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Discovering the Efficient Organization Structure: Horizontal versus Vertical

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Abstract: Structure analysis is one of the most important issues in corporate management. Pyramid structure, as one of the well-known vertical structure, plays an important part in the corporate organization. Most structures of the traditional organizations such as functional structure and divisional structure are vertical. Recently, due to the development of Information Technology, a new horizontal structure, instead of the vertical one, has been drawn considerable attention. In this paper, we reviewed the efficient organization structure, and found that there are two efficient structures: vertical structure and horizontal structure, depending on the different abilities of each member in any organizations with the comprehensive evaluation measurement. The line structure of vertical organization is efficient when the ability of all members is small. While the ability of all members is large, the star structure of horizontal organization will be efficient. Therefore, this paper provides a theoretical perspective to prove the efficient organization structures and their required conditions.

keywords: vertical structure, horizontal structure, efficient, comprehensive evaluation measurement.

1 Introduction

Organization structure is one of the important factors to determine organizational performance. It means the formal system of task and communication that control, coordinates, and motivates employees in order to achieve organization's goal. Most structures of the conventional organizations, such as functional structure and divisional structure, are vertical. They generally refer to the formal, prescribed hierarchy of authority, or administrative structure. Recently organizational structure has been changed from vertical to horizontal due to the development of informational technology. Most of the empirical studies showed that horizontal structure more effective [1, 2, 3].

The question is what kind of structures is an efficient? And what kind of conditions is required for an efficient structure. The main contribution of this paper is to answer this question, and clarify the relationship between the efficient structures and their condition. And based upon our result, we will discuss the implication.

2 Notations

Suppose that $G = (V(G), E(G))$ is a graph. Through this paper, a graph is always finite, undirected and simple with order $n = |V(G)| (n \geq 2)$ and size $m = |E(G)|$.

For U is any set of vertices, $G - U$ is obtained from G deleting all the vertices in $V(G) \cap U$ and their incident edges. If $U = \{v\}$ is Singleton, we write $G - v$ rather than $G - \{v\}$. As above, $G - \{e\}$ and $G + \{e\}$ are abbreviated to $G - e$ and $G + e$ for $e \in E(G)$.

For $u \in V(G)$, by $N(u) = \{v \mid \{u, v\} \in E(G)\}$, we denote the set of vertices adjacent to u, and call $\deg(u) = \sharp N(u)$ the degree of $u \in V(G)$. We refer to a path in $G = (V(G), E(G))$, by the sequence of its vertices, writing,

$$G(x_0, x_k) = x_0 x_1 \cdots x_k$$

for $x_i \in V(G) (i = 0, 1, \cdots, k)$ and $x_j x_{j+1} \in E(G)$ $(j = 0, 1, \cdots, k-1)$, where x_i are all distinct, and calling $G(x_0, x_k)$ a path from x_0 to x_k in G. And the number of edges of path is its length; above path $G(x_0, x_k)$ has length k.

Assume that $P = x_0 x_1 \cdots x_{k-1}$ is a path and $k \geq 3$, then

$$C \equiv P + x_{k-1} x_0$$

is called a cycle. On the other hand, an acyclic graph, one not containing any cycles, is called a forest. A connected forest is called a tree. Thus, a forest is a graph whose components are tree. Sometimes we consider one vertex of a tree as special, then such vertex is called the root of this tree. While the vertices of degree 1 in a tree but the root of the tree, are its leaves. A tree graph T with fixed root r is written by T_r and then the set of T_r's leaves is written by $L(T_r)$. That is,

$$L(T_r) = \{v \in E(T_r) |\deg(v) = 1,\ v \neq r\}.$$

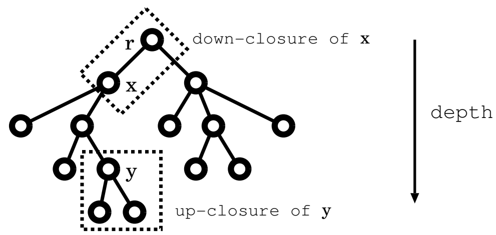


Fig. 1: $x \prec y$ in T_r, down-closure of x, and up-closure of y.

Writing $x \preceq y$ for $x \in T_r(r, y)$, then defines a partial ordering on $V(T_r)$, the tree-order associated with T_r. This ordering will be considered as the expression 'depth': if $x \prec y$, we say x lies below y in T_r, see Fig.1. We call

$$\lceil x \rceil \equiv \{v \in V(T_r) | v \preceq x\}$$

and

$$\lfloor y \rfloor \equiv \{v \in V(T_r) | v \succeq y\}$$

the down-closure of x and the up-closure of y in T_r. Note that the root r is the least element, and that the leaves of T_r are its maximal elements in this partial order.

