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A study of Guaranteed Cost Control of the Manipulator with Passive Revolute Joint

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Abstract : In this paper, we consider a robust control problem of a two link RR manipulator with uncertainty in the joint angle which is caused by several factors. The first purpose is to derive an uncertain LTI system of a two link RR manipulator which includes uncertainty in a rotational angle of each joint. The uncertainty is expressed in a system structure matrix in an explicit form. For this uncertain system, we apply guaranteed cost control. At last, we show the effectiveness of our method by a numerical example.
Keywords: Robust control, Guaranteed cost control, Structured uncertainty, Two link RR manipulator

I. Introduction

In practice, the effect of an uncertainty is a considerable problem. Such an uncertainty is caused by a measurement error, noise in the signal, secular distortion of the device, etc. and makes degradation of the performance index. One of an approach to deal with the influence of uncertainty is to include these effects in the form of the LTI system by a structured uncertainty. By using this uncertain system, it is able to design the robust control system. S.S.Chang and T.K.C.Peng proposed a new robust control method which called the guaranteed cost control [1]. This method guarantees the existence of an upper bound of an uncertainty in the performance index. Yamamoto et al. showed a deviation method for the structured uncertainty that is caused by the higher order terms of the Taylor series expansion [2]. N. Takahashi et al. showed a generalization of the guaranteed cost control problem [3]. Kono et al. extended this problem to the case with cross-term in the performance index [4]. Sato et al. considered the object throwing motion problem that by the manipulator with a passive revolute joint [5]. Takahashi et al. extended the guaranteed cost control problem to the case with an uncertainty in an output matrix [6] and introduced an uncertainty in the angle of a car inverted pendulum system [7]. In this paper, we will introduce two uncertainties in a joint angle of the manipulator system and will apply the guaranteed cost control to our uncertain LTI system. Through the numerical example, we will show the effectiveness of our method. In section II, we will show the formulation of the uncertain LTI system of a two link RR manipulator with a passive revolute joint. And we will apply the guaranteed cost control to the uncertain LTI system to design a robust stable system. In section III, we will give a numerical example to show the effectiveness of our proposed method. At last, we will give a conclusion in section IV.

II. Formulation of uncertain system

In this section, we will give a formulation of the uncertain LTI system of a two link RR manipulator and apply the guaranteed cost control to this uncertain system. The link 1 is connected to the base with a rotational joint 1 and the link 2 is connected to another end point of the link 1 with a rotational joint 2. Each joints and arms have physical parameters illustrated in table 1.

Table.1: Parameters of the Manipulator

Parameters	**Meaning** [unit]
θ_i	Angle of the Joint [rad]
m_i	Mass of the Arm [kg]
I_i	Inertia moment of the Arm [kg $\cdot$ m^2]
l_i	Length of the Arm [m]
l_{Gi}	Distance from the joint to the center of gravity of the Arm [m]
g	Gravity [m/sec^2]
τ_i	Input torque to the joint [N$\cdot$ m]

It is well know that the equation of the motion of a two link RR manipulator is expressed as following nonlinear differential equation.

$$H(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \boldsymbol{h}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + \boldsymbol{g}(\boldsymbol{\theta}) = \boldsymbol{\tau} \tag{1}$$

where

$$\boldsymbol{\theta} = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}, \boldsymbol{\tau} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \tag{2}$$

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Inertia term $H(\boldsymbol{\theta})$ is

$$H(\boldsymbol{\theta}) = \begin{bmatrix} I_1 + m_1 l_{G1}^2 + I_2 + m_2(l_1^2 + l_{G2}^2 + 2l_1 l_{G2} c_2) & I_2 + m_2(l_{G2}^2 + l_1 l_{G2} c_2) \\ I_2 + m_2(l_{G2}^2 + l_1 l_{G2} c_2) & I_2 + m_2 l_{G2}^2 \end{bmatrix} \quad (3)$$

Nonlinear term $\boldsymbol{h}(\boldsymbol{\theta})$ is

$$\boldsymbol{h}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) = \begin{bmatrix} -m_2 l_1 l_{G2}(\dot{\theta}_2^2 + \dot{\theta}_1\dot{\theta}_2)s_2 \\ -m_2 l_1 l_{G2}\dot{\theta}_1\dot{\theta}_2 s_2 \end{bmatrix} \quad (4)$$

