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Stability Analysis of 3D Grasps by Considering Curvatures and Torsions of Contact Geometry

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Abstract: This paper analyzes 3-dimensional static grasp stability taking into account of contact geometry (metric tensor, curvature, and torsion). Grasp stiffness matrices are derived by replacing each finger with a 3-dimensional spring model. The stability is evaluated by eigenvalues and eigenvectors of the matrices. Any friction condition is considered at each contact point. That is, rolling contact occurs at friction contact point, and sliding contact occurs at frictionless contact point.

Keywords: Grasp stability, 3D spring model, frictionless grasp, frictional grasp, curvature, torsion**.**

I. INTRODUCTION

Magicians and jugglers perform dexterous and fine manipulation of multiple objects by their own hands. Multi-finger robotic hands also have potential ability executing the manipulation, because the hands have been developed and improved on the basis of human hands.

Stability is the tendency of a system to return to an equilibrium state when the system is displaced from this state. Yamada et al. [1][2][3] explored static grasp stability of single object and multiple objects. Arimoto and Yoshida [4] discussed dynamic stability by considering spring-mass-damper systems. In these analyses, curvature effect at contact point was considered.

To improve the dexterity of the hands, effect of contact surface geometry (metric tensor, curvature, and torsion) is analyzed in static grasp stability of single object in three dimensions. This analysis is formulated from the viewpoint of potential energy method.

II. PROBLEM FORMULATION

As shown in Fig. 1(a), we suppose that an *n*-fingers robot hand grasp an arbitrary shaped object in three dimensions. We analyze stability of the grasp.

1. Assumptions

For simplicity of discussions, the following conditions are assumed.

(A1) Both an object and fingers are rigid bodies.

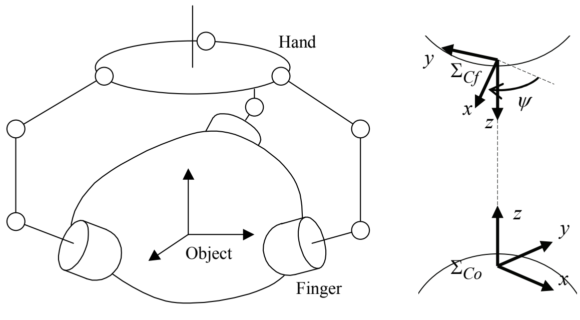
(A2) The contact between two bodies is of single contact.

(A3) Initial grasp configuration is in wrench (force and moment) equilibrium.

(A4) Infinitesimal configuration displacement of an object occurs due to external disturbance.

(A5) The relationship between finger configuration displacement and reaction force is replaced with a three-dimensional orthogonal spring model.

In Assumption (A5), the spring stiffness is denoted by $K_i = \mathrm{diag}[k_{xi}, k_{yi}, k_{zi}] \in \Re^{3\times 3}$, where subscript i means finger number. Direction of the stiffness is fixed along the axes of fingertip coordinate frame. In Assumption (A3), the spring is compressed at initial configuration and generates initial fingertip force $\boldsymbol{f}_i = K_i \boldsymbol{x}_{fi0}$.



(a) (b)

Fig. 1. An object grasped by a multi-fingers hand, and torsion between contact coordinate frames.

2. Symbols

The following coordinates and notations are used.

Σ_{bo} : Initial object coordinate frame.

Σ_o : Displaced object coordinate frame.

Σ_{Lo} : Local coordinate frame at contact point on the object surface.

Σ_{Co} : Contact coordinate frame on the object surface.

Σ_{bf} : Initial finger coordinate frame.

Σ_f : Displaced finger coordinate frame.

Σ_{Lf} : Local coordinate frame at contact point on the finger surface.

Σ_{Cf} : Contact coordinate frame on the finger surface.

$\boldsymbol{\varepsilon}_o \in \Re^6$: Configuration (position and orientation) displacement of the grasped object.

