Modeling an Autonomous Underwater Vehicle with Four-Thrusters

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Abstract: In order to reduce the drag forces against a stream for an X4-autonomous underwater vehicle (AUV), a new type of hull shape is considered with an ellipsoid body. The associated dynamical model is derived by using a Lagrangian mechanics, as well as taking account of the effect of added mass and inertia.

Keywords: AUV, underactuated control system, nonholonomic systems, kinematic model, dynamical model.

I. INTRODUCTION

Generally, underwater vehicles can be divided into three, namely, manned submersibles, remotely operated vehicles (ROVs) and autonomous underwater vehicles (AUVs). The latter two are mostly utilized in the oil and gas industries, and for scientific and military applications. AUVs have great importance in underwater tasks due to its ability to navigate in abyssal zones without necessitating a tether that limits the range and maneuverability of the vehicle. Note however that their autonomy property directly affects the design of control system. That is, it requires advanced controllers and specific control schemes for achieving given tasks.

Most of AUVs have six degrees-of-freedom (DOFs) in motion. The vehicle of interest here falls into the class of underactuated AUVs because it has fewer actuators than the DOFs in motion. Many control systems for underwater vehicles have been designed up to now [1], [2] under the restrictions of cost, weight and complexity, keeping some reliability advantages. The vehicle is also a nonlinear system: all equations of motion of the system include highly coupled terms. Some equations of the motion of the system appear as a second-order nonholonomic constraints, and they cannot be integrated to obtain the position. Therefore, such underwater vehicles pertain to nonholonomic systems. Control of nonholonomic systems poses a difficult problem requiring a special control approach depending on the nature of the mechanical system, as stated in Arslan et al. [3].

An X4-AUV with a spherical hull shape was studied by Okamura [4], in which it makes only use of four thrusters to control the vehicle without using any steering rudders, falls into the class of underactuated AUVs and has nonholonomic features. The consideration of nonholomic systems is an interesting study from a theoretical standpoint because, as pointed out in the earlier works of Brockett, they cannot be asymptotically stabilized to a fixed point in the configuration space using continuously differentiable, time-invariant and state feedback control laws. In this paper, to overcome the demerit that in the former X4-AUV studied in [4] the drag forces against a stream are relatively higher than other AUVs, a new type of hull shape is proposed for the X4-AUV with an ellipsoid body that mostly closes to a streamlined shape and has the durability over pressure like a sphere. The ideal streamlined hull shape is known to minimize the drag forces acting on the hull while the X4-AUV is cruising. The corresponding X4-AUV kinematic and dynamic models are also presented here. When the X4-AUV moves underwater, additional forces and moment coefficients are added to account for the effective mass of the fluid that surrounds the robot, which causes an excessive acceleration of the robot, compared to the case where there is no any added mass and moment of inertia.

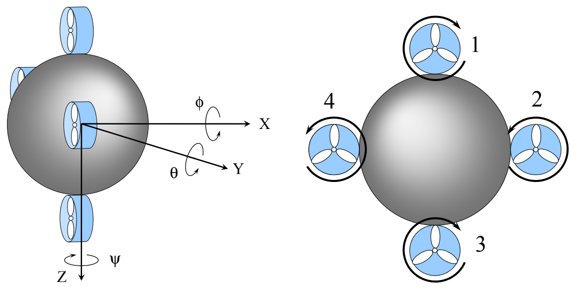
II. MODEL OVERVIEW

1. Former X4-AUV

The X4-AUV with a spherical hull shape is our former type of AUV. It has four thrusters to control the position and attitude changes in the motion [4]. It is categorized in underactuated AUVs and also has nonholonomic features. As shown in Fig. 1, the arrows around the thrusters indicate the rotational direction of the thrusters. The rotational angles consist of roll, pitch and yaw angles commonly used in guidance and navigation.

2. Proposed X4-AUV

The proposed X4-AUV is shown in Fig. 2. Previous work on X4-AUV suggests some limitations on the shape of the vehicle due to the hydrodynamics. A well thought out hull design for an AUV can improve the performances and efficiency of the vehicle. The body length will generally decrease the pressure drag by making the



(a) Side view (b) Front view

Fig. 1. Conventional X4-AUV

body more slender. A general ratio for an ellipsoid is length over diameter, l_E/d_E. This is the slenderness ratio of the body, whose values are in the range between five and ten. A typical value for ellipsoids is five. The induced lift on the ellipsoids having high slenderness ratios (l_E/d_E $\geq$5) is about 90 percent of the induced lift on the infinite cylinder, as reported by Zlotnick and Samuel [5]. In the design of a new X4-AUV, the goal was to make vehicle slenderness with a ratio of 5. Therefore, parameters r_1, r_2 and r_3 that are the lengths of the semi axes of the ellipsoidal vehicle are reduced to r_2= $5r$ and $r_2 = r_3 = r$.

