The Fifteenth International Symposium on Artificial Life and Robotics 2010 (AROB 15th '10),
B-Con Plaza, Beppu,Oita, Japan, February 4-6, 2010

Non-Fragile Control for Trajectory Tracking of Mobile Robots with Time-Delay

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Abstract: This paper is devoted to the problem of non-fragile controller design for the trajectory tracking of mobile robots. Firstly, the model of the mobile robots is exactly linearized via non-linear state feedback and proper coordinate transformation under certain conditions. Secondly, the time-delay is added to the linearized model and the non-fragile controller is designed for the trajectory tracking by employing linear matrix inequalities (LMIs) approach. Finally, simulation examples are included to illustrate the effectiveness of the proposed controller.

Keywords: Mobile Robots; Non-Fragile Control; Trajectory Tracking; Time-Delay; LMIs

I. INTRODUCTION

The mobile robot can be regarded as an effective extension of man's motor ability, and therefore it is sure to be indispensable in the course of recognition and exploration of the world. For example, to achieve the trajectory tracking, the nonlinear and nonholonomic motion of the mobile robot must be considered. Oriolo et al [1]. showed that dynamic feedback linearization was an efficient design the tool to solve trajectory tracking problem. Wang et al [2]. proposed novel, asymptotical, nonlinear adaptive trajectory tracking control laws based on Lyapunov stability theory. Dixon et al [3]. developed a velocity field for the constrained mobile robot trajectory, and formulated a differentiable controller for global asymptotic velocity field tracking. In [4], an adaptive controller for trajectory tracking was developed based on the learning ability of wavelet network(WN). In [5], the fuzzy control solution was introduced to resolve the robustness of mobile robot trajectory tracking problem. Liu et al [6]. used an artificial potential field to navigate the mobile robot in a novel simple adaptive tracking controller. To avoid premature convergence and trapping into local minimum, a chaos genetic algorithm based on population high-efficiency mutation (CGAPM) was presented in [7]. And a novel vector field control method based on nonlinear control algorithm was proposed for the mobile robots in [8].

On the other hand, it is well known that the time-delay usually occurs in the practical plants. Its existence may affect the stability of the system seriously and the dynamic performance of the system. Similarly, it is important for the tracking control of mobile robots to consider the time-delay effect, however it has been attracting little attention.

Further, we know that the mobile robot is driven by the DC motor. The designed control strategies need to be digitized in order to realize control goal. In this process, the small change of the control parameters exists and may cause control failure, or even destroy the system. L.H.Keel et al [9]. indicated that the traditional controller design method like optimal control and robust control only lead to fragile controller. That means that small offset of controller gain coefficient will be likely to damage the stability of the closed-loop system and degrade the performance. This requires that the designed controller gain coefficient should have sufficient adjustable redundancy or non-fragility in order to meet different performance requirements.

In this paper, we design a non-fragile controller by using LMI approach for the mobile robots with time-delay. The paper is organized as follows. In Section 2, the equation of a nonholonomic mobile robot is linearized via state feedback and the model of the mobile robots with time-delay is introduced based on the linearized model. The non-fragile trajectory tracking controller is designed in Section 3. And finally, the effectiveness of designed controller is verified by the simulation in Section 4.

II. PROBLEM FORMULATION AND PRELIMINARIES

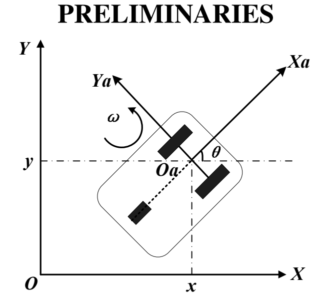

Fig. 1 The planar graph of a mobile robot

In Fig. 1, XOY is the world coordinate system, $X_aO_aY_a$ is the coordinate system fixed the mobile robot body ,Oa is the center of the axle of two driving wheels, (x,y) indicates the coordinate of the robot in world coordinate system and θ is the angle of moving direction (right angle to the wheel axis). v is the linear velocity of the robot and ω is its angular velocity.