$\boldsymbol{\varepsilon}_f \in \Re^6$: Configuration (position and orientation) displacement of the finger.

$$\boldsymbol{\varepsilon} := [\boldsymbol{x}^T, \boldsymbol{\xi}^T]^T = [x, y, z, \xi, \eta, \zeta]^T \quad (1)$$

$\boldsymbol{\alpha}_o \in \Re^2$: Displacement parameter of contact point on the object surface.

$\boldsymbol{\alpha}_f \in \Re^2$: Displacement parameter of contact point on the finger surface.

$\psi \in \Re$: Torsion between two objects (Fig.1(b)).

${}^aT_b \in \Re^{4\times4}$: Homogeneous matrix expressing configuration of Σ_a in Σ_b.

$${}^aT_b = \left[\begin{array}{c|c} {}^aR_b & {}^a\boldsymbol{p}_b \\ \hline 0 & 1 \end{array}\right] \quad (2)$$

Symbols ${}^a\boldsymbol{p}_b$ and ${}^aR_b$ are position vector and rotation matrix. The other homogeneous matrices are similarly defined.

3. Contact Geometry

To derive contact frame velocity on the surfaces, we use contact surface geometry (metric tensor M_C, curvature K_C, and torsion T_C)[5]. Configuration of contact coordinate frame Σ_C in local coordinate frame Σ_L is depend on contact displacement parameter $\boldsymbol{\alpha}$. Hence, body velocity of Σ_c with respect to Σ_L is obtained as

$$\hat{V}_{L,C}(t) := [{}^LT_C(t)]^{-1}[{}^L\dot{T}_C(t)]$$
$$= \left[\begin{array}{cc|c|c} 0 & -T_C M_C\dot{\boldsymbol{\alpha}} & K_C M_C\dot{\boldsymbol{\alpha}} & M_C\dot{\boldsymbol{\alpha}} \\ T_C M_C\dot{\boldsymbol{\alpha}} & 0 & & 0 \\ \hline \multicolumn{2}{c|}{-(K_C M_C\dot{\boldsymbol{\alpha}})^T} & 0 & \\ \hline \multicolumn{3}{c|}{0} & 0 \end{array}\right] \quad (3)$$

The symbols ${}^LT_C$, M_C, K_C, T_C, $\boldsymbol{\alpha}$ mean ${}^{Lo}T_{Co}$, M_{Co}, K_{Co}, T_{Co}, $\boldsymbol{\alpha}_o$ in case of object surface, and mean ${}^{Lf}T_{Cf}$, M_{Cf}, K_{Cf}, T_{Cf}, $\boldsymbol{\alpha}_f$ in case of finger surface.

III. FINGER POSITION DISPLACEMENT

In this section, we omit subscripts of the number of finger for simple description. Finger configuration displacement ${}^{bf}T_f$ is derived by the following homogeneous matrix:

$$\begin{aligned} {}^{bf}T_f(\boldsymbol{\varepsilon}_f) &= [{}^{bf}T_{bo}][{}^{bo}T_o(\boldsymbol{\varepsilon}_o)][{}^oT_{Lo}][{}^{Lo}T_{Co}(\boldsymbol{\alpha}_o)] \\ &\times [{}^{Co}T_{Cf}(\psi)][{}^{Lf}T_{Cf}(\boldsymbol{\alpha}_f)]^{-1}[{}^fT_{Lf}]^{-1} \end{aligned} \quad (4)$$

Hence, finger position displacement $\boldsymbol{x}_f \in \Re^3$ and finger orientation displacement ${}^{bf}R_f \in \Re^{3\times3}$ is derived by the following form:

$$\boldsymbol{x}_f = [I_3, 0_{3\times1}][{}^{bf}T_f(\boldsymbol{\varepsilon}_f)]\begin{bmatrix} 0_{3\times1} \\ 1 \end{bmatrix} \quad (5)$$

$${}^{bf}R_f(\boldsymbol{\xi}_f) = [I_3, 0_{3\times1}][{}^{bf}T_f(\boldsymbol{\varepsilon}_f)]\begin{bmatrix} I_3 \\ 0_{1\times3} \end{bmatrix} \quad (6)$$