Gravity term $\boldsymbol{g}(\boldsymbol{\theta})$ is

$$\begin{aligned} \boldsymbol{g}(\boldsymbol{\theta}) &= \begin{bmatrix} -m_1 g l_{G1} s_1 - m_2 g(l_1 s_1 + l_{G2} s_{12}) \\ -m_2 g l_{G2} s_{12} \end{bmatrix} \\ &= \begin{bmatrix} g_1(\boldsymbol{\theta}) \\ g_2(\boldsymbol{\theta}) \end{bmatrix} \end{aligned} \quad (5)$$

where we denote $\sin(\theta_1 + \theta_2) = s_{12}$ and $\cos(\theta_1 + \theta_2) = c_{12}$. If we assume that $\dot{\theta}_1$ and $\dot{\theta}_2$ takes very small value, thus $\dot{\theta}_2^2, \dot{\theta}_1\dot{\theta}_2 \to 0$ and the nonlinear term becomes $\boldsymbol{h}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) = \mathbf{0}$. From Eq. (1), we have

$$H(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \boldsymbol{g}(\boldsymbol{\theta}) = \boldsymbol{\tau} \quad (6)$$

By multiplying from the left by $H(\boldsymbol{\theta})^{-1}$, we get

$$\ddot{\boldsymbol{\theta}} = -H(\boldsymbol{\theta})^{-1}\boldsymbol{g}(\boldsymbol{\theta}) + H(\boldsymbol{\theta})^{-1}\boldsymbol{\tau} \quad (7)$$

Here we define

$$H(\boldsymbol{\theta}) = \begin{bmatrix} h_{11} & h_{12} \\ h_{12} & h_{22} \end{bmatrix}$$

Then, we have

$$H(\boldsymbol{\theta})^{-1} = \frac{1}{h_D}\begin{bmatrix} h_{22} & -h_{12} \\ -h_{12} & h_{11} \end{bmatrix}$$

$$h_D = h_{11}h_{22} - h_{12}^2$$

and

$$\begin{aligned} \ddot{\boldsymbol{\theta}}(t) &= \frac{1}{h_D}\begin{bmatrix} -h_{22}g_1(\boldsymbol{\theta}) + h_{12}g_2(\boldsymbol{\theta}) \\ h_{12}g_1(\boldsymbol{\theta}) - h_{11}g_2(\boldsymbol{\theta}) \end{bmatrix} \\ &\quad + \frac{1}{h_D}\begin{bmatrix} h_{22} & -h_{12} \\ -h_{12} & h_{11} \end{bmatrix}\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \end{aligned} \quad (8)$$

We consider that the rotational angle of joint 1 $\theta_1(t)$ and joint 2 $\theta_2(t)$ include uncertainties $\Delta\theta_1$ and $\Delta\theta_2$, respectively.

$$\theta_1(t) = \theta_1^*(t) + \Delta\theta_1 \quad (9)$$
$$\theta_2(t) = \theta_2^*(t) + \Delta\theta_2 \quad (10)$$

where $\theta_1^*(t)$ and $\theta_2^*(t)$ denote nominal angle of joints. Let us assume that $\Delta\theta_1$ and $\Delta\theta_2$ are very small values, then

$$\begin{aligned} \sin\Delta\theta_1 &\to 0 \\ \sin\Delta\theta_2 &\to 0 \\ \cos\Delta\theta_1 &\to \Delta c_1 \\ \cos\Delta\theta_2 &\to \Delta c_2 \end{aligned}$$

Thus, we have

$$\begin{aligned} \sin(\theta_1(t)) &= \sin(\theta_1^*(t))\Delta c_1 + \cos(\theta^*(t)) \\ &= \Delta c_1 \sin(\theta_1^*(t)) \end{aligned} \quad (11)$$
$$\begin{aligned} \cos(\theta_1(t)) &= \cos(\theta_1^*(t))\Delta c_1 - \sin(\theta_1^*(t)) \\ &= \Delta c_1 \cos(\theta_1^*(t)) \end{aligned} \quad (12)$$