Suppose that the wheels of the mobile robot rotate without slipping, the constraint can be denoted by

$$\dot{x} sin\theta - \dot{y} cos\theta = 0. \tag{1}$$

Then the equation of the mobile robot with two independently driving wheels can be obtained

$$\begin{aligned} \dot{x} &= vcos\theta \\ \dot{y} &= vsin\theta \\ \dot{\theta} &= \omega \end{aligned} \tag{2}$$

Introduce an auxiliary variable $\dot{v} = a$, and let

$$X = [x_1 \;\; x_2 \;\; x_3 \;\; x_4]^T = [x \;\; y \;\; \theta \;\; v]^T,$$

$$Y = [y_1 \;\; y_2]^T = [x \;\; y]^T,\ u = \begin{bmatrix} a & \omega \end{bmatrix}.$$

Then the system (2) can be transformed to the following form:

$$\begin{aligned} \dot{X} &= f(X) + g(X)u \\ Y &= h(X) \end{aligned} \tag{3}$$

where

$$f(X) = \begin{bmatrix} x_4cosx_3 \\ x_4sinx_3 \\ 0 \\ 0 \end{bmatrix},\ g(X) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix},$$

$$h(X) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \mathrm{X}$$

Setting

$$\xi(t) = [y_1 \; L_f y_1 \; y_2 \; L_f y_2]^T = [x_1 \; x_4cosx_3 \; x_2 \; x_4sinx_3]^T$$

and $u(t) = [acosx_3 - \omega x_4 sinx_3 \quad asinx_3 + \omega x_4 cosx_3]$, the system (3) becomes:

$$\begin{aligned} \dot{\xi}(t) &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xi(t) + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} u(t) \\ &= \tilde{A}\xi(t) + \tilde{B}u(t) \\ Y(t) &= \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \xi(t) = \tilde{C}\xi(t) \end{aligned} \tag{4}$$

Now, introducing time-delay in mobile robots, we have:

$$\begin{aligned} \dot{\xi}(t) &= \tilde{A}\xi(t) + A_\tau \xi(t-\tau) + \tilde{B}u(t) \\ Y(t) &= \tilde{C}\xi(t) \end{aligned} \tag{5}$$

where τ is constant time-delay. Consider the non-fragile controller:

$$u(t) = (K + \Delta K)\xi(t) \tag{6}$$

where K is the state feedback controller; $\Delta K = E\Delta(t)F$ is the additive gain perturbation of the controller with E,F the real matrices of known appropriate dimension, $\Delta(t)$ Lebesgue measurable matrix and $\Delta^T(t)\Delta(t) \le I$.

Combining the system (5) and controller (6) yields the following closes-loop system:

$$\begin{aligned} \dot{\xi}(t) &= [\tilde{A} + \tilde{B}(K + \Delta K)]\xi(t) + A_\tau \xi(t-\tau) \\ Y(t) &= \tilde{C}\xi(t) \end{aligned} \tag{7}$$

The objective of this paper is to design a non-fragile controller (6) such that the closed-loop system (7) is asymptotically stable.

Lemma 1: [10] Let E,F be the real matrices of appropriate dimensions, and $\Delta(t)$ be time-varying matrix with $\Delta^T(t)\Delta(t) \le I$. Then, for any scalar $\varepsilon > 0$, we have

$$E\Delta(t)F + F^T\Delta^T(t)E^T \le \varepsilon EE^T + \varepsilon^{-1}F^TF.$$

Lemma 2: [11] Let $M > 0, L, Q > 0$ be matrices of appropriate dimension. Then

$$M + L^TQ^{-1}L < 0$$

if and only if $\begin{bmatrix} M & L^T \\ L & -Q \end{bmatrix} < 0$ or $\begin{bmatrix} -Q & L \\ L^T & M \end{bmatrix} < 0$.

