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Some Properties of Four-Dimensional Parallel Turing Machines

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Abstract

Informally, a parallel Turing machine (PTM) proposed by Wiedermann is a set of identical usual sequential Turing machines (STM's) cooperating on two common tapes − storage tape and input tape. Moreover, STM's which represent the individual processors of the parallel computer can multiply themselves in the course of computation. On the other hand, during the past about seven years, automata on a four-dimensional tape have been proposed as computational models of four-dimensional pattern processing and several properties of such automata have been obtained. In [1], we proposed a four-dimensional parallel Turing machine (4-PTM), and dealt with a hardware-bounded 4-PTM, which each side-length of each input tape is equivalent. We believe that this machine is useful in measuring the parallel computational complexity of three-dimensional images. In this paper, we continue the study of 3-PTM, which each side-length of each input tape is equivalent, and investigate some accepting powers of it.

Key Words: computational complexity, four-dimensional automaton, hardware-bounded computation, nondeterminism, parallel Turing machine

1 Introduction

A parallel Turing machine (PTM) is a set of identical sequential Turing machines (STM's) cooperating on two common tapes − storage tape and input tape [8]. Moreover, STM's which represent the individual processors of the parallel computer can multiply themselves in the course of computation. In [8] it is shown, for example, that every PTM can be simulated by an STM in polynomial time, and that the PTM cannot be simulated by any sequential Turing machine in linear space.

In [2,4,6,7], two- or three-dimensional version of PTM was investigated. On the other hand, due to the advances in many application areas such as moving image processing, computer animation, and so on, it has become increasingly apparent that the study of four-dimensional pattern processing has been of crucial importance. Thus, we think that the study of four-dimensional automata as a computational model of four-dimensional pattern processing has also been meaningful. From this viewpoint, we first introduced four-dimensional automata [3,5]. In [1], we proposed a four-dimensional parallel Turing machine (4-PTM), and investigated its some properties. Especially, we dealt with a hardware-bounded 4-PTM, a variant of the 4-PTM, which each side-length of each input tape is equivalent. The hardware-bounded 4-PTM is a 4-PTM, the number of whose processors is bounded by a constant or variable depending on the size of inputs. The investigation of hardware-bounded 4-PTM's is more useful than that of 4-PTM's from the practical point of view. In this paper, we continue the study of 4-PTM [1], and investigate some accepting powers of its parallel computational model, which each side-length of each input tape is equivalent.

2 Preliminaries

Definition 2.1. Let Σ be a finite set of symbols, a *four-dimensional tape* over Σ is a four-dimensional rectangular array of elements of Σ. The set of all four-dimensional tapes over Σ is denoted by $\Sigma^{(4)}$. Given a tape $x \in \Sigma^{(4)}$, for each integer j ($1 \leq j \leq 4$), we let $l_j(x)$ be the length of x along the jth axis. The set of all $x \in \Sigma^{(4)}$ with $l_1(x) = n_1$, $l_2(x) = n_2$, $l_3(x)=n_3$ and $l_4(x)=n_4$ is denoted by $\Sigma^{(n_1,n_2,n_3,n_4)}$. When $1 \leq i_j \leq l_j(x)$ for each j ($1 \leq j \leq 4$), let $x(i_1, i_2, i_3, i_4)$ denote the symbol in x with coordinates (i_1, i_2, i_3, i_4). Furthermore, we define

$$x[(i_1, i_2, i_3, i_4),(i'_1, i'_2, i'_3, i'_4)],$$

only when $1 \leq i_j \leq i'_j \leq l_j(x)$ for each integer j $(1 \leq j \leq 4)$, as the four-dimensional input tape y satisfying the following conditions:

(1) for each j $(1 \leq j \leq 4)$, $l_j(y) = i'_j - i_j + 1$;
(2) for each r_1, r_2, r_3, r_4 $(1 \leq r_1 \leq l_1(y),\ 1 \leq r_2 \leq l_2(y),\ 1 \leq r_3 \leq l_3(y),\ 1 \leq r_4 \leq l_4(y))$, $y\ (r_1,r_2,r_3,r_4) = x\ (r_1{+}i_1{-}1,\ r_2{+}i_2{-}1,\ r_3{+}i_3{-}1,\ r_4{+}i_4{-}1)$. (We call $x[(i_1,i_2,i_3,i_4),(i'_1,i'_2,i'_3,i'_4)]$ the $[(i_1,i_2,i_3,i_4),(i'_1,i'_2,i'_3,i'_4)]$-*segment* of x.)

