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Selection of parameters in design of Real-Time Spectral Analysis

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Abstract

A new analytical method with high speed processing in time-frequency domain was presented. In this method, sine and cosine wave with an established frequency, and with multiple periods are used, and we call these waves "cutting out wave". We call the frequency "established frequency", and call the number of periods of the cutting out wave "number of periods". The inner product of the cutting out wave and signal are calculated, and signal element with a frequency near the established frequency is detected. We call the unit that detects the signal element "auditory cell". There are many auditory cells, and those auditory cells have the established frequency which differs little by little. The design of this method is the arrangement of auditory cells. There are three parameters in the design, and these parameters are sampling frequency, number of periods and increasing rate of the established frequencies. In this paper, we show that selection of these parameters.

Keywords: Magnitude and phase characteristic, Time-frequency domain

1 Introduction

A new analytical method in time-frequency domain was presented by the authors [2]-[4]. In this method, processing in search of spectrum is performed by using one input data. Since all processing are perfectly independent, complete parallel processing is possible, and so if parallel processing computer will be brought to realization, real-time processing is realized. We call this analytical method Real-Time Spectral Analysis Method. In this method, sine and cosine waves for multiple periods with a certain frequency are used. We call these waves "cutting out wave" and call this frequency "established frequency", and call number of periods of the cutting out waves "number of periods". The inner product of the cutting out wave and signal are calculated, and a signal element with a frequency near the established frequency is detected. We call the unit that detects the signal element "auditory cell". There are many auditory cells, and those auditory cells have the established frequency which differs little by little. The design of this method is the arrangement of auditory cells. There are three parameters in the design, and these parameters are sampling frequency, number of periods and increasing rate of the established frequencies. In this paper, we show that selection of these parameters.

2 Analysis theory

2.1 Calculation of the decomposition wave in discrete system

The algorithm uses the inner products of the cutting out waves and signal. F_s is a sampling frequency and T_s is the sampling interval. f_j is the established frequency of the cutting out wave (angular frequency $\lambda_j = 2\pi f_j$). Here $j(=1,\cdots,n)$ is the cutting out wave number and n is the number of the cutting out waves. There are T periods (T is natural number) in the cutting out wave, here T is the number of periods. The length of the cutting out wave is $q_j(=T/f_j)$ and N_j denote the number of data included in the cutting out wave. Then the cutting out wave can be expressed as follows:

$$\begin{aligned} &s_j(l)=\sin(\lambda_j l T_s) \quad c_j(l)=\cos(\lambda_j l T_s) \\ &s_j(-l)=-s_j(l) \quad c_j(-l)=c_j(l) \\ &s_j(k+N_j)=s_j(k) \quad c_j(k+N_j)=c_j(k) \\ &k\text{ ; natural number} \quad l=1,\cdots,N_j \end{aligned} \tag{1}$$

Here we consider only sine waves as cutting out wave because the expansion for cosine waves is similar to the case of sine waves. A signal at time kT_s is $x(k)$. By using the signal which dated back to the past, the inner product of the cutting out wave and the signal at time kT_s can be written as follows:

$$Y_s^k(j)=\frac{2}{N_j}\sum_{l=1}^{N_j} x(k-l)s_j(l-k) \tag{2}$$

The inner product at time $(k+1)T_s$ is

$$\begin{aligned} Y_s^{k+1}(j) &= \frac{2}{N_j}\sum_{l=1}^{N_j} x(k+1-l)s_j(l-k-1) \\ &= \frac{2}{N_j}x(k)s_j(-k)+Y_s^k(j)-\frac{2}{N_j}x(k-N_j)s_j(N_j-k) \end{aligned} \tag{3}$$