From (4) and (5), finger position displacement depends on displacements $\boldsymbol{\varepsilon}_o$ and $\boldsymbol{\alpha} := [\boldsymbol{\alpha}_o^T, \psi, \boldsymbol{\alpha}_f^T]^T \in \Re^5$.

$$\boldsymbol{x}_f = \boldsymbol{x}_f(\boldsymbol{\varepsilon}_o, \boldsymbol{\alpha}). \quad (7)$$

IV. GRASP STABILITY

1. Potential Energy of the Grasp

From Assumption (A5), potential energy stored in the i-th finger spring is obtained by

$$\begin{aligned} &U_i(\boldsymbol{\varepsilon}_o, \boldsymbol{\alpha}_i) \\ &= \tfrac{1}{2}\{\boldsymbol{x}_{fi0} + \boldsymbol{x}_{fi}(\boldsymbol{\varepsilon}_o, \boldsymbol{\alpha}_i)\}^T K_i \{\boldsymbol{x}_{fi0} + \boldsymbol{x}_{fi}(\boldsymbol{\varepsilon}_o, \boldsymbol{\alpha}_i)\} \end{aligned} \quad (8)$$

Parameter $\boldsymbol{\alpha}_i$ depends on both object configuration $\boldsymbol{\varepsilon}_o$ and contact friction condition, because each finger is replaced by 3D spring model.

$$\boldsymbol{\alpha}_i = \boldsymbol{\alpha}_i^{fc}(\boldsymbol{\varepsilon}_o) \quad (9)$$

This constraint is formulated in the following subsections. Hence, the potential energy is written as

$$U_i^{fc}(\boldsymbol{\varepsilon}_o) := U_i(\boldsymbol{\varepsilon}_o, \boldsymbol{\alpha}_i^{fc}(\boldsymbol{\varepsilon}_o)) \quad (10)$$

Total potential energy of the grasp is given by

$$U(\boldsymbol{\varepsilon}_o) = \sum_{i=1}^{n} U_i^{fc}(\boldsymbol{\varepsilon}_o) \quad (11)$$

From Appendix, gradient G and hessian H of the grasp are calculated by

$$G_i^{fc} := \left.\frac{\partial U_i^{fc}(\boldsymbol{\varepsilon}_o)}{\partial \boldsymbol{\varepsilon}_o}\right|_0 = [I_6, Q_i^{fc}]G_i \in \Re^6 \quad (12)$$

$$\begin{aligned} H_i^{fc} &:= \left.\frac{\partial^2 U_i^{fc}(\boldsymbol{\varepsilon})}{\partial \boldsymbol{\varepsilon}_o \partial \boldsymbol{\varepsilon}_o^T}\right|_0 \\ &= [I_6, Q_i^{fc}]H_i\begin{bmatrix} I_6 \\ [Q_i^{fc}]^T \end{bmatrix} + \left.\frac{\partial^2 (U_{i,\alpha}^T \boldsymbol{\alpha}_i^{fc})}{\partial \boldsymbol{\varepsilon}_o \partial \boldsymbol{\varepsilon}_o^T}\right|_0 \in \Re^{6\times6} \end{aligned} \quad (13)$$

where

$$G_i := \begin{bmatrix} U_{i,\varepsilon} \\ U_{i,\alpha} \end{bmatrix} \in \Re^{11},\ H_i := \begin{bmatrix} U_{i,\varepsilon\varepsilon} & U_{i,\varepsilon\alpha} \\ U_{i,\alpha\varepsilon} & U_{i,\alpha\alpha} \end{bmatrix} \in \Re^{11\times 11},$$