Definition 2.2. *Four-dimensional parallel Turing machine* (denoted by 4-PTM) is a 10-tuple $M = (Q, E, U, q_s, q_0, \Sigma, \Gamma, F, \delta_n, \delta_f)$, where

(1) $Q = E \cup U \cup \{q_0\}$ is a finite set of *states*;
(2) E is a finite set of *nondeterministic states*;
(3) U is a finite set of *fork states*;
(4) q_s is the *quiescent state*;
(5) $q_0 \in Q$ - $\{q_s\}$ is the *initial state*;
(6) Σ is a finite *input alphabet* ($\# \notin \Sigma$ is the *boundary symbol*);
(7) Γ is a finite *storage tape alphabet* containing the special *blank* symbol B;
(8) $F \subseteq Q - \{q_s\}$ is the set of *accepting states*;
(9) $\delta_n : E \times (\Sigma \cup \{\#\}) \times \Gamma \rightarrow 2^{(Q-\{q_s\})\times(\Gamma-\{B\})\times D_{in}\times D_s}$ (where D_{in} = {east, west, south, north, up, down, future, past, no move} and D_s = {left, right, no move}) is a *next nondeterministic more function*; and
(10) $\delta_f : U \times (\Sigma \cup \{\#\}) \times \Gamma \rightarrow \cup_{1\leq k\leq\infty} ((Q - \{q_s\}) \times (\Gamma - \{B\}) \times D_{in} \times D_s)$ is a *next fork more function* with the restriction that for each $q \in U$, each $a \in \Sigma \cup \{\#\}$, and each $A \in \Gamma$, if $\delta(q, a, A) = ((p_1, c_1, d_{11}, d_{21}), (p_2, c_2, d_{12}, d_{22}), \ldots, (p_k, c_k, d_{1k}, d_{2k}))$, then $c_1 = c_2 = \ldots = c_k$.

As shown in Figure 1, M has a read-only four-dimensional rectangular input tape with boundary symbols "#'s", and one semi-infinite storage tape (extended to the right), initially filled with the blank symbols. Furthermore, M has infinite processors, $P_1, P_2, \ldots$, each of which has its input head and storage-tape head. M starts in the situation that (1) the processors P_1 is in the initial state q_0 with its input head on the upper northwestmost corner of the first cube of the input tape and with its storage-tape head on the leftmost cell of the storage tape, and (2) each of other processors is in the quiescent state q_s with its input head on the upper northwestmost corner of the first cube of the input tape and with its storage-tape head on the leftmost cell of the storage tape.

Seven-way four-dimensional parallel Turing machine (denoted by $SV4$-PTM) is a 4-PTM, input heads of whose processors cannot move in the past

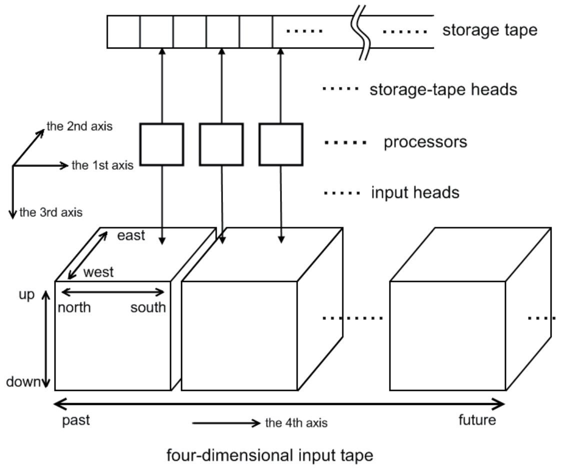

Figure 1: Four-dimensional parallel Turing machine.

direction. In this paper, we are concerned with three-dimensional parallel Turing machines, which each side-length of each input tape is equivalent. Let $L : \mathbf{N} \rightarrow \mathbf{N}$ and $H : \mathbf{N} \rightarrow \mathbf{N}$ be functions. A 4-PTM ($SV4$-PTM) M is called $L(n)$ *space-bounded* if for any $n \geq 1$ and for any input tape x with $l_1(x) = l_2(x) = l_3(x) = l_4(x) = n$, M on x uses at most $L(n)$ cells of the storage tape, and M is $H(n)$ *hardware-bounded* if for any $n \geq 1$ and for any input tape x with $l_1(x) = l_2(x) = l_3(x) = l_4(x) = n$, M on x activates at most $H(n)$ processors. We use the following notations:

