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Parameter Estimation of Chaotic Systems by Nonlinear Time-Varying Evolution PSO Method

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Abstract: An important issue in nonlinear science is parameter estimation for Lorenz chaotic system. Much attention has attracted increasing interests for the identification in various research fields, which could be essentially formulated as a multi-dimensional optimization problem. A novel evolutionary computation algorithm, nonlinear time-varying evolution particle swarm optimization (NTVEPSO) is employed to estimate the parameters. In the NTVEPSO method, the nonlinear time-varying evolution functions are determined by using matrix experiments with an orthogonal array, in which a minimal number of experiments would have an effect that approximates the full factorial experiments. The NTVEPSO method and other PSO methods are then applied to identify of Lorenz chaotic system. Numerical simulation and the comparisons demonstrate the feasibility and the superiority of the proposed NTVEPSO method.

Keywords: Particle swarm optimization, nonlinear time-varying evolution, chaotic system, Lorenz chaotic system.

I. INTRODUCTION

Synchronization and control of chaotic systems have been investigated intensely in various fields during recent years[1–3]. Many of the proposed approaches only work under the assumption that the parameters of chaotic systems are known in advance. In real world, the parameters may be difficult to determine due to the complexity of chaotic systems. Therefore, parameter estimation for chaotic systems has become a hot topic in the past decade[4–9]. Recently, some evolutionary computation algorithms have been successfully applied to real world optimization problems. Several researchers have introduced the evolutionary-based methods into parameter estimation for chaotic systems[10–13].

Particle swarm optimization (PSO) has evolved recently as an important branch of stochastic techniques to explore the search space for optimization[14]. Nowadays, PSO has been developed to be real competitors with other well-established techniques for evolutionary-based optimization methods[15-17]. In this paper, parameter estimation for chaotic systems is formulated as a multi-dimensional optimization problem, and a nonlinear time-varying evolution PSO (NTVEPSO) approach is employed to solve the problem. Numerical simulation based on Lorenz system and comparisons with results obtained by several existed PSO methods verify the feasibility and the validity of the NTVEPSO approach.

II. PROBLEM FORMULATION

Considering the following n-dimensional chaotic system:

$$\dot{\boldsymbol{X}} = F(\boldsymbol{X}, \boldsymbol{X}_0, \boldsymbol{Q}_0) \tag{1}$$

where $\boldsymbol{X} \in R^n$ denotes the state vector, $\boldsymbol{X}_0$ denotes the initial state, and $\boldsymbol{Q}_0$ is a set of original parameters.

When estimating the parameters, suppose the structure of the system is known in advance, and thus the estimated system can be described as follows:

$$\dot{\hat{\boldsymbol{X}}} = F(\boldsymbol{X}, \boldsymbol{X}_0, \boldsymbol{Q}) \tag{2}$$

where $\dot{\hat{\boldsymbol{X}}} \in R^n$ denotes the state vector, and $\boldsymbol{Q}$ is a set of estimated parameters. Therefore, the problem of parameter estimation can be formulated as the following optimization problem:

$$\text{Min} \quad J = \frac{1}{M}\sum_{K=1}^{M}\left\| \boldsymbol{X}_K - \hat{\boldsymbol{X}}_K \right\|^2 \tag{3}$$

where M denotes the length of data used for parameter estimation, $\boldsymbol{X}_K$ and $\hat{\boldsymbol{X}}_K$, $K = 1, 2, \ldots, M$, denote state vectors of the original and the estimated systems at time K, respectively.

Obviously, the parameter estimation for chaotic systems is a multi-dimensional continuous optimization problem, where the decision vector is $\boldsymbol{Q}$ and the optimization goal is to minimize J. The principle of parameter estimation for chaotic systems in sense of optimization can be illustrated with Fig. 1.

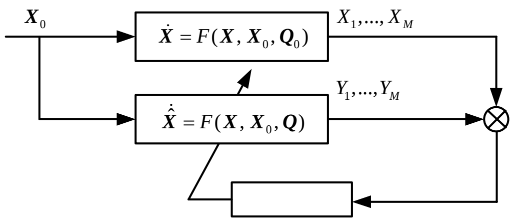

Fig. 1 The parameter estimation scheme for chaotic systems

Due to the unstable dynamic behavior of chaotic systems, the parameters are not easy to obtain. In addition, there are often multiple variables in the problem and multiple local optima in the landscape of *J*, so traditional optimization methods are easy to trap in local optima and it is difficult to achieve the global optimal parameters.

