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Failure of pseudo-periodic surrogates

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Abstract: A surrogate test is a method for revealing properties of time series data. To judge whether or not a pseudo-periodic time series has deterministic properties beyond the pseudo-periodicity, some methods to generate surrogate data have been proposed. Luo's test is one of such methods. In this report, Luo's test and its problem will be discussed. On that test, surrogate datasets are produced by adding the original data to their time-shift. So, the pseudo-periodicity of the time series will be presumably preserved, but fine structure related to the determinism is destroyed. The test gives correct results for many ordinary data. However, Luo's test also provides wrong results for certain time series, for example, time series of Rössler chaos and phase-shifted sinusoidal waves. To overcome this problem, we propose an alternative method based on the Poincaré section.
Keywords: surrogate test, time series, pseudo-periodicity, Luo's test, Poincaré section, deterministic chaos.

1. Introduction

A surrogate test is hypothesis testing which is often used for revealing properties of the time series data. To show that the time series has a certain property, one often chooses a null-hypothesis. Then, one generates surrogate datasets that are randomized so that they preserve the properties of the null-hypothesis. Then, the original dataset and its surrogate datasets are compared by using a test statistic. If the test statistic for the original dataset is out of the range specified with the surrogate datasets, then the null-hypothesis is rejected. Otherwise, it cannot be rejected.

Many methods to generate surrogate data have been proposed. For example, random-shuffled surrogates [1] test the serial dependence, while iterative amplitude adjusted Fourier transform surrogates [2] test the nonlinearity of time series.

To judge whether or not a pseudo-periodic time series has deterministic properties beyond the pseudo-periodicity, some methods to generate surrogate data have been proposed. For example, Small's test [3] is well-known. Luo's test [4] is another of such methods. In this report, the test of Luo and its problem will be shown.

2. Tested data and results

On the test of Luo, surrogate datasets are produced by adding the original data to their time-shift. So, the pseudo-periodicity of the time series will be presumably preserved, but fine structure related to the determinism is destroyed.

The test gives correct results for many ordinary data. However, Luo's test also provides wrong results for certain time series, for example, time series of Rössler chaos and phase-shifted sinusoidal waves. We will show that in what follows.

In the Fig. 1, a tested time series, which is the Rössler attractor, is shown. The original data have 15000 points which were generated by Eq. (1). The initial values were $x_0 = y_0 = z_0 = 1$, the sampling rate was 0.05 unit times, and the first 1000 points were discarded.

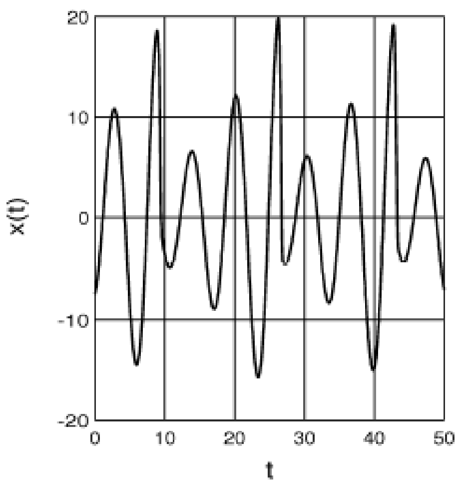

Fig. 1: Time series of Rössler attractor generated by Eq.(1).

$$\begin{cases} \dfrac{dx}{dt} = -y - z \\ \dfrac{dy}{dt} = x + 0.15y \\ \dfrac{dz}{dt} = 0.2 + z(x - 10) \end{cases} \quad \cdots(1)$$

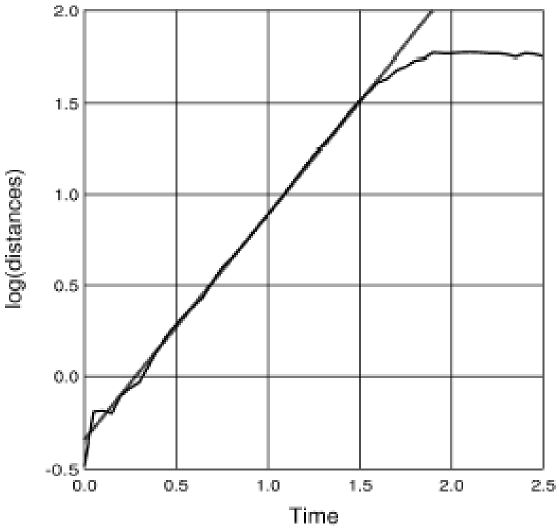

Fig. 2: Exponential increase of distances on Rössler attractor.