III. NON-FRAGILE CONTROLLER DESIGN

The following theorem gives the necessary and sufficient condition for existence of non-fragile state feedback controller for system (5) when the controller has additive gain perturbation.

Theorem 1: Given positive scalar τ, the system (7) is asymptotically stable if there exist scalar $\varepsilon > 0$, matrices $P > 0, Q > 0$ and R, such that the following linear matrix inequalities hold.

$$\begin{bmatrix} S + S^T & \varepsilon\tilde{B}E & A_\tau Q^{-1} & P^{-1} & P^{-1}F^T \\ * & -\varepsilon I & 0 & 0 & 0 \\ * & * & -Q^{-1} & 0 & 0 \\ * & * & * & -Q^{-1} & 0 \\ * & * & * & * & -\varepsilon I \end{bmatrix} < 0 \tag{8}$$

where $S = \tilde{A}P^{-1} + \tilde{B}R$, $*$ expresses the corresponding symmetric terms. And further, the state feedback controller may be taken as $K = RP$.

Proof: For the system (7), define a common Lyapunov function candidate

$$V = \xi^T(t)P\xi(t) + \int_{t-\tau}^{t} \xi^T(s)Q\xi(s)ds \tag{9}$$

where P, Q are solutions of equation (8).

$$\begin{aligned} \dot{V} &= \begin{bmatrix} \xi(t) \\ \xi(t-\tau) \end{bmatrix}^T \begin{bmatrix} P\hat{A} + \hat{A}^TP + Q & PA_\tau \\ A_\tau^TP & -Q \end{bmatrix} \begin{bmatrix} \xi(t) \\ \xi(t-\tau) \end{bmatrix} \\ &= \begin{bmatrix} \xi(t) \\ \xi(t-\tau) \end{bmatrix}^T M \begin{bmatrix} \xi(t) \\ \xi(t-\tau) \end{bmatrix} \end{aligned} \tag{10}$$

where $\hat{A} = \tilde{A} + \tilde{B}(K + \Delta K) = \tilde{A} + \tilde{B}(K + E\Delta(t)F)$, $M = \begin{bmatrix} P\hat{A} + \hat{A}^TP + Q & PA_\tau \\ A_\tau^TP & -Q \end{bmatrix}$. If $M < 0$, then the system (7) is asymptotically stable.

Pre- and post- multiplying M by diagonal matrix $\begin{bmatrix} P^{-1} & 0 \\ 0 & I \end{bmatrix}$ and its transpose respectively, and let $R = KP^{-1}$, we can obtain

$$\begin{aligned} & \begin{bmatrix} P^{-1} & 0 \\ 0 & I \end{bmatrix} M \begin{bmatrix} P^{-1} & 0 \\ 0 & I \end{bmatrix} \\ = & \begin{bmatrix} \hat{A}P^{-1} + P^{-1}\hat{A}^T + P^{-1}QP^{-1} & A_\tau \\ A_\tau^T & -P^{-1}QP^{-1} \end{bmatrix} \\ = & \begin{bmatrix} S + S^T + \hat{B} + \hat{B}^T + P^{-1}QP^{-1} & A_\tau \\ A_\tau^T & -Q \end{bmatrix} \\ < & \ 0 \end{aligned} \tag{11}$$

where $S = \tilde{A}P^{-1} + \tilde{B}R$ and $\hat{B} = \tilde{B}E\Delta(t)FP^{-1}$. According to Lemma 2, we obtain

$$\begin{bmatrix} S + S^T + \hat{B} + \hat{B}^T & A_\tau & P^{-1}Q \\ A_\tau^T & -Q & 0 \\ QP^{-1} & 0 & -Q \end{bmatrix} < 0 \tag{12}$$