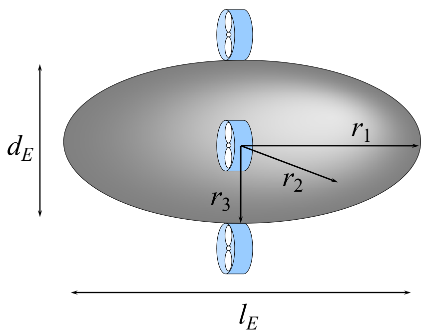


Fig. 2. Coordinate systems

III. MODEL DERIVATION

1. Definition of Coordinate Systems

In order to describe the motion of the underwater vehicle, a special reference frame must be established. There have two coordinate systems: i.e., inertial coordinate system (or fixed coordinate system) and motion coordinate system (or body-fixed coordinate system). The coordinate frame {E} is composed of the orthogonal axes {E$_x$, E$_y$, E$_z$} and is called as an inertial frame. This frame is commonly placed at a fixed place on Earth. The axes E$_x$ and E$_y$ form a horizontal plane and E$_z$ has the direction of the gravity field. The body fixed frame {B} is composed of the orthogonal axes {X, Y, Z} and attached to the vehicle. The body axes, two of which coincide with principle axes of inertia of the vehicles, are defined in Fossen [6] as follows:

X is the longitudinal axis (directed from aft to fore)
Y is the transverse axis (directed to starboard)
Z is the normal axis (directed from top to bottom)

Figure 3 shows the coordinate systems of AUV, which consist of a right-hand inertial frame {E} in which the downward vertical direction is to be positive and right-hand body frame {B}.

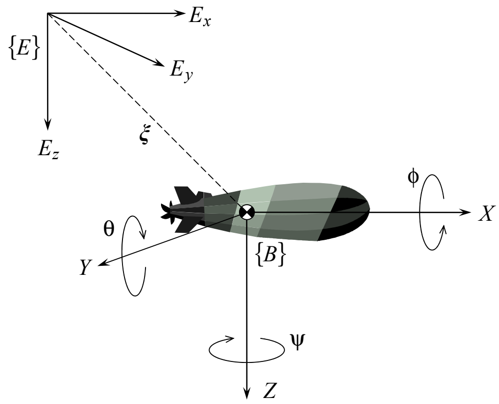


Fig. 3. Frames of X4-AUV

Letting $\boldsymbol{\xi} = [x \quad y \quad z]^T$ denote the mass center of the body in the inertial frame, defining the rotational angles of X-, Y- and Z-axis as $\boldsymbol{\eta} = [\phi \quad \theta \quad \psi]^T$, the rotational matrix R from the body frame {B} to the inertial frame {E} can be reduced to:

$$R = \begin{bmatrix} c\theta c\psi & s\phi s\theta c\psi - c\phi s\psi & c\phi s\theta c\psi + s\phi s\psi \\ c\theta s\psi & s\phi s\theta s\psi + c\phi c\psi & c\phi s\theta s\psi - s\phi c\psi \\ -s\theta & s\phi c\theta & c\phi c\theta \end{bmatrix} \tag{1}$$

where $c\alpha$ denotes $\cos\alpha$ and $s\alpha$ is $\sin\alpha$.

