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How equity norms evolve? - An evolutionary game theory approach to distributive justice

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Abstract: A Nash demand game (NDG) has been applied to explain moral norms of distributive justice. In NDG, two players simultaneously make demands and receive them unless the sum of the demands exceeds the amount of the resource. Otherwise, they obtain nothing. This paper proposes the demand-intensity game (D-I game), which adds an "intensity" dimension to NDG in order to discuss various scenarios for the evolution of norms concerning distributive justice. We show basic analyses of the D-I game in game theory and then evolutionary simulations. Descriptive/evolutionary approaches show that three types of norms could evolve mainly dependent on the conflict cost in the game: egalitarianism, "wimpy" libertarianism and libertarianism in decreasing order of the cost. Although the wimpy libertarianism is classified as the libertarianism in the sense of claiming the full resource, it can achieve an egalitarian division without conflict cost as a result.

Keywords: Distributive justice, Evolutionary game theory, Nash demand game

1 INTRODUCTION

Game theoretic models have been successfully used in the studies on the evolution of moral norms which promote cooperation mainly in humans. Classical game theory is based on a normative theory of rational choice and prescribes what people ought rationally to choose ("normative approach"). Recently, researchers in various fields have tried a different approach based on evolutionary game theory that dispenses with strong assumption about rationality. Rather than asking what moral norms ought to be, they aim at describing how people will in fact choose or how can the existing norms have evolved ("descriptive/evolutionary approach") [1].

The Nash Demand Game (NDG) [2] has been widely employed to explain the emergence of moral norms, especially the evolutionary bases of *distributive justice* [3], [4], [5], etc. In NDG, each player simultaneously makes a demand and each obtains the claimed demand if the aggregated demand between both is no less than the full amount of resource. Otherwise, they obtain nothing. Every pair of the claimed demands that total 100% of the resource is a strict Nash equilibrium; however, people intuitively make the 50% demand [6]. Skyrms provided a game theoretic account of how norms of fair division might have evolved using replicator dynamics as a descriptive/explanatory approach [4]. With a finer step size of the demand than 0.1 and a higher probability of playing between similar strategies, the population always evolved into the fair division equilibrium. However, the assumption of correlated interactions of strategies have been criticized for the reason that it has no actual grounding in reality [7]. Instead, fair division has been achieved by spacial models [5] and two-population models [8].

Skyrmsian approach is evolutionary generalist as it entirely omits the psychological mechanisms, in contrast to evolutionary psychology, which emphasizes particular psychological factors of human behaviors [7]. This paper proposes the demand-intensity game (D-I game), which adds an "intensity" dimension related to some psychological factor (e.g. *bold* or *timid*) to NDG in order to discuss various scenarios for the evolution of norms concerning distributive justice, while keeping such simplicity that it can be analyzed by the concepts and tools of game theory. In NDG, if the sum of the demands exceeds the amount of resource, they obtain nothing. This rigidity in evaluation of the conflict cost is weakened depending on the intensity values of the players.

2 D-I GAME

Similar to NDG, the D-I game is a two-player one-shot game and deals with the problem of allocating a limited resource between two players as shown in Fig. 1, in which d_0, i_0, d_1 and i_1 represent self demand, self intensity, the other's demand and the other's intensity, respectively.

Each player has a strategy $S(d, i)$ noted as a set of parameters d and i $(0 \leq d, i \leq 1)$. The parameter d represents the demand, which is an demanding amount in supposing a total amount of the resource is 1. If the total demand between the two players is not over 1 (the full amount of resource), each player gains the demand as a reward, which equates to NDG.