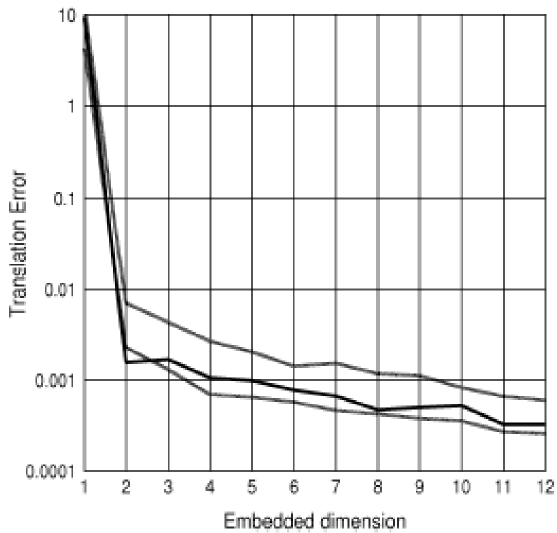

Fig. 3: Results of Luo's test on Rössler attractor.

On the certain parameters, the Rössler attractor behaves as non-chaotic time series. Therefore, we confirmed that the tested time series generated from the above equations show the positive Lyapunov exponent by using Kantz's method [5]. The increase of distances between a point and neighborhood points on the delay coordinates, is shown in Fig. 2. The Lyapunov exponent of the data is calculated by the slope of the scaling region in the logarithmic plot. The Fig. 2 shows that the exponent is 1.24, or a positive value. Our tested dataset is likely to be of deterministic chaos.

The 39 surrogate datasets were produced by Luo's method from the above dataset. We compared the original dataset and its surrogates by Wayland's method [6]. In this method, neighboring points $x_{n_i(t)}$ $(i = 1, 2, \cdots, k)$ are chosen for each selected point x_t $(t \in T \subset \{1, 2, \cdots, N\})$ in a time series $\{x_t : t = 1, 2, \cdots, N\}$, where n_i is the time index for the i-th nearest point for x_t and we set $n_0(t) = t$. As the second step, for each selected point x_t $(t \in T)$, we calculate the translation errors $v_i(t) = x_{n_i(t)+1} - x_{n_i(t)}$ and their average $v(t) = \frac{1}{k+1}\sum_{i=0}^{k} v_i(t)$. As for the third step, we calculate the normalized mean difference between the translation errors and their average, i.e., $e(t) = \frac{1}{k+1}\sum_{i=0}^{k} \frac{\|v_i(t)-v(t)\|^2}{\|v(t)\|^2}$. The statistic we use is the median of $\{e(t) : t \in T\}$.

The results are shown in Fig. 3. In the figure, a thick line which shows the test statistic for the original dataset is inside of the two broken lines which show the upper and lower values for the test statistic obtained from the surrogate datasets. The results of Luo's test mean that *the chaotic data have no deterministic property beyond the pseudo-periodicity*, which should not be correct.

As another example, we applied the same method to a phase-shifted sinusoidal wave. The original data which were generated by Eq. (2) are shown in Fig. 4, in which $\eta(t)$ is a random value chosen for each t from the range of $[-0.1, 0.1]$ uniformly. The data represent a noisy periodic orbit with pseudo-periodicity.

$$\begin{cases} x(t) = \sin\theta(t) \\ \theta(t+1) = \theta(t) + \omega + \eta(t) \\ \eta(t) \in [-0.1, 0.1] \end{cases} \quad \cdots(2)$$

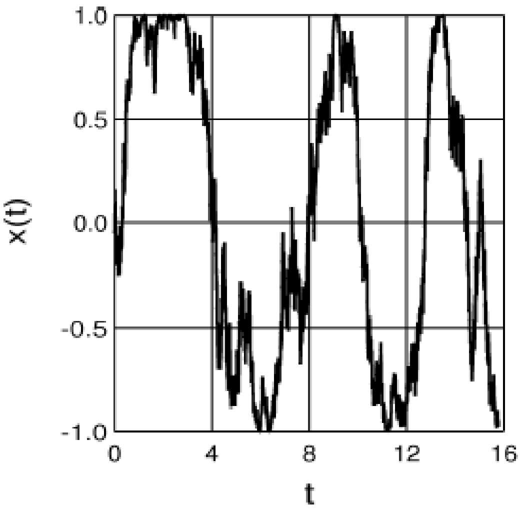

Fig. 4: Phase-shifted sinusoidal wave

The 39 surrogate datasets were also prepared by using Luo's method, and the test statistic for each surrogate was obtained. The results shown in Fig. 5 imply that the dataset has determinism beyond pseudo-periodicity, which should be wrong too.