2. Mass and Inertia Matrix

A phenomenon that affects underwater vehicles is added mass. When a body moves underwater, the immediate surrounding fluid is accelerated along with the body. This affects the dynamics of the vehicle in such a way that the force required to accelerate in the water can be modeled as an added mass. Added mass is a fairly significant effect and is related to the mass and inertial values of the vehicle. M is a mass matrix of the body, J is an inertia matrix of the body, m_b is a mass of the vehicle, J_b is an inertia matrix of the vehicle and I is a 3×3 identity matrix. From the characteristics of added mass, M and J can be written as

$$M = \text{diag}(m_1, m_2, m_3) = m_b I + M_f \tag{2}$$
$$J = \text{diag}(I_x, I_y, I_z) = J_b + J_f \tag{3}$$

Here, the added mass matrix M_f and the added inertia matrix J_f are defined by

$$M_f = \text{diag}(\bar{A}, \bar{B}, \bar{C}) \tag{4}$$
$$J_f = \text{diag}(\bar{P}, \bar{Q}, \bar{R}) \tag{5}$$

The added mass and inertia can be determined by the following relations [8].

$$\begin{aligned} \bar{A} &= \frac{\alpha}{2-\alpha}\rho V \\ \bar{B} &= \frac{\beta}{2-\beta}\rho V \\ \bar{C} &= \frac{\gamma}{2-\gamma}\rho V \end{aligned} \tag{6}$$

$$\begin{aligned} \bar{P} &= \frac{1}{5}\frac{(r_2^2-r_3^2)(\gamma-\beta)}{2(r_2^2-r_3^2)+(r_2^2+r_3^2)(\beta-\gamma)}\rho V \\ \bar{Q} &= \frac{1}{5}\frac{(r_3^2-r_1^2)(\alpha-\gamma)}{2(r_3^2-r_1^2)+(r_3^2+r_1^2)(\gamma-\alpha)}\rho V \\ \bar{R} &= \frac{1}{5}\frac{(r_1^2-r_2^2)(\beta-\alpha)}{2(r_1^2-r_2^2)+(r_1^2+r_2^2)(\alpha-\beta)}\rho V \end{aligned} \tag{7}$$

where r_i $(i = 1, 2, 3)$ is a semi axis of the ellipsoid body along each axis, $V = (4/3)\pi r_1 r_2 r_3$ is a volume of the vehicle and ρ is a density of the fluid. Additionally, the α, β and γ are defined by

$$\alpha = r_1 r_2 r_3 \int_0^\infty \frac{d\lambda}{(r_1^2+\lambda)\Delta} \tag{8}$$

$$\beta = r_1 r_2 r_3 \int_0^\infty \frac{d\lambda}{(r_2^2+\lambda)\Delta} \tag{9}$$

$$\gamma = r_1 r_2 r_3 \int_0^\infty \frac{d\lambda}{(r_3^2+\lambda)\Delta} \tag{10}$$

$$\Delta = \sqrt{(r_1^2+\lambda)(r_2^2+\lambda)(r_3^2+\lambda)} \tag{11}$$

where λ is the eccentricity of the Y-Z axis cutting plane for the ellipsoid body, in which the formula is given in [7]. From equations (4)–(11), the components of M_f and J_f are reduced to

$$\begin{aligned} \bar{A} &\approx 0.0591\rho V = 3.94\pi r^3 \\ \bar{B} &= \bar{C} \approx 0.894\rho V = 5.96\pi r^3 \\ \bar{P} &= 0 \\ \bar{Q} &= \bar{R} \approx 3.640\rho V = 24.3\pi r^3 \end{aligned}$$

3. Dynamic Model

This section describes the dynamic model of the X4-AUV depicted in Fig. 2 by using a Lagrangian method. Assume that the body remains in a neutral buoyancy state at which the gravity and the buoyancy are balanced. We can first obtain the following kinetic energy formula,

$$T = T_{trans} + T_{rot} \tag{12}$$

where T_{trans} and T_{rot} are the translational kinetic energy and the rotational kinetic energy, which are given by

$$T_{trans} = \frac{1}{2}\dot{\boldsymbol{\xi}}^T M \dot{\boldsymbol{\xi}} \tag{13}$$

$$T_{rot} = \frac{1}{2}\dot{\boldsymbol{\eta}}^T J \dot{\boldsymbol{\eta}} \tag{14}$$

so that the kinetic energy can be rewritten in terms of M and J such that

$$T = \frac{1}{2}\dot{\boldsymbol{\xi}}^T M \dot{\boldsymbol{\xi}} + \frac{1}{2}\dot{\boldsymbol{\eta}}^T J \dot{\boldsymbol{\eta}} \tag{15}$$

From the assumption of the balance between buoyancy and the gravity, the potential energy, U is reduced to

$$U = 0 \tag{16}$$

Taking a generalized coordinate as $\boldsymbol{q} = [\boldsymbol{\xi}^T \quad \boldsymbol{\eta}^T]^T$, the Lagrangian L satisfies the following equations:

$$L = T - U \tag{17}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = F \tag{18}$$