Otherwise, the conflicted part of the resource (d_0+d_1-1) is divided according to the newly introduced parameter i, the intensity of the demand, as $1/2 + (i_0 - i_1)/2 : 1/2 + (i_1 - i_0)/2$. For example, $(i_0, i_1) = (0, 0)$, $(0.5, 0.5)$, $(1, 0.5)$ or $(1, 0)$ makes the conflicted part divided as $1 : 1$, $1 : 1$,

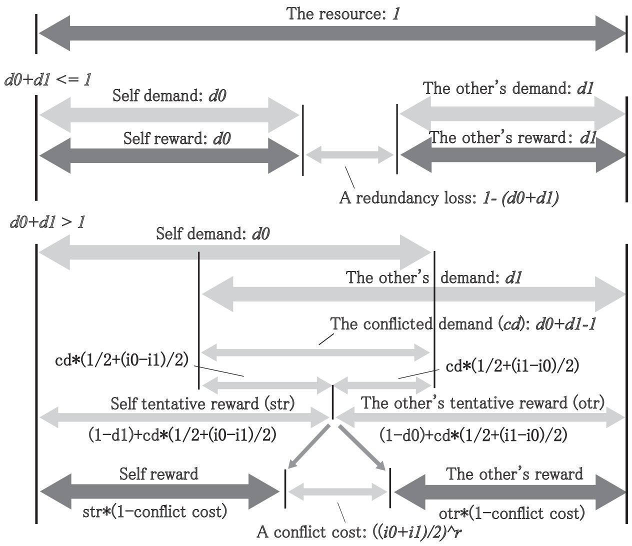

Fig. 1. Rewards in the D-I game.

$0.75 : 0.25$ or $1 : 0$, respectively. Finally, divided resources are reduced in reversely proportional to the sum of i_0 and i_1 as conflict cost. The larger the combined intensity between the players, the smaller rewards both gain. If the sum is maximum ($i_0 = i_1 = 1$), no reward is gained, as is the case with NDG. On the other hand, both share the resource without a loss of a conflict when the combined intensity is minimum ($i_0 = i_1 = 0$). Therefore, the game features two dilemmas: demand and intensity. Each player wants to receive more reward than the other and at the same time wants to avoid the conflict cost for the demand and the intensity.

The reward in the D-I game is shown as Eq. 1. (Fig. 1).

$$R(d_0, i_0, d_1, i_1) \quad = \quad \begin{cases} d_0 & (d_0 + d_1 \leq 1) \\ \{(1-d_1) + (d_0 + d_1 - 1)(\frac{1}{2} + \frac{i_0 - i_1}{2})\}(1 - cost) & \\ & (else) \end{cases} \quad (1)$$

$$cost \quad = \quad \left(\frac{i_0 + i_1}{2}\right)^r \quad (2)$$

r specifies the game structure in terms of the conflict cost as shown in Eq. 2. At $r = 0$, the conflict cost is maximized and the game is equivalent to NDG.

We refer to a strategy of d below 0.5 as "generous" and above 0.5 as "greedy." Specifically, the cases that d is 0, 0.5 and 1 are defined as "unselfish," "even" and "selfish," respectively. The parameter i, the intensity of the demand, represents how strong people claim their own demand in a conflict. The strategy is referred to as "timid" if i is less than 0.5, and "bold" if i is more than 0.5. Specifically, the intensities i of 0, 0.5 and 1 are defined as "wimpy," "moderate" and "belligerent, " respectively.

Regarding d, there are two typical strategies: $d = 0.5$ and $d = 1$. Although debatable, we simply associate the former with egalitarianism and the latter with libertarianism. When $r = 0$, all strategies with $d = 0.5$ are ESS independently of i. Previous studies on a descriptive/evolutionary approach intended to describe how people could evolve this egalitarian norm. However, the strategies with $d = 0.5$ are not ESS except $r = 0$ and the norm becomes weaker as r increases. This is because a decrease of conflict cost (an increase of r) makes greed attractive.

When both players use the same greed strategy ($d_0 = d_1 > 0.5$), the reward is 0.5 at $i_0 = 0$ and decreases monotonically as i_0 increases when $r > 0$. Therefore, it is notable that ideal society in the sense of equality and efficiency can be achieved by not only a pure egalitarianism norm ($S(0.5, *)$) but also an eventual equality norm based on libertarianism ($S(1, 0)$) in the D-I game as shown later.