$$U_{i,\varepsilon} := \left.\frac{\partial U_i(\boldsymbol{\varepsilon}_o,\boldsymbol{\alpha}_i)}{\partial \boldsymbol{\varepsilon}_o}\right|_0,\ U_{i,\alpha} := \left.\frac{\partial U_i(\boldsymbol{\varepsilon}_o,\boldsymbol{\alpha}_i)}{\partial \boldsymbol{\alpha}_i}\right|_0,$$

$$U_{i,\varepsilon\varepsilon} := \left.\frac{\partial^2 U_i(\boldsymbol{\varepsilon}_o,\boldsymbol{\alpha}_i)}{\partial \boldsymbol{\varepsilon}_o \partial \boldsymbol{\varepsilon}_o^T}\right|_0,\ U_{i,\varepsilon\alpha} := \left.\frac{\partial^2 U_i(\boldsymbol{\varepsilon}_o,\boldsymbol{\alpha}_i)}{\partial \boldsymbol{\varepsilon}_o \partial \boldsymbol{\alpha}_i^T}\right|_0,$$

$$U_{i,\alpha\varepsilon} := \left.\frac{\partial^2 U_i(\boldsymbol{\varepsilon}_o,\boldsymbol{\alpha}_i)}{\partial \boldsymbol{\alpha}_i \partial \boldsymbol{\varepsilon}_o^T}\right|_0,\ U_{i,\alpha\alpha} := \left.\frac{\partial^2 U_i(\boldsymbol{\varepsilon}_o,\boldsymbol{\alpha}_i)}{\partial \boldsymbol{\alpha}_i \partial \boldsymbol{\alpha}_i^T}\right|_0,$$

$$Q_i^{fc} := \left.\frac{\partial(\boldsymbol{\alpha}_i^{fc})^T}{\partial \boldsymbol{\varepsilon}_o}\right|_0 \in \Re^{6\times 5}. \quad (14)$$

Because vector G_i and matrix H_i are independent of contact friction, these terms are calculated using (4), (5), and (8). But matrix Q_i^{fc} is dependent of contact friction condition. To obtain hessian H_i^{fc}, we derive Q_i^{fc} in the following sections.

2. Frictionless Contact

In case of frictionless contact, a body slides on other bodies' surface. If once object configuration moves due to external disturbance, finger moves on the object surface. If each finger motion is stable, finger position moves to local minimum of its potential energy. In this case, we can write displacement parameters $\boldsymbol{\alpha}_o$ and ψ as functions of $\boldsymbol{\alpha}_f$. Hence, we have the following constraint:

$$\boldsymbol{h}^T = \frac{\partial U(\boldsymbol{\varepsilon}_o,\boldsymbol{\alpha}(\boldsymbol{\alpha}_f))}{\partial \boldsymbol{\alpha}_f^T} = \frac{\partial U(\boldsymbol{\varepsilon}_o,\boldsymbol{\alpha})}{\partial \boldsymbol{\alpha}^T} A_f = 0 \quad (15)$$

where

$$A_f := \begin{bmatrix} \left[[{}^{Lf}R_{Lo}] \begin{bmatrix} \Omega K_{Co} \\ T_{Co} \end{bmatrix} M_{Co}, \boldsymbol{u}_3 \right]^{-1} \begin{bmatrix} \Omega K_{Cf} \\ T_{Cf} \end{bmatrix} M_{Cf} \\ I_2 \end{bmatrix}, \in \Re^{5\times 2}$$

$$\Omega := \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \in \Re^{2\times 2},\ \boldsymbol{u}_3 := [0,0,1]^T \in \Re^3 \quad (16)$$