Next, we apply a same operation to $\theta_2(t)$,

$$\sin(\theta_2(t)) = \Delta c_2 \sin(\theta_2^*(t)) \quad (13)$$
$$\cos(\theta_2(t)) = \Delta c_2 \cos(\theta_2^*(t)) \quad (14)$$

Moreover, about $\theta_1(t) + \theta_2(t)$,

$$\begin{aligned} &\sin(\theta_1(t) + \theta_2(t)) \\ &= \sin(\theta_1(t))\cos(\theta_2(t)) + \cos(\theta_1(t))\sin(\theta_2(t)) \\ &= \Delta c_1 \Delta c_2 \sin(\theta_1^*(t))\cos(\theta_2^*(t)) \\ &\quad + \Delta c_1 \Delta c_2 \cos(\theta_1^*(t))\sin(\theta_2^*(t)) \end{aligned} \quad (15)$$

Taking a first-order of the Taylor expansion around $\theta_1^*(t) = 0$ and $\theta_2^*(t) = 0$, we can approximate Eqs. (11)-(15) as follows [7]

$$\begin{aligned} \sin(\theta_1(t)) &\approx \Delta c_1 \theta_1^*(t) \\ \cos(\theta_1(t)) &\approx \Delta c_1 \\ \sin(\theta_2(t)) &\approx \Delta c_2 \theta_2^*(t) \\ \cos(\theta_2(t)) &\approx \Delta c_2 \\ \sin(\theta_1(t) + \theta_2(t)) &\approx \Delta c_1 \Delta c_2(\theta_1^*(t) + \theta_2^*(t)) \end{aligned}$$

In vertue of the above results, the first row element of term in the left-hand side of Eq. (8) becomes

$$-h_{22}g_1(\boldsymbol{\theta}) + h_{12}g_2(\boldsymbol{\theta}) = \bar{h}_{11}\theta_1^* + \bar{h}_{12}\theta_2^* \quad (16)$$

where

$$\begin{aligned} \bar{h}_{11} &= \Delta c_1 g(h_{22}(m_1 l_{G1} + m_2 l_1) \\ &\quad + \Delta c_2 m_2 l_{G2}(h_{22} - h_{12})) \end{aligned} \quad (17)$$
$$\bar{h}_{12} = \Delta c_1 \Delta c_2 m_2 g l_{G2}(h_{22} - h_{12}) \quad (18)$$

The second row element of term in the left-hand side of Eq. (8) becomes

$$h_{12}g_1(\boldsymbol{\theta}) - h_{11}g_2(\boldsymbol{\theta}) = \bar{h}_{21}\theta_1^* + \bar{h}_{22}\theta_2^* \quad (19)$$

where

$$\begin{aligned} \bar{h}_{21} &= \Delta c_1(-h_{12}g(m_1 l_{G1} + m_2 l_1) \\ &\quad + \Delta c_2 m_2 g l_{G2}(h_{11} - h_{12})) \end{aligned} \quad (20)$$

$$\bar{h}_{22} = \Delta c_1 \Delta c_2 m_2 g l_{G2}(h_{11} - h_{12}) \quad (21)$$

h_{11}, h_{12} and h_{22} are given as follows

$$\begin{aligned} h_{11} &= I_1 + m_1 l_{G1}^2 \\ &\quad + I_2 + m_2(l_1^2 + l_{G2}^2 + \Delta c_2 2 l_1 l_{G2}) \\ h_{12} &= I_2 + m_2(l_{G2}^2 + \Delta c_2 l_1 l_{G2}) \\ h_{22} &= I_2 + m_2 l_{G2}^2 \end{aligned}$$

Thus we can transform Eq. (8) to following form

$$\begin{aligned} \ddot{\boldsymbol{\theta}}(t) &= \frac{1}{h_D}\begin{bmatrix} \bar{h}_{11} & \bar{h}_{12} \\ \bar{h}_{21} & \bar{h}_{22} \end{bmatrix}\boldsymbol{\theta}(t) \\ &\quad + \frac{1}{h_D}\begin{bmatrix} h_{22} & -h_{12} \\ -h_{12} & h_{22} \end{bmatrix}\boldsymbol{\tau} \end{aligned} \quad (22)$$