3 GAME THEORETIC ANALYSES

There are many efficient strategies in the D-I game according to a Nash equilibrium analysis as in NDG. An ESS (Evolutionary Stable Strategy) analysis leads to only three types of norms in these efficient strategies. The three norms (ESS) are egalitarianism (Norm A: $S(0.5, *)$, when $r = 0$), libertarianism (Norm B: $S(1, i^*)$ (i^* depends on r as shown in Fig. 3), when $r \geq 0.5$) and "wimpy" libertarianism (Norm C: $S(1, 0)$, when $0 < r < 0.773$). Fig. 2 shows the three norms in two dimensions of strategy: demand and intensity. The left, middle and right panels illustrate the cases of $r = 0$, $0 < r \leq 1$ and $r = 1$, respectively.

Norm A is represented as an ESS group of even strategies ($d = 0.5$) with any intensity value only when $r = 0$ (NDG setting) as shown in the left panel of Fig. 3. When $r > 0$, the egalitarian strategies are divided into two groups according to the the value of the intensity (X and Y in the middle and right panels of Fig. 3). The "bolder" egalitarian strategies (X) can be invaded only by all the egalitarian strategies (X+Y), while the "timider" strategies (Y) can be invaded not only by all the egalitarian strategies (X+Y) but also by other strategies. As r grows from 0 to 1, the range of bolder strategies (X) narrows from $0 \leq i \leq 1$ to $0.438 < i \leq 1$, and it finally vanishes when $r = 4.67$.

Although both Norm B and Norm C have a selfish property ($d = 1$, libertarianism), Norm B exists as an ESS under lower-cost conditions ($r \geq 0.5$) while Norm C exists as an ESS under higher-cost conditions ($0 < r < 0.773$) as shown in Fig. 3. Norm B bifurcates into two ESS at $r = 0.5$, $i = 0.25$. One of the bifurcated ESS becomes "timider" along with increased r, and the other "bolder." The "timider" ESS exists in $0.5 \leq r < 0.571$, whereas the "bolder" ESS exists till infinity. It should be noted that a society with Norm C is ideal from the aspect of equality and efficiency as in

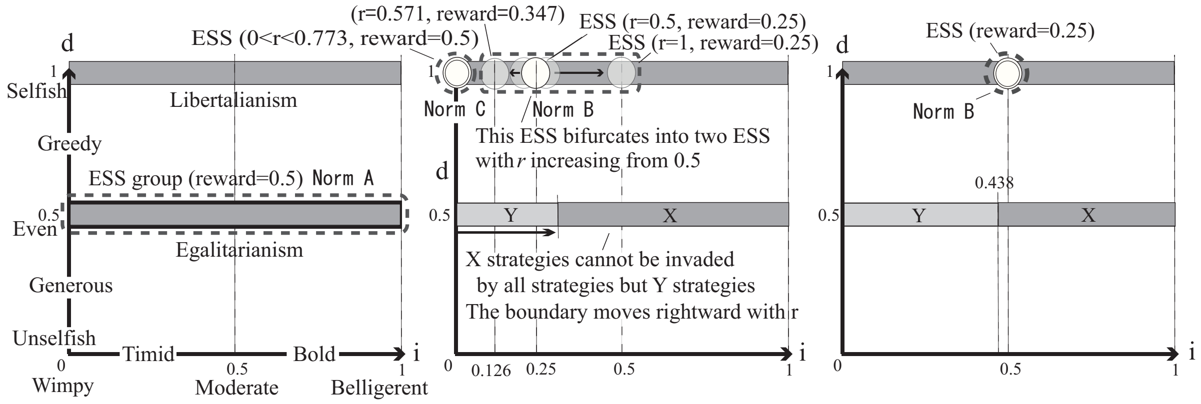

Fig. 2. Three types of norms in strategy space (Norm A: egalitarianism, Norm B: libertarianism, Norm C: "wimpy" libertarianism). The left, middle and right panels correspond to the cases of $r = 0$, $0 < r \leq 1$ and $r = 1$, respectively.