III. NONLINEAR TIME-VARYING EVOLUTION PSO APPROACH

1. Review of some PSO methods

Particle swarm optimization, first introduced by Kennedy and Eberhart[14], is based on observations of the social behavior of animals, such as bird flocking, fish schooling and the swarm theory. PSO is initialized with a population of random solutions. Each individual (called a particle) is assigned with a random velocity and evolves according to the flying experiences of its own and companions. The particles then fly through hyperspace and approach the global optimum. In PSO algorithm, each particle keeps track of its own position and velocity in the problem space. At each iteration, the new positions and velocities of the particles are updated using the following two equations:

$$P_i(k+1) = P_i(k) + V_i(k+1) \quad \text{for } i = 1, 2, \cdots, m \tag{4}$$

$$V_i(k+1) = V_i(k) + c_1 \cdot r_1 \cdot (P_i^l(k) - P_i(k)) + c_2 \cdot r_2 \cdot (P^g - P_i(k)) \tag{5}$$

where m is the number of particles in a population, k is the number of current iteration, c_1 and c_2 are acceleration coefficients, r_1 and r_2 are random numbers between 0 and 1, $P_i(k)$, $P_i^l(k)$, and $V_i(k)$ are the position, the local best, and the velocity of *i*th particle at iteration k, P^g is the global best of all particles.

Several researchers have put much effort to improve the original version of PSO since the introduction of the PSO method in 1995[14]. Shi and Eberhart[18] used a linearly varying inertia weight over iterations. The mathematical representations of this PSO method are given as shown in (4) and

$$V_i(k+1) = \omega(k) \cdot V_i(k) + c_1 \cdot r_1 \cdot (P_i^l(k) - P_i(k)) + c_2 \cdot r_2 \cdot (P^g - P_i(k)) \quad \text{for } i = 1, 2, \cdots, m \tag{6}$$

where the acceleration coefficients c_1 and c_2 are fixed, r_1 and r_2 are two random numbers. The inertia weight starts with a high value $\omega_{\max}$ and linearly decreases to $\omega_{\min}$ at the maximal number of iterations. From hereafter, this PSO algorithm will be referred to as the time-varying inertia weight factor method (TVIWPSO).

Eberhart and Shi[19] found that the TVIWPSO method is not very effective in tracking dynamic systems. Considering the dynamic nature of real-world applications, they proposed a random inertia weight factor to track dynamic systems. In their method, the representations are the same as those in the TVIWPSO method except that the inertia weight factor changes randomly. In the rest of this paper, this algorithm will be referred to as the RANDWPSO method.

An automation strategy for the PSO with time-varying acceleration coefficients was proposed[20]. The objective is to enhance the global search in the early part of the optimization and to encourage the particles to converge toward the global optimum at the end of the search. In their method, the representations are the same as those in the TVIWPSO method except that the acceleration coefficients change according to linear time-varying evolution. From hereafter, this algorithm will be referred to as the TVACPSO method.

A time-varying nonlinear function modulated inertia weight adaptation was proposed by Chatterjee and Siarry[21]. In this method, the acceleration coefficients are also fixed. However, the inertia weight starts with a high value $\omega_{\max}$ and nonlinearly decreases to $\omega_{\min}$ at the maximal number of iterations. This means that the representations are the same as those in the TVIWPSO method except that the inertia weight factor changes according to

$$\omega(k) = \omega_{\min} + \left(\frac{iter_{\max} - iter}{iter_{\max}} \right)^{\alpha} \cdot (\omega_{\max} - \omega_{\min}) \tag{7}$$

where $iter_{\max}$ is the maximal number of iterations and $iter$ is the current number of iterations.

2. PSO-NTVE method based on orthogonal arrays

In this section, based on the concept presented[20,21], an NTVEPSO method is proposed. In the proposed PSO method, the inertia weight is given as described in (7). The cognitive parameter c_1 starts with a high value $c_{1\max}$ and nonlinearly decreases to $c_{1\min}$. Meanwhile, the social parameter c_2 starts with a low value $c_{2\min}$ and nonlinearly increases to $c_{2\max}$. This means that the mathematical expressions are given as shown in (4), (7), and

$$V_i(k+1) = \omega(k) \cdot V_i(k) + c_1(k) \cdot r_1 \cdot (P_i^l(k) - P_i(k)) + c_2(k) \cdot r_2 \cdot (P^g - P_i(k)) \quad \text{for } i = 1, 2, \cdots, m \tag{8}$$

$$c_1(k) = c_{1\min} + \left(\frac{iter_{\max} - iter}{iter_{\max}} \right)^{\beta} \cdot (c_{1\max} - c_{1\min}) \tag{9}$$

$$c_2(k) = c_{2\max} + \left(\frac{iter_{\max} - iter}{iter_{\max}} \right)^{\gamma} \cdot (c_{2\min} - c_{2\max}) \tag{10}$$

where α, β, and γ are constant coefficients.

