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Properties of Hopfield model with the zero-order synaptic decay

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Abstract: In this paper, we investigate the effect of synaptogenesis on memories in the brain, using the abstract associative memory model, Hopfield model with the zero-order synaptic decay. Using the numerical simulation, we demonstrate the possibility that synaptogenesis plays a role in maintaining recent memories embedded in the network while avoiding overloading. For the network consisting of 1000 units, it turned out that the minimum decay rate to avoid overloading is 0.02, and the optimal decay rate to maximize the storage capacity is 0.08. We also show that the average numbers of replacement synapses at each learning step corresponding to these two values are 1187 and 21024, respectively.

Keywords: synaptogenesis, zero-order synaptic decay, associative memory, Hopfield model, forgetting process, overloading

1 INTRODUCTION

Memory was considered to consist of three processes: (i) learning and (ii) keeping information in the network, and then, (iii) recalling it when needed. Hopfield model [1], the best-known associative memory model, could reproduce the form of memory in the brain. By changing the synaptic strengths, the model stored memory patterns in the network. Thereafter, this network could retrieve the stored pattern even if given its noisy version. The properties of the associative memory model have been investigated over several decades [1]-[4].

The ordinary Hopfield model has a critical number of memory patterns that can be stably stored, $0.138N$, where N is the system size. The additional learning of new patterns beyond the value overloads the network system, and makes no patterns retrievable [4].

As a matter of fact, the past associative memory model missed the fourth factor of memory, i.e., forgetting old information. If the dynamics of the synaptic strength in the network have decay or saturation, overloading does not occur. The forgetting model, an alternative associative memory model to avoid overloading, was proposed by Mézard et al. [5]. In the forgetting model, every time the network learns a new pattern, the synaptic strength decays in proportion to itself. In this scheme, the old memory traces are erased exponentially with time. However, as yet there is no conclusive experimental evidence for the existence of such a system in the brain.

Recently, it has been reported that synaptogenesis, birth of synapses, continues to take place in certain regions of the postnatal brain including the hippocampal regions [6],[7]. Synaptogenesis, i.e., replacement of old synapses with new ones, seems to be crucial for the formation of neural networks. Thus, it affects neural network functions, especially memory formation. Furthermore, according to the previous neurophysiological experiment, synapses with smaller strength tend to be replaced with higher probability [7]. This raises questions about how this synaptogenesis affects memories in the brain. Should synaptogenesis, which breaks the memory circuit, be a negative factor for memories embedded in the network?

In this paper, we investigate the effect of this synaptogenesis on memories, modeling it mathematically: Hopfield model with the zero-order synaptic decay. In our model, every time the network learns a new pattern, all the synaptic strengths decay a constant value , i.e., decay rate α. This decay process represents the characteristics of the synaptogenesis more exactly than the forgetting model: The smaller synaptic strength is, the more easily synaptogenesis occurs.

From a computational perspective, we demonstrate the possibility that synaptogenesis plays a role in maintaining recent memories embedded in the network while avoiding overloading. Moreover, we show that the storage capacity (maximal number of retrievable patterns in the network) of our model depends on the decay rate. For the network consisting of 1000 units, it turned out that the minimum decay rate to avoid overloading, $\alpha_{\min}$, is 0.02, and that the optimal decay rate to maximize the storage capacity, α_{opt}, is 0.08. Finally, we show how many synapses are replaced with new ones at each learning step when the decay rate takes these two critical values. For the network consisting of 1000 units, it turned out that the average number of replacement synapses corresponding to $\alpha_{\min}$ is 1187, and that corresponding to α_{opt} is 21024.

This paper is organized as follows: Section 2 gives a formulation of the zero-order synaptic decay process in the Hopfield model. Section 3 shows the numerical simulation results using our model. Section 4 gives our conclusions.