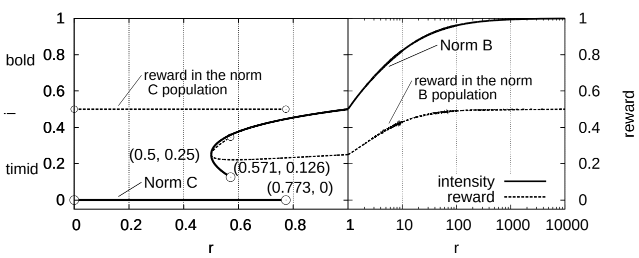

Fig. 3. Intensity i of two types of ESS (Norm B and Norm C). $d = 1$ (libertarianism). Rewards in the ESS population.

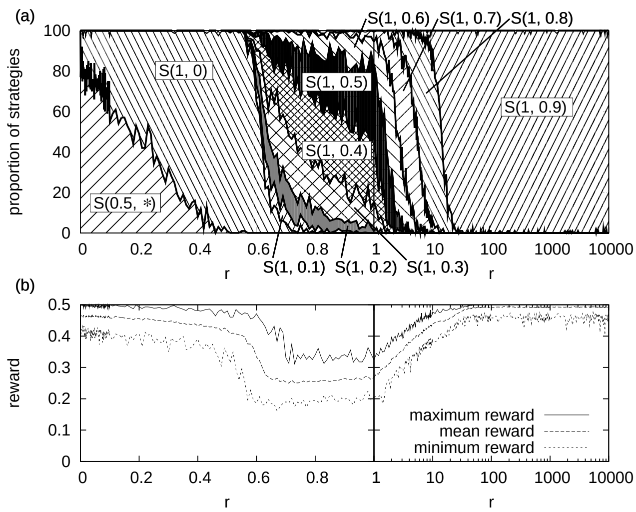

Fig. 4. (a) Proportion of the most common strategy in a population at the last generation over all 100 trials. (b) Average reward in the population. The graphs are the maximum, the mean and the minimum of the rewards over all 100 trials.

the case with Norm A, in the sense that each obtains the maximum reward (0.5) in the ESS population of selfish and wimpy strategies ($d = 1$ and $i = 0$). On the other hand, a society with Norm B is inefficient in the sense that each obtains less than 0.5 in the ESS population of "greed" and "bolder" strategies. The reward each obtains approaches 0.5 as r approaches infinity, in other words, the conflict cost approaches 0.

4 EVOLUTIONARY SIMULATIONS

We performed evolutionary simulations using a genetic algorithm. The population was composed of N individuals represented as the strategy parameters, d and i. The fitness of each individual was defined proportional to the total amount of rewards in playing the D-I game with all other members in the population. The strategy parameters d and i were and discretized into steps of size S_d and S_i in the range of $[0, 1]$, respectively. The initial populations consisted of N individuals with randomly selected d and i.

New individuals were generated by the three genetic operations: fitness-proportionate selection, crossover with a rate R_c, which simply exchanged the parent's intensity values, and mutation with a rate R_m, which selected another value for d or i, keeping them in the range of $[0, 1]$. We show the result with $N = 100$, $S_d = S_i = 0.1$, $R_c = 0.5$, and $R_m = 0.05$.

Fig. 4 (a) illustrates a proportion of the most popular strategy at the last generation (1500th generation) over all 100 trials. For small r, the egalitarian strategies ($d = 0.5$) remained as the most common strategies until the last generation in many trials. While the proportion over all 100 trials decreased along with a growth of r, libertarian strategies ($d = 1$) became the most common strategies in more trials. The intensity of the libertarian strategies became larger as r increases.