From equations (15) and (16), it follows that

$$L = \frac{1}{2}(\dot{\xi}^T M \dot{\xi} + \dot{\eta}^T J \dot{\eta}) \tag{19}$$

Noting the representation of Lagrangian in (19), each derivative (or partial derivative) in (18) can be reduced to

$$\frac{\partial L}{\partial \dot{\boldsymbol{q}}} = M\dot{\boldsymbol{\xi}} + J\dot{\boldsymbol{\eta}} \tag{20}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\boldsymbol{q}}}\right) = \begin{bmatrix} M\ddot{\boldsymbol{\xi}} \\ J\ddot{\boldsymbol{\eta}} + \dot{J}\dot{\boldsymbol{\eta}} \end{bmatrix} \tag{21}$$

$$\frac{\partial L}{\partial \boldsymbol{q}} = \frac{1}{2}\begin{bmatrix} 0 \\ \frac{\partial}{\partial \boldsymbol{\eta}}(\dot{\boldsymbol{\eta}} J \dot{\boldsymbol{\eta}}) \end{bmatrix} \tag{22}$$

The equations of motion of X4-AUV due to Lagrangian mechanics become

$$\begin{bmatrix} M\dot{\boldsymbol{\xi}} \\ J\dot{\boldsymbol{\eta}} + \dot{J}\dot{\boldsymbol{\eta}} \end{bmatrix} - \begin{bmatrix} 0 \\ \frac{1}{2}\frac{\partial}{\partial \boldsymbol{\eta}}(\dot{\boldsymbol{\eta}} J \dot{\boldsymbol{\eta}}) \end{bmatrix} = F \tag{23}$$

where the generalized force F is given by

$$F = \begin{bmatrix} F_\xi \\ \boldsymbol{\tau} \end{bmatrix} \tag{24}$$

$$\boldsymbol{\tau} = \begin{bmatrix} \tau_\phi & \tau_\theta & \tau_\psi \end{bmatrix}^T \tag{25}$$

Here, F_ξ and $\boldsymbol{\tau}$ are the translational force and the rotational torque for the AUV.

Letting u_1 be an input to control X-directional translational motion for the AUV and f_i be a thrust generated by each thruster. Then, u_1 is defined by

$$u_1 = f_1 + f_2 + f_3 + f_4 \tag{26}$$

Introducing the unit vector $\boldsymbol{e}_X$ for the X-direction at the body frame, the corresponding translational force F_ξ is reduced to

$$\begin{aligned} F_\xi &= R\boldsymbol{e}_X u_1 \\ &= \begin{bmatrix} \cos\theta\cos\psi \\ \cos\theta\sin\psi \\ -\sin\theta \end{bmatrix} [f_1 + f_2 + f_3 + f] \end{aligned} \tag{27}$$

Defining the torque generated by each motor as τ_{Mi}, the moment around each axis at the body frame is given by

$$\tau_\phi = \sum_{i=1}^{4} \tau_{Mi} \tag{28}$$

$$\tau_\theta = (f_1 - f_3)l \tag{29}$$

$$\tau_\psi = (f_2 - f_4)l \tag{30}$$

where l denotes the distance between the center of gravity of the body and each thruster. The equations of motion are divided into the translational motion and the rotational motion, respectively,

$$\begin{aligned} M\ddot{\boldsymbol{\xi}} &= F_\xi \\ &= \begin{bmatrix} \cos\theta\cos\psi \\ \cos\theta\sin\psi \\ -\sin\theta \end{bmatrix} [f_1 + f_2 + f_3 + f] \end{aligned} \tag{31}$$

$$\begin{aligned} J\ddot{\boldsymbol{\eta}} + (\dot{J}\dot{\boldsymbol{\eta}} - \frac{1}{2}\frac{\partial}{\partial\boldsymbol{\eta}}(\dot{\boldsymbol{\eta}}^T J\dot{\boldsymbol{\eta}}) &= \boldsymbol{\tau} \\ &= \begin{bmatrix} \sum_{i=1}^{4} \tau_{Mi} \\ (f_1 - f_3)l \\ (f_2 - f_4)l \end{bmatrix} \end{aligned} \tag{32}$$