2 MODELING THE SYNAPTOGENESIS

2.1 Network dynamics

We begin by formulating a recurrent neural network with N units and $N(N-1)$ synapses. We assume that all the units work synchronously at discrete time $t = 1, 2, \cdots$. The network dynamics are determined by

$$s_i(t+1) = \mathrm{sgn}\Big(\sum_{j=1(\neq i)}^{N} J_{ij} s_j(t)\Big), \quad (1)$$

where $s_i(t)$ is the state of unit i at discrete time t. J_{ij} denotes the strength of the synapse connecting unit j to i, and it is symmetrical, i.e., $J_{ij} = J_{ji}$. We assume that the network is assumed to have no self-interaction, $J_{ii} = 0$. In Eq.(1), the sign function $\mathrm{sgn}(x)$ denotes the next state $s_i(t+1)$ of unit i as

$$\mathrm{sgn}(x) = \begin{cases} 1 & x \geq 0, \\ -1 & \text{otherwise.} \end{cases} \quad (2)$$

According to Eqs.(1) and (2), when a weighted sum of its inputs $\sum_{j=1(\neq i)}^{N} J_{ij} s_j(t)$ exceeds 0 (threshold), the next state $s_i(t+1)$ becomes 1, representing the neuronal firing. On the other hand, $s(t+1) = -1$ represents that the unit is not firing at $t+1$. Equation (1) defines the time evolution of the system state. For any symmetric connection matrix $J_{ij}; J_{ij} = J_{ji}$, the network system has finite possible states. Starting from any arbitrary initial state, the system state of the Hopfield model always reaches an equilibrium or a periodic solution, and the period is known to be no more than 2.

2.2 Associative memory with the zero-order synaptic decay

Each element ξ_i^μ of the μ-th memory pattern $\boldsymbol{\xi}^\mu$, which is stored in the network, takes ± 1, and is generated independently with the probability,

$$\mathrm{Prob}[\xi_i^\mu = \pm 1] = \frac{1}{2}. \quad (3)$$

Here, we consider the following learning dynamics of the synaptic strength J_{ij}, reflecting the characteristics of the synaptogenesis. The synapse which has its small strength $\|J_{ij}\|$ tends to be replaced with new ones. We assume that if the synaptogenesis occurs, the new synapses rebuilds the connection to all the units except for self-coupling. We use the following synaptic decay process, which is equivalent to the above replacement procedure of synapses.

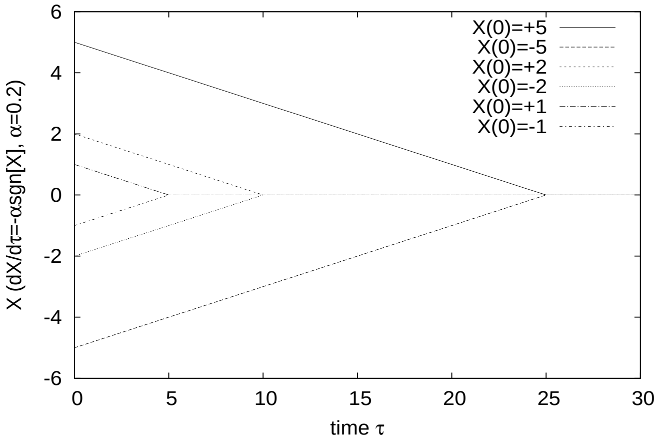


Fig.1: Dynamics of the zero-order synaptic decay process (the second term of the right-side Eq.(4), ordinate). The quantity τ (abscissa) denotes time. Convergence speed (i.e., decay rate $\alpha = 0.2$) is constant with the sign and the value of $X(\tau)$. Thus, the smaller the synaptic strength is, the faster it converges to zero.

$$\Delta J_{ij} = \xi_i^\mu \xi_j^\mu - \alpha \mathrm{sgn}[J_{ij}], \quad (4)$$

where α is the decay rate. The index μ of ξ_i^μ denotes the learning step. Note that $\mu = 1$ corresponds to the latest learning step. The larger serial number $\mu > 1$ signifies that pattern $\boldsymbol{\xi}^\mu$ was stored earlier than $\boldsymbol{\xi}^1$.

The first term of the right-side Eq.(4) denotes the Hebbian learning in the Hopfield model. Every time the network learns a new pattern $\boldsymbol{\xi}^\mu$, J_{ij} is increased by ξ_i^μ and ξ_j^μ, not depending on the global structure of the state.