Rewrite the above equation

$$\begin{bmatrix} S + S^T & A_\tau & P^{-1}Q \\ A_\tau^T & -Q & 0 \\ QP^{-1} & 0 & -Q \end{bmatrix} + \Sigma_1\Delta(t)\Sigma_2 + (\Sigma_1\Delta(t)\Sigma_2)^T < 0 \tag{13}$$

where $\Sigma_1 = \begin{bmatrix} \tilde{B}E & 0 & 0 & 0 \end{bmatrix}^T, \Sigma_2 = \begin{bmatrix} FP^{-1} & 0 & 0 & 0 \end{bmatrix}$. Then, from the Lemma 1, if there exists scalar $\varepsilon > 0$, we have

$$\begin{bmatrix} S + S^T & A_\tau & P^{-1}Q \\ A_\tau^T & -Q & 0 \\ QP^{-1} & 0 & -Q \end{bmatrix} + \varepsilon\Sigma_1\Sigma_1^T + \varepsilon^{-1}\Sigma_2^T\Sigma_2 < 0 \tag{14}$$

Note that Lemma 2 again, we have

$$\begin{bmatrix} S + S^T & \varepsilon\tilde{B}E & A_\tau & P^{-1}Q & P^{-1}F^T \\ * & -\varepsilon I & 0 & 0 & 0 \\ * & * & -Q & 0 & 0 \\ * & * & * & -Q & 0 \\ * & * & * & * & -\varepsilon I \end{bmatrix} < 0 \tag{15}$$

Pre- and post-multiplying (15) by diagonal matrix $diag(I, I, Q^{-1}, Q^{-1}, I)$ and its transpose respectively, the upper formula can be expressed by the following

$$\begin{bmatrix} S + S^T & \varepsilon\tilde{B}E & A_\tau Q^{-1} & P^{-1} & P^{-1}F^T \\ * & -\varepsilon I & 0 & 0 & 0 \\ * & * & -Q^{-1} & 0 & 0 \\ * & * & * & -Q^{-1} & 0 \\ * & * & * & * & -\varepsilon I \end{bmatrix} < 0 \tag{16}$$

And the state feedback controller is $K = RP$. This completes the proof. ■

Obviously, if $\Delta(t) = 0$, Theorem 1 degenerates to the normal state feedback control.

Now applying the designed controller K in Theorem 1 to the mobile robots (5), we obtain

$$u(t) = K\xi = \begin{bmatrix} acos\theta - \omega vsin\theta \\ asin\theta + \omega vcos\theta \end{bmatrix}.$$

IV. NUMERICAL EXAMPLES

Consider the systems (5)-(7) with

$$A_\tau = \begin{bmatrix} 0.01 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0.02 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tau = 0.2,$$

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, F = \begin{bmatrix} 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 \end{bmatrix}$$

$$\Delta_1 = -I = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \Delta_2 = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

From Theorem 1, we get the non-fragile controller by employing the LMI Toolbox in MATLAB

$$K_{nf} = \begin{bmatrix} -4.4453 & -4.8047 & -1.4766 & -1.5234 \\ -1.4766 & -1.5234 & -4.4453 & -4.8047 \end{bmatrix}$$

and the normal state feedback controller is

$$K_{normal} = \begin{bmatrix} -2.9687 & -3.2813 & 0 & 0 \\ 0 & 0 & -2.9687 & -3.2813 \end{bmatrix}.$$

When the two controllers exist the perturbation Δ_1, they all can make the system stable, see Fig.2 and Fig.3. Using the normal control with the perturbation Δ_2, the system becomes unstable (see Fig.4), while the non-fragile controller with the perturbation Δ_2 still makes the system stable (see Fig.5).