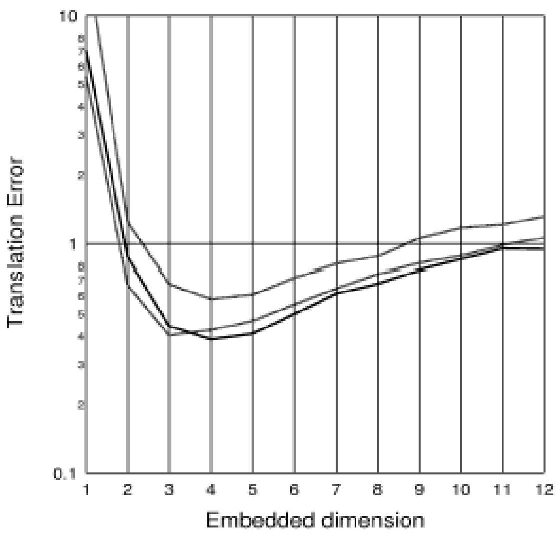

Fig. 5: Results of Luo's test on phase-shifted sinusoidal waves.

3. Proposal method

So far, we found that Luo's method does not classify phase-shifted pseudo-periodicity data correctly. To overcome this weak point, we propose an alternative method based on the Poincaré section.

If a time series has the determinism beyond the pseudo-periodicity, some structures should be found on the Poincaré section. Therefore, by comparing the original data with its random shuffle surrogates on the Poincaré section, one will be able to judge whether or not time series has fine deterministic properties beyond the pseudo-periodicity.

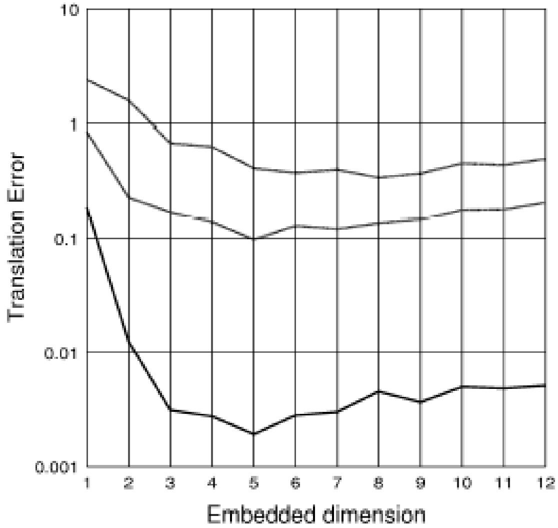

Fig. 6: Results of random shuffle surrogate on the Poincaré section (Rössler chaos).

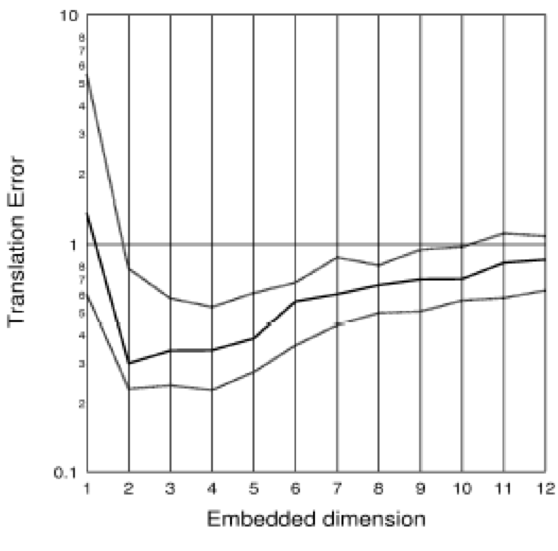

Fig. 7: Results of random shuffle surrogate on the Poincaré section (phase-shifted sinusoidal wave).

We applied the method based on the Poincaré section to the datasets of the Rössler chaos and phase-shifted sinusoidal waves. The results are plotted in Figs. 6 and 7, respectively. Because we found serial dependence on the Poincaré section of the Rössler attractor, but not on the Poincaré section of the phase-shifted sinusoidal waves, we confirmed that this method provides correct results on the tested datasets.

4. Conclusion

We examined the performance of Luo's test for generating pseudo-periodic surrogates and found that it sometimes provides wrong results for "phase-shifted" pseudo-periodicity data. Therefore, we proposed an alternative method that tests serial dependence on a Poincaré section. We showed that the proposed method provides consistent results on the tested datasets.

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