From Eq.(3), the inner product at time $(k+1)T_s$ can be obtained by adding the product of the input data at time

kT_s and the cutting out wave to the inner product at time kT_s, and subtracting the third term. Since the second and third term have already been calculated, the necessary calculation in Eq.(3) is a multiplication in the first term. When it is $k = N_j$, third term on the right side of Eq.(3) is $s_j(0)$, but this can be found $s_j(N_j)$ by using Eq.(1). In the case of a cosine cutting out wave, the inner product $Y_c^k(j)$ is found using a similar way. By using these inner products, the following is calculated:

$$Y_{out}^k(j) = \sqrt{Y_s^k(j)^2 + Y_c^k(j)^2} \tag{4}$$

The unit that outputs $Y_{out}^k(j)$ detects a signal component of certain frequency (close to the established frequency f_j); below, this unit is called an auditory cell, and $Y_{out}^k(j)$ is spectrum. Since there are two equations of inner product in the auditory cell, the multiplication number of times that is necessary to update of an inner product is twice. Since number of auditory cells is n, total multiplication number is $2n$. Auditory cells are independent each other, and two equations are also independent. If parallel processing computer will be brought to realization, calculation number of times that need to calculates spectrums by Eq.(4) is 2 multiplications and one square root calculation. Even if number of auditory cells becomes large, the calculation number of times does not change. By using the inner products expressed by discrete system, we show below a procedure to get decomposition waves.

Step1 Calculate the inner products $Y_s^k(j)$ and $Y_c^k(j)$ at time kT_s

Step2 multiply $Y_s^k(j), Y_c^k(j)$ by $-s_j(k), c_j$ and the following is calculated:

$$Y_{ss}^k(j) = -Y_s^k(j)s_j(k), \quad Y_{cc}^k(j) = Y_c^k(j)c_j(k)$$

Step3 By using $Y_{ss}^k(j)$ and $Y_{cc}^k(j)$, the following is calculated:

$$Y_{cout}^k(j) = Y_{ss}^k(j) + Y_{cc}^k(j) \tag{5}$$

Step4 k is set to $k+1$, it returns to Step 1, and repeats below. $Y_{cout}^k(j)$ is the decomposition wave, and we call it "actual decomposition wave". The reconstruction wave is made by adding those decomposition waves, and we call this wave "actual reconstruction wave".

2.2 The signal detection range of the hearing cell

The established frequency is f and the length of the cutting out wave is q (=T/f). The wave that length for $T+1$ and $T-1$ periods becomes q can not be detected by this auditory cell. The detection width of the auditory cell is designated as B. The following relationship $B = 2f/T$ or $Bf/T = 2$ is formed. That is, the area made from spectrum width B and time length T/f becomes two and constant.

2.3 Calculation of the decomposition wave in continuous system

Consider a signal that is a wave with angular frequency ω, magnitude A, and phase ϕ. The signal at time kT_s is shown as follows:

$$x(k-l) = A\sin(\omega(k-l)T_s) \tag{6}$$

Eq.(6) is substituted for Eq.(2), and by using $s_j(k)$ in Eq.(1), the inner product is expressed as follow:

$$\begin{aligned}\hat{Y}_s^k(j) &= -\frac{2}{N_j}\sum_{l=1}^{N_j}\sin(\omega(k-l)T_s+\phi)\sin(\lambda_j(k-l)T_s) \\ &= -\frac{2}{N_j}A\left\{\cos(\phi)\sum_{l=1}^{N_j}\sin(\omega(k-l)T_s\sin(\lambda_j(k-l)T_s)\right. \\ &\quad \left.+\sin(\phi)\sum_{l=1}^{N_j}\cos(\omega(k-l)T_s)\sin(\lambda_j(k-l)T_s)\right\}\end{aligned} \tag{7}$$

Writing $t = kT_s, \tau = lT_s, q_j = T/f_j$, the following is obtained by minimization of T_s:

$$\begin{aligned}\tilde{Y}_s(j,t) &= -\frac{2f_j}{T}A\cos(\phi)\int_0^{q_j}\sin(\omega(t-\tau))\sin(\lambda_j(t-\tau)) \\ &-\frac{2f_j}{T}A\sin(\phi)\int_0^{q_j}\cos(\omega(t-\tau))\sin(\lambda_j(t-\tau))\end{aligned} \tag{8}$$