On the other hand, the second term of Eq.(4) denotes the zero-order synaptic decay process, modeling the synaptogenesis. Every time the network learns a new pattern, J_{ij} decays at a constant rate α. Figure 1 illustrates the dynamics of the second term of Eq.(4). The smaller the synaptic strength is, the faster it converges to zero. On the other hand, the larger synaptic strength takes more time to converge to zero. If the sign of the synaptic strength inverts ($J_{ij} \geq 0$ and $J_{ij} + \Delta J_{ij} < 0$, or $J_{ij} \leq 0$ and $J_{ij} + \Delta J_{ij} > 0$), then the synaptic strength is reset ($J_{ij} = 0$). This procedure represents that the synapse with the small strength dies and the new one is born.

3 NUMERICAL SIMULATION

For simplicity, we assumed that neither number of units nor synapses does change over time in the following numerical simulation. The network consisting of 1000 neurons learned M memories, $\boldsymbol{\xi}^1, \cdots, \boldsymbol{\xi}^M$, one by one. The maximum value of M was set to 400 (i.e., $0.4N$), which was much larger than the critical value of the ordinary Hopfield model: $0.138N$. All of the initial synaptic strengths were set to zero, $\{J_{ij}\} = 0$. Every time the network learned a memory pattern, the synaptic strength $\|J_{ij}\|$ was weaken by the decay rate α. The smaller $\|J_{ij}\|$ is, the more frequently synaptogenesis occurs, i.e., $J_{ij} = 0$. Each stored pattern $\boldsymbol{\xi}^\mu$ ($\mu = 1, 2, \cdots, M$) was given as a initial state $\boldsymbol{s}(0)$ in the network dynamics. We assume that the system is forced to stop when the present state $s_i(t)$ equals the second to last one $s_i(t-2)$ in the network dynamics, considering the periodical solution of the network dynamics. Simulations were carried out on a computer by varying the decay rate α.

First, we investigate how many patterns are retrievable in the network with the zero-order synaptic decay. Because the synaptic decay process implies erasing the old memory traces gradually, a limit exists on the number of retrievable patterns. As the criterion of successful recall, we used the overlap m^μ between the μ-th memory pattern $\boldsymbol{\xi}^\mu$ and the stationary system state $\boldsymbol{s}$:

$$m^\mu = \frac{1}{N} \sum_{i=1}^{N} \xi_i^\mu s_i. \tag{5}$$

If $m^\mu \geq 0.8$, we regarded $\boldsymbol{\xi}^\mu$ as the retrievable pattern, and we counted it. The results are shown in Fig.2, which plots the number of retrievable patterns as a function of the number of stored ones, M. According to Fig.2, we see that the larger the decay rate α is, the earlier forgetting starts: for $\alpha = 0$, forgetting started at around $M = 140$; for $\alpha = 0.2$, it started at around $M = 30$; for $\alpha = 0.4$, it started at around $M = 10$. Our interest is in the properties of the network when the number of retrievable patterns is nearly saturated. The network for $\alpha = 0$, i.e., the ordinary Hopfield model could not recall any memory pattern when $M > 220$. On the other hand, for $\alpha \neq 0$, the Hopfield model with the zero-order synaptic decay could recall an almost constant number of memories when $M > 40$.

Second, we investigate how old stored patterns are retrievable in the network with the zero-order synaptic decay. Figure 3 plots the overlap m^μ as a function of the learning step μ for two values of the decay rate α. As shown in Fig.3, only the recent memories could be recalled correctly ($m^\mu \geq 0.8$). Moreover, Fig.3 predicts a phase transition phenomenon depending on the learning step μ: If the learning step is smaller than the storage capacity ($\mu < \psi$), the memory retrieval state ($m^\mu \approx 1$) is stable. On the other hand, if the learning step exceeds the storage capacity ($\mu > \psi$), the memory retrieval state becomes unstable, and the so-called spin-glass state ($m^\mu \approx 0$) appears.