The egalitarian ESS prevailed at $r = 0$ with 100% trials and the "wimpy" libertarian ESS prevailed for $0 < r < 0.773$ with the percentage of trials as shown in Fig. 4 (a). The egalitarian strategies remained until the last generation

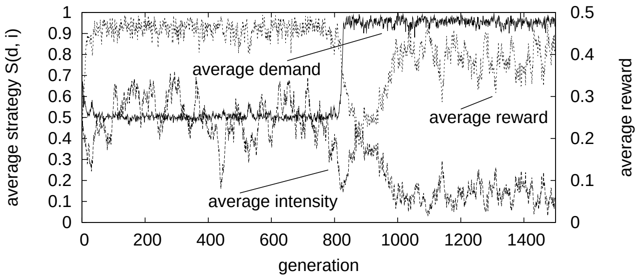

Fig. 5. Average demand, average intensity and average reward in a population through an evolutionary simulation for $r = 0.6$.

in some evolutionary simulations for $r > 0$. Fig. 5 shows the average demand, the average intensity and the average reward in a population for $r = 0.6$. Some trials kept egalitarian strategies until certain generation even when $r = 0.6$. The egalitarian strategies can be stabilized for $r > 0$ because a strategy distribution in a population is diverse through evolutionary simulations and thus the bolder egalitarian strategies (X in Fig. 2) can obtain a stability for some situations.

On the other hand, there was no coexistence between egalitarian strategies and libertarian strategies. Here we consider the reason of it. The generation in Fig. 5 sees a quick transition from an egalitarian to a libertarian population at about 800th generation. It is possible reason that a libertarian strategy obtains three times as much as an egalitarian strategy's reward when they play the game with each other under the same intensity. The difference is lager when i is smaller. Once libertarian strategies, especially the wimpy libertarian strategy, invade in a population, egalitarian strategies obtain the lower reward and can not remain in the population.

The most common strategy was various for around $0.6 < r < 10$ with a low reward in a population as shown in Fig. 4. The various strategies include not only the libertarian ESS but also other libertarian strategies. Furthermore, libertarian strategies coexisted with other libertarian strategies in trials for the range of r. It might be due to a little difference in the rewards between similar libertarian strategies.

For $0 \leq r < 0.6$, an ideal society was achieved with the successful reward in the population (0.5) by egalitarian strategies or the "wimpy" libertarian strategy. Fig. 4 (b) illustrates the average reward in a population at the last generation. The most successful trial for $0 \leq r < 0.6$ achieved the reward of around 0.5 ("maximum reward" in Fig. 4 (b)). When r approaches infinity (no conflict cost), an ideal society was also achieved, as a matter of course.

5 CONCLUSION

We proposed the D-I game, which adds an "intensity" dimension to NDG in order to discuss various scenarios for the evolution of norms concerning distributive justice. We did game theoretic analyses of the D-I game and performed evolutionary simulations. Descriptive/evolutionary approaches (an ESS analysis and evolutionary simulations) show evolution into three types of norms: egalitarianism, libertarianism and wimpy libertarianism. While the wimpy libertarianism is classified as the libertarianism for claiming the full resource, it can also achieve an egalitarian division in a population without conflict cost as a result.

A level of conflict cost has a large influence on what kind of norms emerge: the egalitarianism, the wimpy libertarianism and the libertarianism in decreasing order of the cost (which could be interpreted as a psychological cost). People may not feel conflict cost pressure so much when we share sweets among family members. We may act like a libertarian in this case and we may demand the whole sweets strongly. If the members are friends, we may feel more cost pressure. Then, we may act like a wimpy libertarian. If the share resource is money, we may feel more cost pressure than sharing the sweets and we may make an equal demand strictly as an egalitarian.

We believe that the D-I game provides us with a useful framework to study dynamics of distributive justice from an emergence perspective, beyond the question of whether strategies demanding equal share can dominate the population or not.

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