Derivative of (15) is calculated as

$$\frac{\partial \boldsymbol{h}^T}{\partial \boldsymbol{\varepsilon}_o} = \left[I_3, \frac{\partial \boldsymbol{\alpha}^T}{\partial \boldsymbol{\varepsilon}_o} \right] \begin{bmatrix} \dfrac{\partial^2 U(\boldsymbol{\varepsilon}_o,\boldsymbol{\alpha})}{\partial \boldsymbol{\varepsilon}_o \partial \boldsymbol{\alpha}^T} \\ \dfrac{\partial^2 U(\boldsymbol{\varepsilon}_o,\boldsymbol{\alpha})}{\partial \boldsymbol{\alpha} \partial \boldsymbol{\alpha}^T} \end{bmatrix} [A_f] = 0 \quad (17)$$

We require other three constraints, because two constraints exist in (15) and five constraints exist in (9). From Assumption (A5), hence, we use the following finger orientation constraint ($\boldsymbol{\xi}_f = 0$):

$${}^{bf}R_f(\boldsymbol{\varepsilon}_o,\boldsymbol{\alpha}) = I_3 \quad (18)$$

From (17) and (18), Q^{fc} is obtained as

$$Q^{fs} := -\left[\begin{array}{c|c} \begin{matrix} 0 \\ [{}^{bf}R_{bo}]^T \end{matrix} & U_{\varepsilon\alpha} A_f \end{array} \right] \times \left[\begin{array}{c|c} \begin{matrix} M_{Co} \begin{bmatrix} \Omega K_{Co} \\ T_{Co} \end{bmatrix}^T [{}^{bf}R_{Lo}]^T \\ \hline \boldsymbol{u}_3^T [{}^{bf}R_{Lf}]^T \\ \hline -M_{Cf} \begin{bmatrix} \Omega K_{Cf} \\ T_{Cf} \end{bmatrix}^T [{}^{bf}R_{Lf}]^T \end{matrix} & U_{\varepsilon\varepsilon} A_f \end{array} \right]^{-1} \in \Re^{6\times 5} \quad (19)$$

where superscript "*fs*" means finger sliding. And we have

$$\left.\frac{\partial^2 (U_{i,\alpha}^T \boldsymbol{\alpha}_i^{fs})}{\partial \boldsymbol{\varepsilon}_o \partial \boldsymbol{\varepsilon}_o^T}\right|_0 = 0 \quad (20)$$

Hence, stiffness matrix of frictionless contact finger is given by

$$H_i^{fs} = [I_6, Q_i^{fs}] \begin{bmatrix} U_{i,\varepsilon\varepsilon} & U_{i,\varepsilon\alpha} \\ U_{i,\alpha\varepsilon} & U_{i.\alpha\alpha} \end{bmatrix} \begin{bmatrix} I_6 \\ [Q_i^{fs}]^T \end{bmatrix} \quad (21)$$

3. Frictional Contact

In case of frictional contact, a body rolls on other bodies' surface. In this case, relative velocity of two bodies at contact point becomes zero.

$$V_{Co,Cf}^b(t) = 0 \quad (22)$$

From (18) and (21), we have

$$Q^{fr} := -\begin{bmatrix} 0_{3\times 2} & 0_{3\times 3} \\ 0_{3\times 2} & [{}^{bf}R_{bo}]^T \end{bmatrix} \times \left[\begin{array}{c|c} M_{Co} R_\psi(\psi_0) & M_{Co} \begin{bmatrix} \Omega K_{Co} \\ T_{Co} \end{bmatrix}^T [{}^{bf}R_{Lo}]^T \\ \hline 0_{1\times 2} & \boldsymbol{u}_3^T [{}^{bf}R_{Lf}]^T \\ \hline -M_{Cf} & -M_{Cf} \begin{bmatrix} \Omega K_{Cf} \\ T_{Cf} \end{bmatrix}^T [{}^{bf}R_{Lf}]^T \end{array} \right]^{-1}, \in \Re^{6\times 5}$$

$$R_\psi(\psi_0) := \begin{bmatrix} \cos\psi_0 & \sin\psi_0 \\ \sin\psi_0 & -\cos\psi_0 \end{bmatrix} \quad (23)$$

where superscript "*fr*" means finger rolling and ψ_0 is initial torsion between the object and the finger. And we have

$$\left.\frac{\partial^2 (U_{i,\alpha}^T \boldsymbol{\alpha}_i^{fr})}{\partial \boldsymbol{\varepsilon}_o \partial \boldsymbol{\varepsilon}_o^T}\right|_0 = 0 \tag{24}$$