Suppose that $\Sigma = \{\sigma_1, \sigma_2, \cdots, \sigma_n\} (n \geq 2)$ and $\mathcal{A}(\sharp\mathcal{A} \geq 1)$ are finite sets. For a given Σ, we call $(\Sigma, \{\phi_i\}_{i \in \mathcal{A}})$ an evaluation system, if

$$\phi_i : \Sigma \to \mathbf{R}^+ \equiv \{x \in \mathbf{R} \mid x > 0\} \quad \text{for } i \in \mathcal{A}.$$

We call $\phi_i(\sigma)$ the personal ability of $\sigma \in \Sigma$ with respect to $i \in \mathcal{A}$.

For a given $(\Sigma, \{\phi_i\}_{i \in \mathcal{A}})$, let T_r be a tree graph with $V(T_r) = \Sigma$. Then we shall evaluate the rooted tree T_r by

$$\Phi(T_r) = \sum_{i \in \mathcal{A}} \sum_{l \in L(T_r)} \prod_{v \in T_r(r,l)} \phi_i(v). \tag{2.1}$$

We call $\Phi(T_r)$ the ability value of T_r with respect to the evaluation system $(\Sigma, \{\phi_i\}_{i \in \mathcal{A}})$. Our purpose is to find the most efficient organization structure tree which maximize its ability value for a given $(\Sigma, \{\phi_i\}_{i \in \mathcal{A}})$.

Through this paper, we shall substitute (Σ, ϕ) for $(\Sigma, \{\phi\})$ if $\sharp\mathcal{A} = 1$. Since $\sharp A$ means the number of evaluation measures, thus $\sharp\mathcal{A} = 1$ indicates that its number is one. This paper should treat only this special case. Then we have the following.

Lemma 1 *Suppose that T_r is an efficient tree for a given (Σ, ϕ). Then we see that $x \prec y$ implies $\phi(x) \geq \phi(y)$.*

Proof of lemma 1 For a given (Σ, ϕ), let T_r be an efficient tree. Assume that $\phi(x) < \phi(y)$ holds for some $x, y \in \Sigma$ with $x \prec y$ in T_r. And that T'_r is the tree by interchanging x and y in T_r. Then we get

$$\begin{aligned} & \Phi(T'_r) - \Phi(T_r) \\ = & \left(\frac{\phi(y)}{\phi(x)} - 1\right) \sum_{\substack{l \in L(T_r) \\ l \succeq x,\ y \notin T_r(r,l)}} \prod_{v \in T_r(r,l)} \phi(v) > 0. \end{aligned}$$

This contradicts that T_r is an efficient tree for (Σ, ϕ). Thus we get $\phi(x) \geq \phi(y)$ if $x \prec y$ in T_r. □

Lemma 1 suggests that our efficient trees are suitable for a hierarchical model of the group Σ whose personal abilities are given by $\{\phi(\sigma)\}_{\sigma \in \Sigma}$, when the number of evaluation measures is only one. In this article, we shall show that our efficient trees must be the following three types under our special setting of $\sharp\mathcal{A} = 1$.

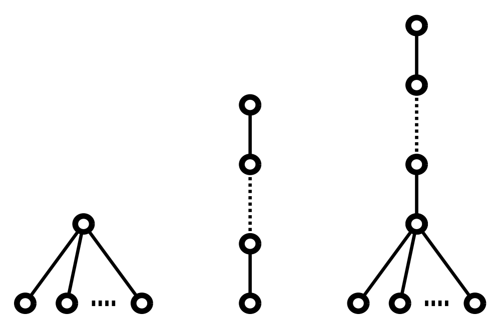


Fig. 2: Three types of our efficient trees when $\sharp\mathcal{A} = 1$.

Actually, two or more evaluation measures exist, so the overall evaluation value of the organization should obtain the expression (2.1). Thus, there might be the most efficient tree besides the types in Fig.2. For example, let us set $\Sigma = \{1,\ 2,\ 3,\ 4\}$, and put

$$\phi_1(1) = \phi_1(2) = 3, \qquad \phi_1(3) = \phi_1(4) = 1/2,$$
$$\phi_2(1) = \phi_2(2) = 1/2, \qquad \phi_2(3) = \phi_2(4) = 3.$$

Then we get an efficient tree for $(\Sigma, \{\phi_i\}_{i=1,2})$ as follows.