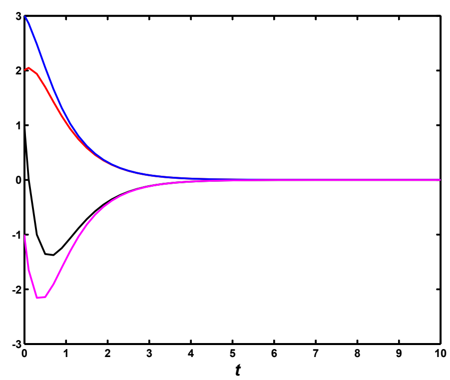

Fig.2 The control effect of the normal controller with Δ_1

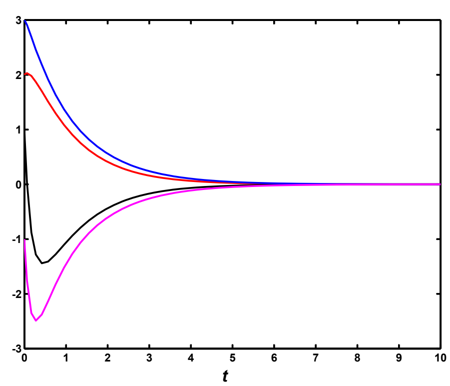

Fig.3 The control effect of the non-fragile controller with Δ_1

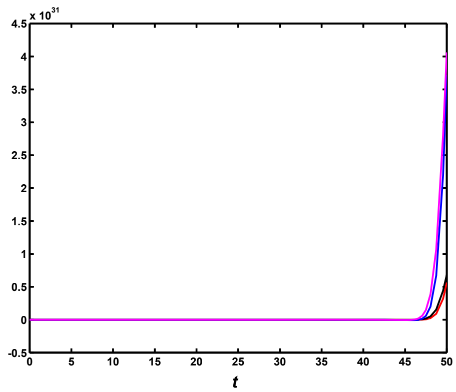

Fig.4 The control effect of the normal controller with Δ_2

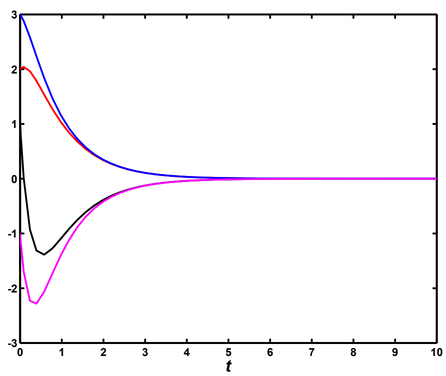

Fig.5 The control effect of the non-fragile controller with Δ_2

Next, we apply the designed non-fragile controller K to the trajectory tracking of the mobile robot system. We hope the mobile robot to make the circular motion:

$$x_d(t) = 0.5cos(0.01t), y_d(t) = 0.5sin(0.01t), \theta_d(t) = t.$$

Fig.6 is the mobile robot motion trajectory using the non-fragile controller on the $x-y$ plane. We can see that the trajectory can converge to the expected trajectory quickly.

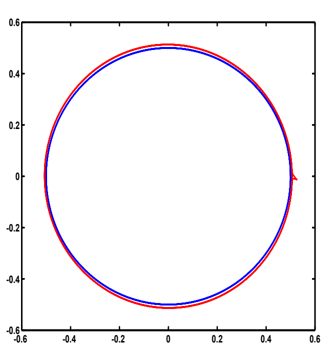

Fig.6 The mobile robot motion trajectory

V. CONCLUSIONS

In this paper, we have introduced non-fragile control to the mobile robots with time-delay for the first time. After exactly linearizing the model of the mobile robots via non-linear state feedback and proper coordinate transformation under certain conditions, we give the model of mobile robots with time-delay. Based on which, the non-fragile controller with the additive gain perturbation is designed in order to make the mobile robots asymptomatically stable. Finally, the proposed non-fragile controller is used to the mobile robots trajectory tracking control. Simulation results show that the designed non-fragile controller has strong robustness to controller gain perturbations which guarantee the fast response and superior control effect.

VI. ACKNOWLEDGMENTS

This work is supported by the National Natural Science Foundation of China (Grant Nos. 60727002, 60774003, 60921001, 90916024), the National 973 Program (No. 2005CB321902), and the COSTIND (No. A2120061303).

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