Hence, stiffness matrix of friction contact finger is given by

$$H_i^{fr} = [I_6, Q_i^{fr}] \begin{bmatrix} U_{i,\varepsilon\varepsilon} & U_{i,\varepsilon\alpha} \\ U_{i,\alpha\varepsilon} & U_{i.\alpha\alpha} \end{bmatrix} \begin{bmatrix} I_6 \\ [Q_i^{fr}]^T \end{bmatrix} \tag{25}$$

3. Grasp Stiffness Matrix

Grasp stiffness matrix is obtained by summation of stiffness matrix of each finger.

$$H = \sum_{i=1}^{n} H_i^{fc} \in \Re^{6\times 6} \tag{26}$$

where friction condition "*fc*" stands for "*fs*" or "*fr*". This condition is assigned depending on each finger friction condition.

V. CONCLUSIONS

This paper has analyzed static stability of three-dimensional grasp of single object from viewpoints of potential energy. And we have the following contributions: (1) Grasp stability is analyzed considering the torsion in addition to the curvature of an object's and fingers' surfaces. (2) In case of frictionless contact, each finger motion is treated as sliding on the object. (3) In case of frictional contact, each finger motion is treated as rolling on the object. (4) Stiffness matrices of mixed friction condition grasps are derived.

The following contributions are omitted due to lack of pages. We can calculate eigenvalues and eigenvectors of the derived matrices. Grasp stability is quantitatively evaluated by using the eigenvalues. Moreover, stable and unstable directions of object displacement are derived. Effectiveness of our proposed method is verified through numerical examples.

Our analyses of frictionless contact are applicable to the following cases: (1) Small friction objects are grasped. For examples, a cube of ice or a cake of soap is grasped. (2) Friction property is unknown before grasping, but object shape is acquired with image sensor.

Our method is applicable to optimization problem for grasp position design and fixture position design.

In our projected work, we will attack grasping multiple objects, consideration of finger rotation, and so on.

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APPENDIX

Total potential energy of the grasp is denoted by $U(\boldsymbol{\varepsilon})$, where $\boldsymbol{\varepsilon}$ stands for independent displacement parameters of the grasp. The grasp is stable if and only if $U(\boldsymbol{\varepsilon})$ is local minimum at $\boldsymbol{\varepsilon} = 0$ (initial configuration). Taylor expansion of $U(\boldsymbol{\varepsilon})$ is written as

$$U(\boldsymbol{\varepsilon}) = U(0) + \boldsymbol{\varepsilon}^T G + \boldsymbol{\varepsilon}^T H \boldsymbol{\varepsilon} + \cdots \tag{27}$$

where G and H are the gradient and the hessian, respectively.

$$G := \left.\frac{\partial U}{\partial \boldsymbol{\varepsilon}}\right|_0, \quad H := \left.\frac{\partial^2 U}{\partial \boldsymbol{\varepsilon} \partial \boldsymbol{\varepsilon}^T}\right|_0 \tag{28}$$

Hence, the grasp is stable if the following two conditions are satisfied.

(C1) $G = 0$.

(C2) H is positive definite.

Condition (C1) is satisfied by Assumption (A3). Hence, the grasp stability can be evaluated by Condition (C2).

Therefore, this paper derives independent parameter $\boldsymbol{\varepsilon}$, potential energy $U(\boldsymbol{\varepsilon})$, and hessian *H*. In this paper, the hessian is called as a grasp stiffness matrix, because the grasp system is replaced with spring model.