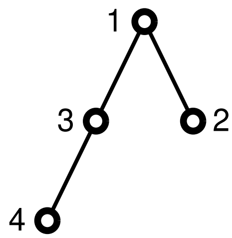


Fig. 3: An efficient tree for $(\Sigma, \{\phi_i\}_{i=1,2})$.

3 Results for $\sharp\mathcal{A} = 1$

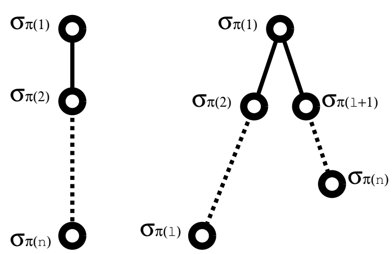
In this section, we shall discuss a structure of an efficient tree for a given (Σ, ϕ). Without loss of generality, we may assume that

$$\phi(\sigma_1) \geq \phi(\sigma_2) \geq \cdots \geq \phi(\sigma_n) \qquad (3.1)$$

for $\Sigma = \{\sigma_1, \sigma_2, \cdots, \sigma_n\}$. Firstly, we shall examine what kind of situation would be better if organization structure tree branch off. For a given $(\{\sigma_1, \cdots, \sigma_n\}, \phi)(n \geq 3)$, let us $\{\sigma_{\pi(1)}, \cdots, \sigma_{\pi(n)}\}$ be a permutation of $\{\sigma_1, \cdots, \sigma_n\}$ satisfying with

$$\sigma_{\pi(1)} = \sigma_1,$$
$$\phi(\sigma_{\pi(2)}) \geq \phi(\sigma_{\pi(3)}) \geq \cdots \geq \phi(\sigma_{\pi(l)}),$$
$$\phi(\sigma_{\pi(l+1)}) \geq \phi(\sigma_{\pi(l+2)}) \geq \cdots \geq \phi(\sigma_{\pi(n)})$$

and $2 \leq l \leq n-1$. Let us set two organization structure trees T_{σ_1} and T'_{σ_1} as shown in Fig.4.



(a) T_{σ_1} (b) T'_{σ_1}

Fig. 4: Two organization structure trees T_{σ_1} and T'_{σ_1} for (Σ, ϕ)

Then, we see that

$$\begin{aligned} &\Phi(T_{\sigma_1}) - \Phi(T'_{\sigma_1}) \\ = \ &\phi(\sigma_1)\left(\prod_{i=2}^{l}\phi(\sigma_{\pi(i)})\prod_{i=l+1}^{n}\phi(\sigma_{\pi(i)}) \right. \\ &\left. -\prod_{i=2}^{l}\phi(\sigma_{\pi(i)}) - \prod_{i=l+1}^{n}\phi(\sigma_{\pi(i)})\right). \end{aligned}$$

Therefore we have the following lemma.

Lemma 2 *Under the assumption (3.1), for T_{σ_1} and T'_{σ_1} in Fig.4, we get*

$$\Phi(T_{\sigma_1}) - \Phi(T'_{\sigma_1}) \begin{cases} \geq 0 & \text{if } \phi(\sigma_n) \geq 2 \\ \leq 0 & \text{if } \phi(\sigma_2) \leq 1. \end{cases}$$

By using lemma 2 repeatedly, we obtain the following conclusions.

Theorem 1 *Under assumption (3.1), we have the followings.*

(a) If $\phi(\sigma_n) \geq 2$ holds. Then we see that an efficient graph is the path graph $\sigma_1\sigma_2\cdots\sigma_n$ in Fig.5.

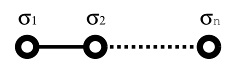


Fig. 5: T_{σ_1} as a path graph.

(b) If $\phi(\sigma_2) \leq 1$ holds. Then we see that an efficient graph is the star graph with center σ_1 in Fig.6.

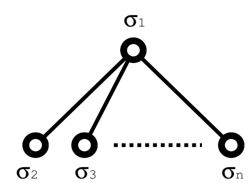


Fig. 6: T_{σ_1} as a star graph.

The typical form of the efficient tree is a path graph or star graph which appeared in Theorem 1. In general, we can show that the form of the efficient tree becomes the form which seems to have put these two figures together, when $\sharp\mathcal{A} = 1$.