- $D4$-$PTM(L(n), H(n))$: the class of sets of cubic tapes accepted by $L(n)$ space-bounded and $H(n)$ hardware-bounded deterministic 4-PTM's
- $N4$-$PTM(L(n), H(n))$: the class of sets of cubic tapes accepted by $L(n)$ space-bounded and $H(n)$ hardware-bounded nondeterministic 4-PTM's
- $DSV4$-$PTM(L(n), H(n))$: the class of sets of cubic tapes accepted by $L(n)$ space-bounded and $H(n)$ hardware-bounded deterministic $SV4$-PTM's
- $NSV4$-$PTM(L(n), H(n))$: the class of sets of cubic tapes accepted by $L(n)$ space-bounded and $H(n)$ hardware-bounded nondeterministic $SV4$-PTM's

3 Seven-Way versus Eight-Way

This section investigates a relationship between the accepting powers of $SV4$-PTM's and 4-PTM's.

Theorem 3.1. *Let $H : \mathbf{N} \to \mathbf{N}$ be a function such that $\binom{H(n)}{2} < \frac{n}{2} (n \geq 2)$. Furthermore, Let $L : \mathbf{N} \to \mathbf{N}$ and H' : $\mathbf{N} \to \mathbf{N}$ be functions such that*

(1) $\exists n_0 \in \mathbf{N}$, $\forall n \geq n_0$ $[\binom{H'(n)}{2} \leq \binom{H(n)}{2}]$, *and*
(2) *max* $\{ H'(n)^2 \binom{H(n)}{2} log\, n,$
$H'(n)^2 \binom{H(n)}{2} log\, L(n),$
$L(n)H'(n)\binom{H(n)}{2} \} = o(n).$

Then,
$D4$-PTM(1,2) $-$ $NSV4$-$PTM(L(n), H'(n)) \neq \phi$.

Proof: Let $T_1 = \{x \in \{0, 1, 2\}^{(4)} | \exists n \geq 3[l_1(x) = l_2(x) = l_3(x) = l_4(x) = n$ & there exists an odd number i $(3 \leq i \leq n)$ such that
(i) $x[(1, 1, 1, 1), (n, n, n, i-1)] \in \{0, 1\}^{(4)}$,
(ii) $x[(1, 1, 1, i), (n, n, n, n)] \in \{2\}^{(4)}$, and
(iii)$\forall j (1 \leq j \leq i-1)$ [the jth cube of x is identical with the (i-j)th cube of x]]}.

It is easily seen that $T_1 \in D4$-PTM(1,2). On the other hand, by using the idea as in the proof of Theorem 3.1 in [1], we can show that $T_1 \notin NSV4$-$PTM(L(n), H'(n))$ for any $L(n)$ and any $H'(n)$ in the theorem. The proof is obtained by replacing $V(n)$ (in the proof of Theorem 3.1 in [1]) with $T_1(n)$, where for large $n \geq 2\binom{H(n)}{2} + 1$, let

$$T_1(n) = \{x \in \{0, 1, 2\}^{(4)} |$$
$$l_1(x) = l_2(x) = l_3(x) = l_4(x) = n \;\&$$
$$x[(1, 1, 1, 1), (n, n, n, 2\tbinom{H(n)}{2})] \in \{0, 1\}^{(4)} \&$$
$$x[(1, 1, 1, 2\tbinom{H(n)}{2} + 1), (n, n, n, n)] \in \{2\}^{(4)} \&$$
$$\forall i (1 \leq i \leq \tbinom{H(n)}{2})[\text{the } i\text{th cube of } x \text{ is}$$
$$\text{identical with the } (2\tbinom{H(n)}{2} + 1 - i)\text{th cube of } x]\}.$$

□

Corollary 3.1. *For any integer $k \geq 1$,*

$$D4\text{-}PTM(1,2) - NSV4\text{-}PTM(o(n), k) \neq \phi.$$

Letting $H(n) = \lceil n^{\frac{1}{4}} \rceil, L(n) = 1$, and $H'(n) = \lceil n^{\frac{1}{5}} \rceil$ ($\lceil r \rceil$ means the smallest integer greater than or equal to r.) in Theorem 3.1, we also have

Corollary 3.2.

$$D4\text{-}PTM(1,2) - NSV4\text{-}PTM(1, \lceil n^{\frac{1}{5}} \rceil) \neq \phi.$$

4 Determinism versus Nondeterminism

This section investigates a relationship between the accepting powers of deterministic and nondeterministic seven-way 4-PTM's.