In the case of a cosine cutting out wave, the inner product $\tilde{Y}_c(j,t)$ is obtained by using similar way. The following is obtained by multiplying $\tilde{Y}_s(j,t)$ and $\tilde{Y}_c(j,t)$ by $-s_j(t)$ and $c_j(t)$.

$$\overline{Y}_{ss}(j,t) = -\tilde{Y}_s(j,t)s_j(t), \quad \overline{Y}_{cc}(j,t) = \tilde{Y}_c(j,t)c_j(t)$$

Here, $s_j(t) = \sin(\lambda_j t)$, $c_j(t) = \cos(\lambda_j t)$. Writing $\omega_j^+ = \omega - \lambda_j$, $\omega_j^- = \omega + \lambda_j$, and if we write

$$G_+(j) = \frac{2f_j}{T}\left\{\frac{1}{\omega_j^-}\sin(\frac{\omega_j^-}{2}q_j)\cos(\frac{\omega_j^-}{2}) + \frac{1}{\omega_j^+}\sin(\frac{\omega_j^+}{2}q_j)\cos(\frac{\omega_j^+}{2}q_j)\right\}$$

$$H_+(j) = \frac{2f_j}{T}\left\{\frac{1}{\omega_j^+}\sin^2(\frac{\omega_j^+}{2}q_j) + \frac{1}{\omega_j^-}\sin^2(\frac{\omega_j^-}{2}q_j)\right\}$$

Sum of two inner products is following:

$$\begin{aligned}Y_{out}^+(j,t) &= \overline{Y}_{ss}(j,t) + \overline{Y}_{cc}(j,t) \\ &= A\{G_+(j)\sin(\omega t+\phi) - H_+(j)\cos(\omega t+\phi)\}\end{aligned} \tag{9}$$

Furthermore, if we write

$$\Gamma_+(j) = \sqrt{G_+^2(j) + H_+^2(j)}, \quad \Psi_+(j) = \tan^{-1}(-H_+(j)/G_+(j))$$

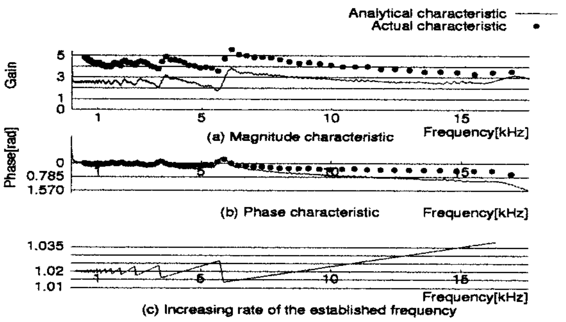

Fig. 1. Characteristic and increasing rate of the established frequency

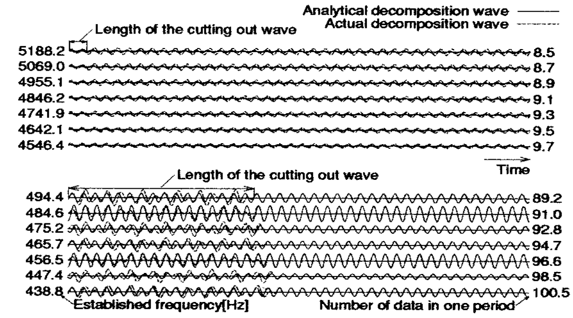

Fig. 2. Analytical decomposition waves and actual decomposition waves

Eq.(9) can be written as

$$Y_{out}^{+}(j,t) = A\Gamma_{+}(j)\sin(\omega t + \phi + \Psi_{+}(j)) \tag{10}$$

Eq.(10) is the decomposition wave, and essentially equal to Eq.(5). We call $Y_{out}^{+}(j,t)$ "analytical decomposition wave".