For a given (Σ, ϕ) and its organization structure tree T_r, let us define

$$D_3(T_r) = \{\sigma \in \Sigma |\, \deg(\sigma) \geq 3 \text{ if } \sigma \neq r, \\ \deg(\sigma) \geq 2 \text{ if } \sigma = r\}.$$

Theorem 2 *Assume that T_r is an efficient tree for a given (Σ, ϕ). Then we see the followings.*

(a) $\sharp D_3(T_r)$ is equal to 0 or 1.

(b) Putting $D_3(T_r) = \{x\}$ when $\sharp D_3(T_r) = 1$. Then $\{\sigma \in \Sigma \mid \sigma \succ x \text{ in } T_r\} = L(T_r)$ holds.

The general form of the most efficient tree which theorem 2 insists on, is shown in Fig.7. It is obvious that the upper-half is a path graph, and the lower-half is the star graph.

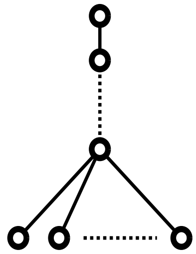


Fig. 7: General form of the efficient tree.

Proof of theorem 2 Assume that $\sharp D_3(T_r) \geq 2$. Then we can find $x, y \in D_3(T_r)$ satisfying with $x \prec y$ and $x, y \in T_r(r, l)$ for some $l \in L(T_r)$. If $\Phi(T_r(r,x)) < \Phi(T_r(r,y))$, put

$$T'_r = T_r - x\sigma + y\sigma$$

for some $\sigma \in N(x)$ with $\sigma \succ x$ and $\sigma \notin T_r(x,y)$. Then we see that

$$\begin{aligned} & \Phi(T'_r) - \Phi(T_r) \\ = & \Big(\Phi(T_r(r,y)) \\ & \quad -\Phi(T_r(r,x))\Big)\Phi(\lceil\sigma\rceil) > 0. \qquad (3.2) \end{aligned}$$

Since T_r is the efficient tree under the assumption of theorem, (3.2) implies contradiction.

Next, if $\Phi(T_r(r,x)) \geq \Phi(T_r(r,y))$, put

$$T'_r = T_r + \bigcup_{\sigma \in N(y),\ \sigma \succ y} (x\sigma - y\sigma)\,.$$

Then we see that

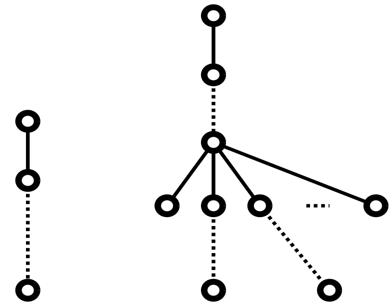
$$\begin{aligned} \Phi(T'_r) - \Phi(T_r) = \Phi(T_r(r,x)) + \\ \Big(\Phi(T_r(r,x)) - \Phi(T_r(r,y))\Big)\Phi(\lceil y\rceil)/\phi(y) > 0 \end{aligned}$$

which implies contradiction. Therefore, we get (a) of theorem 2, by reduction to absurdity.

From the discussion mentioned above, we see that $\sharp D_3(T_r)$ is equal to 0 or 1. To prove (b) of theorem 2, thus, we have only to think about the case of $\sharp D_3 = 1$ in Fig.8.

Put $D_3(T_r) = \{x\}$. Assume that there exist $y \succ x$ with $y \notin L(T_r)$. If $\phi(y) > 1$, put

$$T'_r = T_r + \bigcup_{\sigma \in N(x),\ \sigma \succ x,\ \sigma \neq y} (y\sigma - x\sigma)\,.$$



(a) $\sharp D_3 = 0$ (b) $\sharp D_3 = 1$

Fig. 8: Two kind of possible form

Then we see that

$$\begin{aligned} \Phi(T'_r) - \Phi(T_r) = \Phi(T_r(r,x))(\phi(y) - 1) \\ \times \sum_{\sigma \in N(x),\ \sigma \succ x,\ \sigma \neq y} \Phi(\lceil\sigma\rceil) > 0, \end{aligned}$$

which implies contradiction. Next, if $\phi(y) \leq 1$, put

$$T'_r = T_r - y\sigma + x\sigma,$$

for $\sigma \in N(y)$ with $\sigma \succ y$. Then we see that

$$\begin{aligned} & \Phi(T'_r) - \Phi(T_r) \\ = & \Phi(T_r(r,x))\frac{1 - \phi(y)}{\phi(y)}\Phi(\lceil y\rceil)\Phi(T_r(r,y)) > 0 \end{aligned}$$

which implies contradiction. Therefore, we get (b) of theorem 2, by reduction to absurdity. □

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