Let us define a state vector $\boldsymbol{x}(t)$ and an input vector $\boldsymbol{u}(t)$ are

$$\boldsymbol{x}(t) = \begin{bmatrix} \theta_1^*(t) \\ \dot{\theta}_1^*(t) \\ \theta_2^*(t) \\ \dot{\theta}_2^*(t) \end{bmatrix}, \boldsymbol{u}(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$

Consequently, we obtain a following uncertain LTI system.

$$\dot{\boldsymbol{x}}(t) = A(\xi)\boldsymbol{x}(t) + B(\zeta)\boldsymbol{u}(t) \quad (23)$$

where, an input matrix $A(\xi)$ and an output matrix $B(\zeta)$ are

$$A(\xi) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \bar{h}_{11}/h_D & 0 & \bar{h}_{12}/h_D & 0 \\ 0 & 0 & 0 & 1 \\ \bar{h}_{21}/h_D & 0 & \bar{h}_{22}/h_D & 0 \end{bmatrix}$$

$$B(\zeta) = \begin{bmatrix} 0 & 0 \\ h_{22}/h_D & -h_{12}/h_D \\ 0 & 0 \\ -h_{12}/h_D & h_{22}/h_D \end{bmatrix}$$

Here we consider that the system matrices are consisted of deterministic elements A_0, B_0 and uncertain elements $\Delta A, \Delta B$.

$$A(\xi) = A_0 + \Delta A \quad (24)$$

$$B(\zeta) = B_0 + \Delta B \quad (25)$$

From $A(\xi)$ and $B(\zeta)$, we can obtain deterministic elements A_0 and B_0 as $(\Delta\theta_1, \Delta\theta_2) = (0, 0)$. A_1 and B_1 are obtained in the condition of $(\Delta\theta_1, \Delta\theta_2) = (max_1, 0)$ and A_2 and B_2 are obtained in the condition of $(\Delta\theta_1, \Delta\theta_2) = (0, max_2)$. Where max_i is a maximum uncertainty of the rotational angle in the joint i. The structured uncertain elements ΔA and ΔB are

$$\Delta A = \sum_{i=1}^{p} \xi_i A_i,\ |\xi_i| \le 1$$

$$\Delta B = \sum_{j=1}^{p} \zeta_j B_j,\ |\zeta_j| \le 1$$

where $p = 1, 2$. ξ_i, ζ_j and A_i, B_j denote the scale and structure of an uncertainty in the system, respectively.

For the uncertain LTI system (23), we apply the guaranteed cost control. The perfoemance index fuction to be minimized is

$$J = \int_0^\infty (\boldsymbol{x}^{\mathrm{T}}(t)Q\boldsymbol{x}(t) + \boldsymbol{u}^{\mathrm{T}}(t)R\boldsymbol{u}(t))dt \quad (26)$$

The stoctastic algebraic Riccati equation based on the eugenvalue upper bound is

$$A_0^{\mathrm{T}}P + PA_0 - PB_0R^{-1}B_0^{\mathrm{T}}P + Q + U_E = \mathbf{O} \quad (27)$$

Upper bound matrix U_E is

$$\begin{aligned} &U_E(\Delta A(\xi), \Delta B(\zeta), P, R) \\ &= \sum_{i=1}^{p} L_i|\Lambda_i|L_i^{\mathrm{T}} + \sum_{j=1}^{p} M_i|\Gamma_i|M_i^{\mathrm{T}} \end{aligned} \quad (28)$$

where $|\cdot|$ denotes the matrix which has absolute value of each elements. L_i, M_i, Λ_i and Γ_i are

$$L_i^{\mathrm{T}}(PA_i + A_i^{\mathrm{T}}P)L_i = \Lambda_i \quad (29)$$

$$M_i^{\mathrm{T}}P(B_iR^{-1}B_0^{\mathrm{T}} + B_0R^{-1}B_i^{\mathrm{T}})PM_i = \Gamma_i \quad (30)$$

where Λ_i and Γ_i are diagonal matrices which have eigenvalues on the diagonal elements. L_i and M_i are orthogonal matrices which constructed from the corresponding orthogonal vectors. From the solution P of Eq. (27), we obtain the feedback gain matrix F as

$$F = -R^{-1}B_0^{\mathrm{T}}P \quad (31)$$

The closed-loop system that the feedback gain is F to be a robust stable system.