3 Reconstruction of the signal

We call the wave made by adding decomposition waves in Eq.(10) "analytic reconstruction wave". It is written as

$$\hat{Y}_{out}(t) = A\left\{\sum_{j=1}^{n}(G_{+}(j))\sin(\omega t+\phi) - \sum_{j=1}^{n}(H_{+}(j)\cos(\omega t+\phi)\right\} \tag{11}$$

Here, if we write

$$Z_G = \sum_{j=1}^{n} G_{+}(j), \qquad Z_H = \sum_{j=1}^{n} H_{+}(j)$$

$$\hat{r} = \sqrt{Z_G^2 + Z_H^2}, \qquad \hat{q} = \tan^{-1}(Z_H / Z_G)$$

Eq.(11) can be represented as

$$\hat{Y}_{out}(t) = A\hat{r}\sin(\omega t + \phi + \hat{q}) \tag{12}$$

It should be noted that the frequency is not the established frequency but the signal frequency ω. Amplitude has changed by $\hat{r}$ times for amplitude of the signal, and Phase $\hat{q}$ has newly generated. We call $\hat{r}$ "analytical magnitude characteristic", and call $\hat{q}$ "analytical phase characteristic". Each characteristics based on the actual reconstruction wave are called "actual magnitude characteristic" and "actual phase characteristic". The conditions of reconstruction is as follow:

Magnitude characteristic; The magnitude ratio $\hat{r}$ is constant regardless of frequency.

Phase characteristic; The newly generated phases $\hat{q}$ are proportional to the frequency. This is linear-phase characteristic.

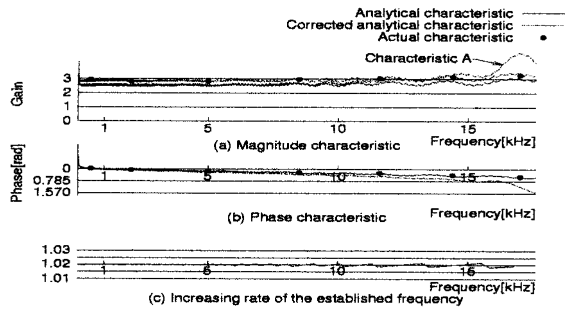

Fig. 3. Corrected actual characteristic and increasing rate of the established frequency

4 Selection of parameters in design

The characteristic to decide the precision of the reconstruction wave depends on three parameters, and those parameters are sampling frequency, the increasing rate of the established frequency and number of periods T. Below, selection of these parameters is shown.

4.1 Sampling frequency

When sampling frequency is 44.1kHz and number of periods T is 10, and the increasing rate μ of the established frequency is 1.02 (2%), the characteristic is shown in Fig.1. A solid line is the analytical characteristic and a mark ● is the actual characteristic. In Fig.1, there are two problems. First, actual reconstruction wave is essentially equal to analytic reconstruction wave, but both of them are different. Second, in Fig.1(c), the increasing rate μ changes like the saw, and according to it, the characteristics are also changing like a saw. At the beginning, we consider about the first problem. In the established frequency 5069Hz, number of data in one period of the cutting out wave becomes 8.7, and because the number of data is too small, the actual decomposition waves do not become approximation of the analytical decomposition waves. Because the both decomposition waves are different, the both reconstruction waves do not become the same. Both of the decomposition waves are shown in Fig.2. The number of the right side in Fig.2 is the number of data in one period of the cutting out wave, and at small number, both of the decomposition waves is different. When the number of data in one period of the cutting out wave is approaching 100, in order to increase the number of data in it, we make the sampli-

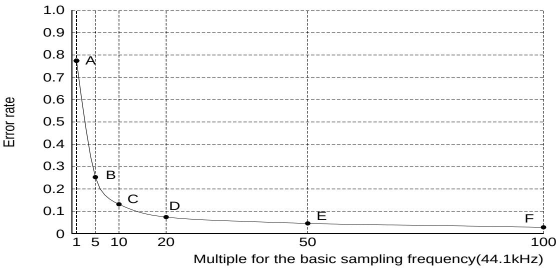

Fig. 4. Error rate between actual magnitude and analytical magnitude characteristic