Figures 2 and 3 show the number of retrievable patterns depends on the decay rate α. Thus, we investigate two critical decay rate: the minimum value to avoid overloading, $\alpha_{\min}$, and the optimal value to maximize the storage capacity, α_{opt}. Figure 4 illustrates the storage capacity ψ of the zero-order decay model as a function of the decay rate α. It turned out

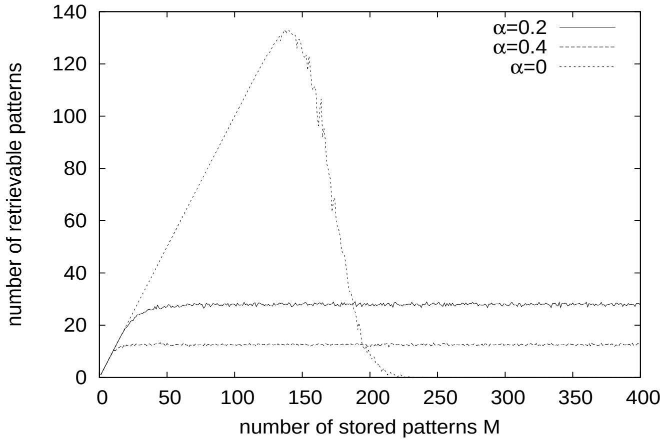


Fig.2: Relation between the number of stored patterns, M (abscissa) and that of retrievable patterns (ordinate). Each curve is the average of 10 samples ($N = 500$), illustrating how many memories so far learned were remembered. Solid curve is the result of the zero-order synaptic decay model with $\alpha = 0.2$, dashed one is that with $\alpha = 0.4$, and dotted one is that with $\alpha = 0$, i.e., the ordinary Hopfield model.

that for $N = 1000$, the minimum decay rate to avoid overloading ($\alpha_{\min}$, a point at the intersection of the abscissa with the solid curve in Fig.4) is 0.02. The optimal decay rate to maximize the storage capacity (α_{opt}, a point at the peak of the solid curve in Fig.4) is 0.08.

Finally, we approximate the number of replacement synapses at each learning step corresponding to these two critical decay rates. In other words, we investigate how many synapses should be replaced with new ones on average at each learning step to avoid overloading, or to maximize the storage capacity. Figure 5 plots the number of replacement synapses at each learning step for $\alpha_{\min}$ and α_{opt}, respectively. According to Fig.5, as time advances, the number of replacement synapses becomes nearly constant. Averaged by the number of stored patterns, M, the minimum number of replacement synapses at each learning step to avoid overloading is 1187, which corresponds to $\alpha_{\min}$. Moreover, the optimal one to maximize the storage capacity is 21024, which corresponds to α_{opt}.

4 CONCLUSION

In order to investigate the effect of synaptogenesis on memories embedded in the neural network, we proposed the Hopfield model with the zero-order synaptic decay. Using the numerical simulation, we demonstrated the possibility that synaptogenesis plays a role in maintaining recent memories while avoiding overloading. Moreover, it turned out that the storage capacity of this model depends on the decay rate α, which corresponds to the number of replacement