The proposed PSO method will encourage particles to wander through the entire search space, instead of clustering around a local optimum, during early iterations of the optimization. On the other hand, the algorithm will expedite convergence toward the global optimum during latter iterations. In this manner, the optimal solution should be obtained in a computation-efficient way.

To determine the optimal combination of α, β, and γ, all combinations must be tested. An $L_{25}(5^6)$ is an orthogonal array that can deal with at most six variables in five possible values with 25 experiments[22,23]. Instead of 5^3 possible combinations, one only needs to perform 25 experiments to determine the optimal combination of α, β, and γ.

IV. SIMULATION RESULTS

Lorenz system, a typical chaotic system[10–12], is adopted as an example in this paper. The mathematical description of Lorenz system is described as follows:

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) \\ \dot{x}_2 = bx_1 - x_1x_3 - x_2 \\ \dot{x}_3 = x_1x_2 - cx_3 \end{cases} \tag{11}$$

where $a = 10, b = 28, c = 8/3$ are the original parameters.

In our simulation, the original Lorenz system firstly evolves freely from a random initial state. Then successive M states ($M = 300$) of both the original system and the estimated system are used to calculate J. The searching ranges are set as follows: $9 \le a \le 11$, $20 \le b \le 30$, and $2 \le c \le 3$. In the Lorenz system (11), three-dimensional parameter are unknown and need to be estimated.

In the simulation for PSO methods, the population size and the maximal iteration number are chosen to be 40 and 100, respectively. Moreover, the particles in PSO methods are all chosen as real numbers in the ranges of *a*, *b*, and *c* in (11). Several parameters in the PSO simulation must be specified first. In the employed PSO methods, the values of $\omega_{\max}$, $\omega_{\min}$, $c_{1\max}$, $c_{1\min}$, $c_{2\max}$, and $c_{2\min}$ in (7), (9), and (10) are set to 0.9, 0.2, 2.5, 0.5, 2.5, and 0.5, respectively. These values are determined based on Ratnaweera et al.[20]. In the orthogonal-array-based NTVEPSO, first, assume that α, β, and γ in (7), (9), and (10) are all within the set $\{0.5, 1, 1.5, 2, 2.5\}$. The values of α, β, and γ are 0.5, 1.5, and 1.5 determined by 25 experiments of orthogonal arrays.

The statistical results obtained by PSO methods are shown in Table 1, in which each algorithm is implemented after 20 times independently. From the results, it is clear that the best, the average, and the worst results obtained by NTVEPSO are better than those obtained by the other PSO methods.

V. CONCLUSION

Parameter estimation for chaotic systems was formulated as a multi-dimensional optimization problem in this paper. A novel orthogonal-array-based evolutionary algorithm, NTVEPSO, was applied to solve such an issue. Numerical simulation and comparisons based on Lorenz system demonstrated the effectiveness and efficiency of NTVEPSO. The future work is to apply PSO for other chaotic systems and to develop more effective and adaptive PSO based approaches.

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Table 1 Statistical results of different approaches for three-dimensional parameter estimation (14) after 20 times.

PSO method	Average result				Best result				Worst result			
	a	*b*	*c*	*J*	*a*	*b*	*c*	*J*	*a*	*b*	*c*	*J*
PSO-TVIW	10.3311	27.7707	2.9425	5.6777	10.0237	28.0430	2.6532	0.0291	10.1359	24.6792	2.000	40.6330
PSO-RANDW	10.3960	27.3898	2.9023	9.7032	10.1720	27.9128	2.6704	0.2364	10.7252	24.8650	2.3745	30.7412
PSO-TVAC	10.5719	27.6857	2.9089	6.2706	10.0798	27.8970	2.6804	0.0677	10.6266	24.0630	2.000	45.9042
PSO-NTVE	10.5804	27.7521	2.9406	**5.4474**	9.9803	28.0279	2.6636	**0.0083**	10.7770	25.0512	2.0000	43.6542

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