Here, the second term of left-hand side in the equation of rotational motion (32) is related to the Coriolis term in general equations of motion, and equal to the following equation that represents the Coriolis torque and the gyro torque:

$$\boldsymbol{\omega} \times J\boldsymbol{\omega} + \sum_{i=1}^{4} J_t(\boldsymbol{\omega} \times \boldsymbol{e}_X)\omega_i \tag{33}$$

where $\boldsymbol{\omega} = [\dot{\phi} \quad \dot{\theta} \quad \dot{\psi}]^T$, J_t is a moment of inertia for the thruster, ω_i is a rotational speed for the thruster i. Since $\boldsymbol{\omega}\times$ is a skew symmetric matrix, it follows that

$$\boldsymbol{\omega} \times J\boldsymbol{\omega} = \begin{bmatrix} -\dot{\theta}\dot{\psi}(I_y - I_z) \\ -\dot{\theta}\dot{\psi}(I_z - I_x) \\ -\dot{\theta}\dot{\psi}(I_x - I_y) \end{bmatrix} \tag{34}$$

$$\boldsymbol{\omega} \times \boldsymbol{e}_X = \begin{bmatrix} 0 \\ \dot{\psi} \\ -\dot{\theta} \end{bmatrix} \tag{35}$$

From the above results, equation (32) can be rewritten as

$$\begin{aligned} J\ddot{\boldsymbol{\eta}} &+ \begin{bmatrix} -\dot{\theta}\dot{\psi}(I_y - I_z) \\ -\dot{\theta}\dot{\psi}(I_z - I_x) \\ -\dot{\theta}\dot{\psi}(I_x - I_y) \end{bmatrix} \\ &+ J_t \begin{bmatrix} 0 \\ \dot{\psi} \\ -\dot{\theta} \end{bmatrix} [\omega_2 + \omega_4 - \omega_1 - \omega_3] \\ &= \begin{bmatrix} \sum_{i=1}^{4} \tau_{Mi} \\ (f_1 - f_3)l \\ (f_2 - f_4)l \end{bmatrix} \end{aligned} \tag{36}$$

Thus, the final dynamical model for the X4-AUV can be summarized by

$$m_1\ddot{x} = \cos\theta\cos\psi\, u_1 \tag{37}$$

$$m_2\ddot{y} = \cos\theta\sin\psi\, u_1 \tag{38}$$

$$m_3\ddot{z} = -\sin\theta\, u_1 \tag{39}$$

$$I_x\ddot{\phi} = \dot{\theta}\dot{\psi}(I_y - I_z) + u_2 \tag{40}$$

$$I_y\ddot{\theta} = \dot{\phi}\dot{\psi}(I_z - I_x) - J_t\dot{\psi}\Omega + lu_3 \tag{41}$$

$$I_z\ddot{\psi} = \dot{\phi}\dot{\theta}(I_x - I_y) - J_t\dot{\theta}\Omega + lu_4 \tag{42}$$

Here, the values of m_1, m_2, m_3, I_x, I_y, and I_z are defined as in Section 3.2. u_1, u_2, u_3, and u_4 are the control inputs for the translational (X-axis) motion, the roll (ϕ-axis) motion, pitch (θ-axis) motion, and yaw (ψ-axis) motion, respectively. Additionally, letting b be a thruster factor, d be a drag factor such as $\tau_{Mi} = d\omega_i^2$, it is found that Ω, u_1, u_2, u_3, and u_4 are described by

$$\begin{aligned} \Omega &= \omega_2 + \omega_4 - \omega_1 - \omega_3 \\ u_1 &= f_1 + f_2 + f_3 + f_4 \\ &= b(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \\ u_2 &= d(-\omega_2^2 - \omega_4^2 + \omega_1^2 + \omega_3^3) \\ u_3 &= f_1 - f_3 = b(\omega_1^2 - \omega_3^2) \\ u_4 &= f_2 - f_4 = b(\omega_2^2 - \omega_4^2) \end{aligned}$$

IV. CONCLUSION

In this paper, for an autonomous underwater vehicle (AUV), called "X4-AUV," a new type of hull shape has been proposed to reduce the drag forces against a stream. After taking account of added mass and inertia for this new type of hull shape considered here, the equations of motion for the X4-AUV were derived by using a Lagrangian mechanics. For the future work, any time-varying or switching control approach should be applied to the present X4-AUV to realize an underactuated control system.

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