Estimating Stochastic Volatility Models of Stock Returns in Chinese Markets

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Abstract: Volatility plays a key role for microstructure issues in the study of financial market. Stochastic Volatility (SV) models have been applied to the behavior study of financial variables. Two stock markets exist in China: Shanghai Stock exchange and Shenzhen Stock exchange. As emerging stock markets, investors are increasingly concerned about volatilities of these two stock markets. We introduce briefly how to estimate SV models using Markov chain Monte Carlo (MCMC) method. In order to do full and comprehensive analyses of the volatilities of stock returns, we estimate SV models using most of the historical data and different data frequencies of the two Chinese markets. We find that estimated values of volatility parameters are very high for all data frequencies. It suggests that stock returns are extremely volatile even at long term intervals in Chinese markets.

Keywords: MCMC Method, Volatility, Stock returns, Chinese markets.

I. INTRODUCTION

Volatility plays a key role for financial market microstructure issue. This gives rise to strong interest in volatility model which describes the evolution of conditional variance of the time-series variable.

There are essentially two types of models for describing the dynamics of volatility. One is GARCH model, and the other is SV models.

GARCH model considers only one source of uncertainty, but SV model introduces the additional innovation to the conditional variance equation so that they are much more flexible than ARCH or GARCH models [1].

There are two stock markets in China: Shanghai Stock exchange and Shenzhen Stock exchange. Many want to know Chinese stock markets' volatility.

In this paper, we primarily apply SV models to describe the behavior of volatility for different frequencies data for Chinese stock returns. In order to compare the results form SV models with other models, we also estimate stock returns' volatility with GARCH models.

The first contribution is that our analyses almost cover all historical data of Chinese stock markets, providing a full description about volatility. The second contribution of this paper is that we set and estimate SV models for different data frequencies, so we give very comprehensive analyses for the behaviors of volatilities of stock returns.

The rest of the paper is outlined as follows. In section 2, we introduce the basic principle and framework of SV models. Section 3 applies SV models, together with GARCH models, to analysis of Chinese stock returns, and then provides and discusses the estimated results. A final section concludes.

II. SV MODELS

1. The structure of volatility and GARCH model

Before discussing how to estimate SV models, we briefly introduce the structure of a volatility and GARCH model. The structure of a volatility model can be described as

$$x_t = \mu_t(\theta) + \varepsilon_t \tag{1}$$

$$\varepsilon_t = \sigma_t(\theta) z_t \tag{2}$$

In (1), the return x_t at time *t* is decomposed into a residual term ε_t and a conditional mean $\mu_t(\theta)$ According (2), the residual term ε_t has volatility conditional on information set available at time *t*-1, denoted σ_t. θ is the vector of unknown parameters. The variable z_t will be assumed to follow some distribution with mean 0 and variance 1.

The conditional variance of a GARCH is

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \sigma_{t-i}^2 + \sum_{j=1}^{q} \beta_j \varepsilon_{t-j}^2 \tag{3}$$

For this model to be well defined and the conditional variance to be positive, the parameters must satisfy the following constraints:

$\alpha_0 > 0$,

$\alpha_i \geq 0$,

$i = 1, \cdots, p$,

$\beta_j \geq 0$, and

$j = 1, \cdots, q$.

2. Stochastic volatility

Jacquier, Polson, and Rossi consider a model where the log σ_t^2 follows an AR (1) process with the error term v_t of volatility σ_t^2 [2, 3].

$$x_t = \mu_t + \varepsilon_t \quad (4)$$

$$\varepsilon_t = \sigma_t z_t \quad (5)$$

$$\ln \sigma_t^2 = \alpha_0 + \alpha_1 \ln \sigma_{t-1}^2 + \sigma_v v_t \quad (6)$$

3. Estimating SV models: MCMC method

SV model is not estimated directly by Maximum Likelihood method because the process σ_t^2 is an unobservable variable. In this paper, we model stochastic volatility using MCMC method.

Tsay gives great details to work for the above described with Gibbs sampling approach [4].

III. STOCHASTIC VOLATILITY MODELS OF STOCK RETURNS IN CHINESE MARKETS: EMPIRICAL ANALYSIS

1. Data

As authoritative statistical indicators widely adopted by domestic and overseas investors in measuring the performance of Chinese stock markets, many indices are compiled and published by Shanghai and Shenzhen stock exchanges. We choose the composite index of Shanghai stock exchange spanning the period from Dec 31, 1991 through Sep 30, 2009, and the component index of Shenzhen exchange, spanning the period from Jan 3, 1994 through Sep 30, 2009[1].