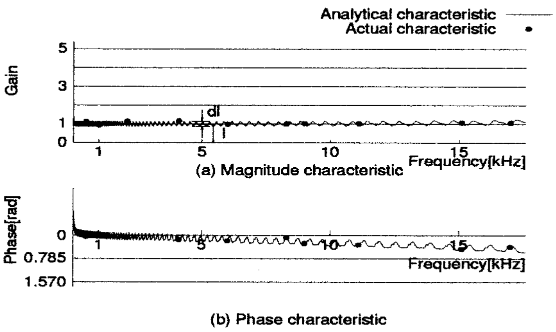

Fig. 5. Fluctuation of gain and phase in case that μ is 1.0527

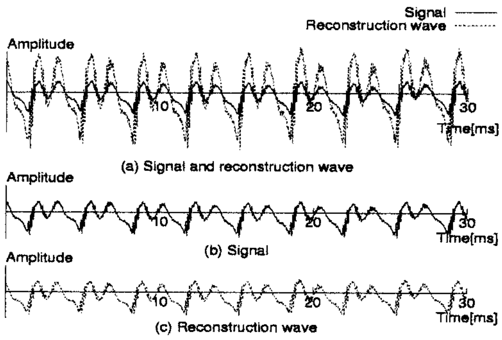

Fig. 6. The waveform of Japanese voices sound /i/ and reconstruction wave

ing frequency big. Here, at the established frequency of 333Hz, we make the sampling frequency 441kHz (10times), and the characteristic in that case is shown in Fig.3. Because the analytical range is designated until 17kHz, If auditory cells are set up by 17Hz, as A in Fig.3(a) shown, a big distortion will occurs. In order to ease this, auditory cells are set up to 25kHz. The actual characteristic is in good agreement with the analytical characteristic. The second problem is caused by the reason that it becomes impossible to maintain μ because the number of data in the cutting out wave decreases. However, because the number of data increases, this problem is improved, too. The error rate between actual and analytic magnitude characteristic for the increase of sampling frequency is shown in Fig.4. It is shown that the error rate decreases as the sampling frequency increases.

4.2 Increasing rate of the established frequency

Characteristic in case that μ is 1.0527 is shown in Fig.5. In the case, there are three auditory cells in B. Since number of auditory cell in B is small, each characteristic changes greatly like the wave. It is better for about ten auditory cells to exist in B. The number of data in B is designated as ν. T and μ are not determined independently but are decided by relationship of $2/T \quad \mu^{\nu}$.

4.2 Number of periods

It is impossible to make B which displays the spread of the spectrum and T which displays following characteristic small simultaneously. Although T should be decided by the situation of signal fluctuation, Usually it is good to make between 10 and 20.

5 The example of analysis using an voice signal

By using the auditory cells which become characteristic of Fig.3, the reconstruction wave of Japanese voice sound /i/ is shown in Fig.6. In Fig.6, the reconstruction wave with high approximation accuracy is obtained.

4. Conclusion

Approximation accuracy of the reconstruction wave when this analytical method is used is largely influenced by setting of the auditory cell. In the setting of the auditory cells, there are three parameters, a sampling frequency F_s and an increasing rate μ of the established frequency and the number of periods T. First, we decide T that decides follow efficiency. Next, we decide the sampling frequency F_s in consideration of the frequency range of the signal processing. Third, we decide increasing rate μ by using the relation of $2/T \quad \mu^{\nu}$. Here, it is better to decide that ν is about 10.

References

[1] C. K. Chui, Multivariate Splines, CBNS-NSF Series in Applied Math. #54, SIAM Publ, Philadelphia, 1988

[2] H. Nakatsuji, and S. Omatu, " Real-Time Spectrl Analysis," Electrical Engineering in Japan, Wiley InterScience , Vol. 161, No. 1, pp. 43-50, 2007

[3] H. Nakatsuji and S. Omatu, "Decomposition and reconsrtruction of signal in Real-time spectral analysis," Electrical Engineering in Japan, Wiley InterScience , Vol. 91, No.11, pp. 1123-1130, 2008

[4] H. Nakatsuji and S. Omatu, "Decomposition and reconsrtruction of signal in High-Speed Processing Method of Real-time spectral analysis," IEEJ Trans.EIS, Vol.128, No. 7, pp. 1168-1175, 2008 (in Japanese).