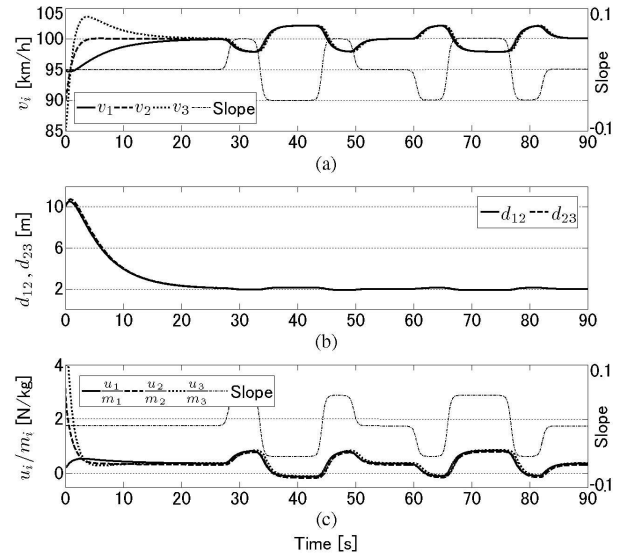


Fig. 8. The computer simulation results of the EVP

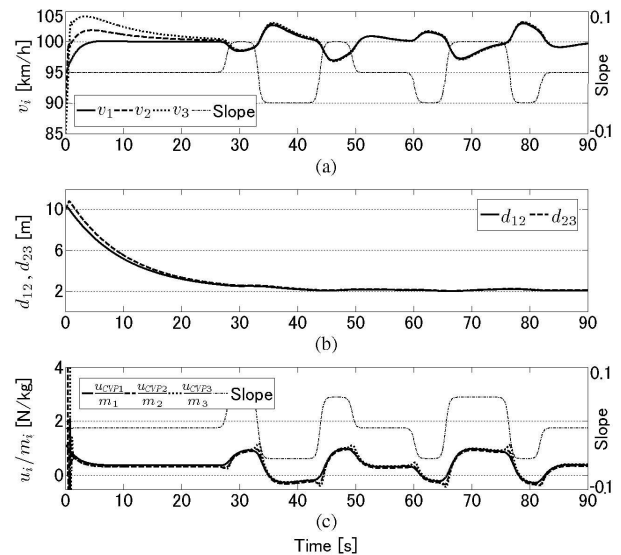


Fig. 9. The computer simulation results of the CVP

- [6] Kuroki M, Mukai M, Kawabe T et al (2009), Vehicular model predictive control for optimal fuel consumption based on engine map (in Japanese). *Proceedings of the 28th Kyushu Annual Congresses of The Society of Instrument and Control Engineers, Fukuoka, Fukuoka Japan, Nov 28-29, 2009, pp. 29-30*