For the observed sequence, *t = 1, ……, T*, we let X_1 to be the last composite index of Shanghai stock exchange and X_2 to be the last component index of Shenzhen stock exchange. Then, we let Y_1 to be the returns of the composite index and Y_2 to be the returns of the component index. Y_1 and Y_2 are defined as follows.

$$Y_{1t} = \ln(X_{1t} / X_{1t-1}) \quad (7)$$

$$Y_{2t} = \ln(X_{2t} / X_{2t-1}) \quad (8)$$

In order to examine the features of stock indexes' returns for different frequencies, we consider three frequencies: daily returns, monthly returns, and quarterly returns. Denote that Y_{1t}^d, Y_{1t}^m, and Y_{1t}^q are Shanghai composite indexes' daily log returns, monthly log returns, and quarterly log returns, respectively. Denote that Y_{2t}^d, Y_{2t}^m, and Y_{2t}^q are Shenzhen component indexes' daily log returns, monthly log returns, and quarterly log returns, respectively. Each log return is measured in percent.

2. Estimating GARCH models

In contrast to SV models' results, we estimate GARCH models for log returns using Maximum Likelihood method.

For parsimony, we let mean equation equal to constant, hence, volatility model is

$$\sigma_t^2 = \alpha_0^G + \alpha_1^G \sigma_{t-1}^2 + \beta^G \varepsilon_{t-1}^2 \quad (9)$$

There are estimated values of α_1^G in Table 1.

Table 1. Estimating GARCH Models

α_1^G		
Y_{1t}^d	Y_{1t}^m	Y_{1t}^q
0.788	0.517	0.696
Y_{2t}^d	Y_{2t}^m	Y_{2t}^q
0.891	0.702	0.838

In Table 1, all estimates of volatility parameter, α_1^G, for different frequencies of two stock markets are statistically significant; in fact, their significant levels are all below 1%. The volatility parameter α_1^G measures the persistence of volatility, namely, volatility clustering. This suggests that volatility of log returns is strongly serially correlated; in other words, this means that a large (positive or negative) return tends to be followed by another large (positive or negative) return.

[1] Data is retrieved on Oct 20, 2009, from http://vip.stock.finance.sina.com.cn/.

3. Estimating SV models

Consider SV models,

$$\ln \sigma_t^2 = \alpha_0^{SV} + \alpha_1^{SV} \ln \sigma_{t-1}^2 + \sigma_v^{SV} v_t \qquad (10)$$

We run the Gibbs sampling for 10000 iterations for daily log returns, but for 2000 iterations for monthly log returns and quarterly log returns because of their small sample size relative to daily log returns. We discard results of the first 100 iterations in order to make remaining samples independent.

Table 2. Estimating SV Models

	α_1^{SV}	
Y_{1t}^d	Y_{1t}^m	Y_{1t}^q
0.903	0.891	0.891
Y_{2t}^d	Y_{2t}^m	Y_{2t}^q
0.889	0.857	0.889

According our estimating, SV models provide improvements in model fitting relative to GARCH models since most of parameters are statistically significant. (Here we do not present the t-statistics of GARCH and SV models for parsimony.) From Table 2, we see that all estimated values of persistent parameters α_1^{SV} are larger than estimated values α_1^{G}, the persistent parameters for GARCH models. This shows that in the case of SV models, the persistence of volatilities is high, relative to the case of GARCH models. In addition to high persistence in volatilities, there are almost identical estimated values of α_1^{SV} for different frequencies in SV models, while estimated values of α_1^{G} for monthly log returns are apparently smaller than estimated values for other frequencies in GARCH models.

4. Discussion

Table 2 shows that log returns are extremely volatile for Chinese stock markets even with a long time interval.

Lu, Ito, and Voges indicate that stock returns of Shenzhen component index exhibit long memory processes [5]. Lu and Ito find that there is a two-way feedback between Chinese two stock markets by using the expectational model to trace the response of a stock market to the other stock market [6]. These studies could suggest that high volatilities of stock markets could give rise to long memory or strong two-way feedback for stock market. Moreover, Lu and Ito have showed Chinese many macroeconomic series seem to turn out to be unstable [7]. Hence, this suggests that macro-economic instability could cause the stock market to be extremely volatile.

IV. CONCLUSION

Volatility is very important issue for studying the behavior of stock markets. There are two stock markets in China: Shanghai Stock exchange and Shenzhen Stock exchange. As emerging stock markets, investors are increasingly concerned about volatilities of the two stock markets.

We estimate SV and GARCH models of Chinese stock markets' returns in this paper.

According to our empirical analyses for Chinese stock markets, we show that SV models provide improvements in model fitting relative to GARCH models since most of parameters are statistically significant. Furthermore, for all data frequencies of Chinese stock markets, we find that estimated values of volatility parameters are very high for all frequencies' data. This implies that log returns are extremely volatile even with a long time interval in Chinese stock markets.

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