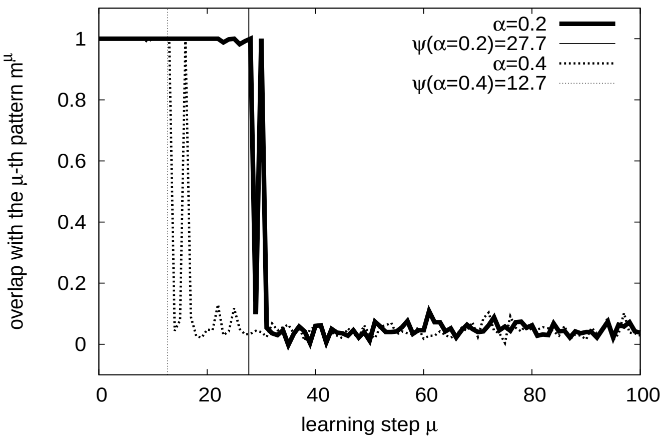


Fig.3: Overlap m^μ (ordinate) between the stationary system state $\boldsymbol{s}$ and the μ-th memory pattern $\boldsymbol{\xi}^\mu$ (abscissa). Note that $\mu = 1$ represents the most recently learned pattern, and that the larger μ is, the older $\boldsymbol{\xi}^\mu$ is. $N = 1000$, and $M = 400$. Heavy solid and dashed curves are the results when $\alpha = 0.2,\ 0.4$ (1 sample), respectively. Vertical solid and dashed curves are the storage capacity ψ when $\alpha = 0.2,\ 0.4$, respectively. Both values are taken from the results shown in Fig.4.

synapses. For $N = 1000$, the minimum decay rate to avoid overloading, $\alpha_{\rm min}$, is 0.02 and it is equivalent to an average of 1187 synapses replaced with new ones at each learning step. The optimal decay rate to maximize the storage capacity, $\alpha_{\rm opt}$, is 0.08 and it is equivalent to an average of 21024 synapses replaced with new ones.

The followings are our possible future works: (i) Since early times, it has also been reported that neurogenesis continues to take place in certain regions of the postnatal brain including the hippocampal regions[8],[9]. Moreover, it has been also demonstrated using numerical simulations that the Hopfield model with unit replacement, mathematically-modeled neurogenesis, avoids overloading and keeps the recent memories[10]. Thus, comparison of the properties between the synaptogenesis (zero-order synaptic decay model) and the neurogenesis (unit replacement model) should be done. (ii) Characteristics of the Hopfield model depend largely on how items are encoded in the pattern vectors to be stored. When most of the components of encoded patterns to be stored are inactivated and only a small share of the components are activated, the encoding scheme is said to be sparse[2],[3]. Investigation how synaptogenesis affects the sparsely encoded network will be done.

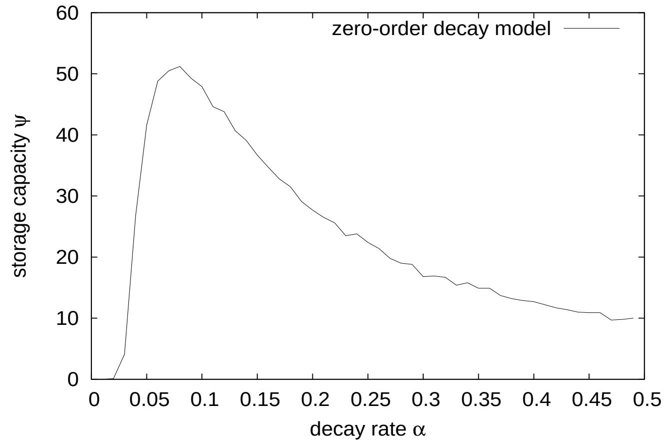
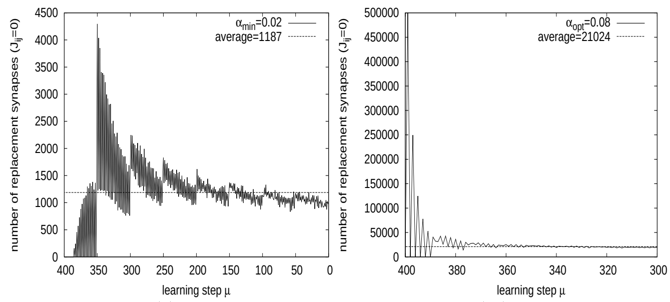


Fig.4: Storage capacity ψ of the zero-order decay model as a function of the decay rate α. Each curve shows the average of 10 samples ($N = 1000$, $M = 400$).



(I) (II)

Fig.5: Number of replacement synapses (ordinate) at each learning step μ (abscissa). Dashed line in each figure shows the averaged number of replacement synapses by the number of stored pattern M. $N = 1000$, and $M = 400$ (1 sample). (I) Minimum number of replacement synapses to avoid overloading corresponding to $\alpha_{\rm min}$. (II) Optimal number of replacement synapses to maximize the storage capacity corresponding to $\alpha_{\rm opt}$.

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