III. Numerical example

In this section, we will show the numerical example. Here we consider that the joint 2 is passive, thus the input matrix $B(\zeta)$ becomes

$$B(\zeta) = \begin{bmatrix} 0 \\ h_{22}/h_D \\ 0 \\ -h_{12}/h_D \end{bmatrix}$$

The weighting matrices are $Q = \text{diag}(1,1,1,1)$ and $R = 1$. The initial value is $\boldsymbol{\theta}(0) = [\ 0.1\ \ 0.0\ \ -0.05\ \ 0.0\ ]$. The uncertainty is $\Delta\theta_1 = \Delta\theta_2 = 0.08$. The values of the physical paratemers are illustrated as in table 2.

Table 2: Parameters

parameter	value	parameter	value
m_1	1	m_2	1
I_1	0.03	I_2	0.03
l_1	0.3	l_2	0.3
l_{G1}	0.15	l_{G2}	0.15
g	9.8		

For this system, we design the closed-loop system by using two method, linear quadratic regulator (LQR) and guaranteed cost control (GCC). The simulation enviroment is MATLAB and SIMULINK. The LQR problem is solved by the `lqr` in the control system toolbox and the GCC problem is solved by the Euler method algorithm of our coded m-file. Figure 1 illustrates the trajectory of the joints angle θ_1 and θ_2 of both simulation results.

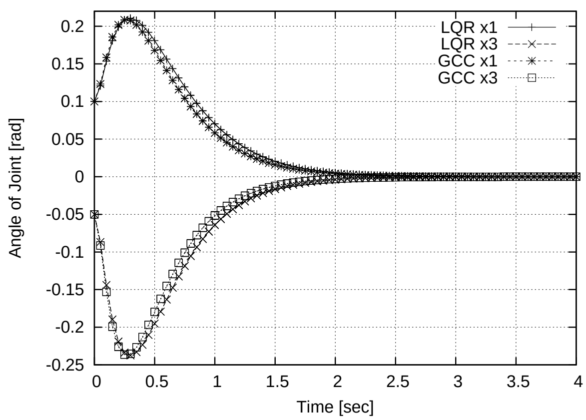

Fig.1 : Simulation Results

The solution of the algebraic Riccati equation P_{LQR} and feedback gain F_{LQR} are (ordinary method)

$$P_{LQR} = \begin{bmatrix} 309.6216 & 80.0140 & 250.8530 & 45.7467 \\ 80.0140 & 20.8360 & 65.4442 & 11.9295 \\ 250.8530 & 65.4442 & 210.2729 & 37.8890 \\ 45.7467 & 11.9295 & 37.8890 & 6.9139 \end{bmatrix}$$

and

$$F_{LQR} = [-47.5721 - 12.6897 - 47.3509 - 8.7623]$$

The eigenvalues of the closed-loop system are

$$(-23.0041, -2.8177, -4.3278 \pm 0.7210i)$$

The solution of the SARE (27) P_{GCC} and feedback gain F_{GCC} are (proposed method)

$$P_{GCC} = \begin{bmatrix} 384.4634 & 99.3605 & 304.8125 & 56.5752 \\ 99.3605 & 25.8774 & 79.3563 & 14.7504 \\ 304.8125 & 79.3563 & 250.4025 & 45.7241 \\ 56.5752 & 14.7504 & 45.7241 & 8.4955 \end{bmatrix}$$

and

$$F_{GCC} = [-54.9208 - 14.5893 - 53.4964 - 9.8803]$$

The eigenvalues of the closed-loop system are

$$(-23.6828, -2.8569, -4.8189 \pm 0.1002i)$$

From the figure 1, we can recognize that our proposed method have designed a robust stable system.

IV. Conclusion

In this paper, we showed the formulation of a uncertain LTI system of a two link RR manipulator with uncertainty in the rotational angle of joints. For this system, we applied the guaranteed cost control to obtain the robust stable system and showed that the simulational result. Future study is to apply the observer to this system.

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