Theorem 4.1. *Let H,L, and H' be functions described in Theorem 3.1. Then,*

$$NSV4\text{-}PTM(1,2) - DSV4\text{-}PTM(L(n), H'(n)) \neq \phi.$$

Proof: Let $T_2 = \{x \in \{0, 1, 2\}^{(4)} | \exists n = 2m+1 \geq 3[l_1(x) = l_2(x) = l_3(x) = l_4(x) = n$ & (there exists an integer i $(3 \leq i \leq n)$ such that
(i) $x[(1, 1, 1, i), (n, n, n, n)] \in \{2\}^{(4)}$,
(ii) $\forall j (1 \leq j \leq i-1)$ $[w_j = x[(1, 1, 1, j), (m, n, n, j)] \in \{0, 1\}^{(4)}$ & $x[(m+1, 1, 1, j), (m+1, n, n, j)] \in \{2\}^{(4)}$ & w'_j=$x[(m+2, 1, 1, j), (n, n, n, j)] \in \{0, 1\}^{(4)}]$,
(iii) $\exists k, \exists l$ $(1 \leq k < l \leq i-1)$ $[w_k = x[(1, 1, 1, k), (m, n, n, k)] \in \{0, 1\}^{(4)}$ &
$x[(m+1, 1, 1, k), (m+1, n, n, k)] \in \{2\}^{(4)}$ &
w'_k=$x[(m+2, 1, 1, k), (n, n, n, k)] \in \{0, 1\}^{(4)}$ &
w_l=$x[(1, 1, 1, l), (m, n, n, l)] \in \{0, 1\}^{(4)}$ &
$x[(m+1, 1, 1, l), (m+1, n, n, l)] \in \{2\}^{(4)}$ &
$w'_l = x[(m+2, 1, 1, l), (n, n, n, l)] \in \{0, 1\}^{(4)}$ &
$w_k = w_l$ & $w'_k \neq w'_l$])]}.

That is, each cube consists of a *tag field* w_j and a *value filed* w'_j. A tape x is in T_2 iff there is a pair of cubes (of x) with the same tag field but different value fields and the bottom cubes of x consist of 2's. Clearly $T_2 \in NSV4$-PTM(1,2).

We below show that $T_2 \notin DSV4$-$PTM(L(n), H'(n))$. We just present the main idea here and leave the details to the reader, as they are quite similar to those of the proof of Theorem 3.1 in [1]. For each integer $n = 2m+1 \geq 2\binom{H(n)}{2} + 1$, let

$T_2(n) = \{x \in T_2 | l_1(x)$=$l_2(x)$=$l_3(x)$=$l_4(x)$=$n$ &
$\forall j (1 \leq j \leq 2\binom{H(n)}{2})$[(w$_j$ consists of the former continuous a 1's and the latter continuous $m(2m+1)(2m+1)-$a 0's by scanning w_j systematically from the first plane to the $(2m+1)$th plane in w_j, from the first column to the $(2m+1)$th column in a plane and from the first row to the mth row in a column) (a = min$\{j,\ 2\binom{H(n)}{2}$+1$-j\}$) and w'_j=$\{0,1\}^{m(2m+1)(2m+1)}$] & x[(1,1,1,$2\binom{H(n)}{2}$+1),(n,n,n,n)] $\in \{2\}^{(4)}$ }.

As in the proof of Theorem 3.1 in [1], there can be constructed a tape in $T_2(n)$ which M will reject, using the fact that there are many words having this tag structure such that the jth cube and the $(2\binom{H(n)}{2} + 1-j)$th cube are identical for $1 \leq j \leq \binom{H(n)}{2}$ (and thus not in T_2). □

Letting $H(n)=\lceil n^{\frac{1}{4}} \rceil$, $L(n)=1$, and $H'(n)=\lceil n^{\frac{1}{5}} \rceil$ in Theorem 4.1, we have

Corollary4.1

$$NSV4\text{-}PTM(1,2) - DSV4\text{-}PTM(1, \lceil n^{\frac{1}{5}} \rceil) \neq \phi.$$

5 Conclusion

This paper investigated some accepting powers of four-dimensional parallel Turing machines, which each side-length of each input tape is equivalent. We conclude the paper by giving some open problems.

(1) What is a hierarchy of the accepting powers of 4-PTM's or $SV4$-PTM's, based on the hardware complexity depending on the input length?

(2) What is a relationship between the accepting powers of deterministic and nondeterministic 4-PTM's with bounded hardware?

(3) For any $k \geq 1$,
$$D4\text{-}PTM(1, k+1) - N4\text{-}PTM(o(logn), k) \neq \phi?$$

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