A Walking Gait Generation Using Stance-leg Actuation

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Abstract: We study a simple two-link model based on Passive Dynamic Walking which can walk on the level ground. It is powered by extending and shortening the telescopic stance-leg. Through the simulation of an easy equivalent instantaneous model, we find that the stance-leg actuation is a way to compensate for the lost energy at the collision. It has stable cyclic walking gait. Besides, the model has mechanical energy feedback.

Keywords: stance-leg actuation, passive dynamic walking, powered walking, mechanical energy.

I. INTRODUCTION

In the early 1990s, McGeer pioneered Passive Dynamic Walking (PDW) robots that can walk down on shallow slopes without actuation and control [1]. This passive robot has a gait which is very natural and energy-efficient. The PDW concept shows us that walking can be sustained only by gravity. These passive robots must walk on a slope which provides a source of energy to compensate for the lost energy at the inelastic collision with floor. However, it is impossible to restore energy by gravity on level ground. So it is important for a robot to restore the lost kinetic energy through other approaches.

When McGeer proposed the PDW, he also put forward some ways to make passive walkers walk on level ground with active sources which have been realized by the followers [2]. These applications compensate for the lost energy with either kinetic energy or potential energy. Recently, Asano and Luo *et al* propose a parametric excitation approach which is a principle to increase the amplitude of vibration by swinging [3]. They demonstrate that the mechanical energy compensation is a good way to achieve energy-efficient and high-speed gait. The Robots lab in Automation Department in Tsinghua University introduced the "virtual slope walking" that compensates for lost energy through extending the stance leg and shortening the swing leg.

In this paper, we propose another way to compensate for the lost energy with mechanical energy. The model we discuss here has telescopic actuators on the legs. The system's energy is increased through up-and-down motion of the mass in the hip by the stance leg's extension and shortening. It is hard to study the model analytically when the leg's length is in the course of changing. But through numerical simulations we can find an equivalent place in the middle of the process of extension where the leg extends instantaneously. So does in the process of shortening. Then we can study this equivalent model which is easy to analyse under the conservation of angular of momentum.

This paper is organized as follows. In section Ⅱ, the model with telescopic legs and its parameters are introduced. In Section III, the walking map is presented using Lagrange Equation. In section IV, we find an easy equivalent model. In section V, the analysis of this equivalent model is conducted to clarify the existence of the fixed point and its stability. Finally, in section VI, we study the mechanical energy of this model.

II. THE MODEL

The model in this paper is a simplified model with only one mass *m* in the hip and two massless telescopic legs connected to the hip. As shown in Fig. 1(a). The other parameters and the whole walking step are shown in the Fig. 1(b). θ is clockwise angle of the stance leg with respect to the vertical, ω is the angular velocity of the stance leg and φ_0 is the counter clockwise inter-leg angle. A step starts when the prior swing leg has just made contact with the ground and the prior stance leg is ready to leave the ground. At this instant both legs have equal length r_s. The new stance leg begins to swing freely. When it gets to a key position where we have $\theta=\theta_{II}$, it begins to extend until its length is equals to r_e. We take this instant as another key position where $\theta=\theta_{III}$. Here we define leg length ratio $\beta= r_s / r_e$. Note that the velocity along the direction of the leg is zero in these two positions. After its length reaches the maximum value r_e, it begins to shorten to r_s at the key position where $\theta=\theta_{IV}$. The velocity along the direction of leg is also zero here. Finally, it swings freely until it strikes the floor, namely the instant V. Here we consider the collision as an inelastic one (no slip and no bounce). After the collision, the swing leg sticks to the ground and the stance leg is about to leave the ground. The transformation of the two legs is the end of one walking step.

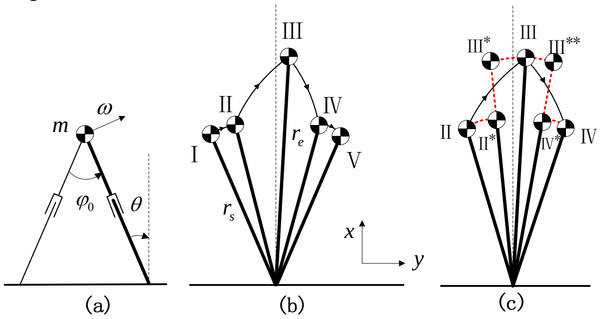

Fig.1. Model and key instants in one walking step

III. THE WALKING MAP

A walking step in the model is considered as a Poincare map or stride function [1], which consists of the dynamic function in swing phases and the strike function at collision. We choose the instant after collision as the Poincare section. Limit cycles are the fixed points of the walking map.

1. Dynamic Equations

In order to obtain the dynamic equations, we create a coordinates where the x-axis is along horizontal direction and y-axis is along the vertical direction with the origin in the stance foot contact point. In this system, the zero potential energy plane is the horizontal plane on the ground.

According to the Lagrange Equation, we have the dynamic equation in free swing phases I to II, IV to V.

$$\ddot{\theta}(t)=\frac{g}{r_s}\sin\theta(t) \tag{1}$$

We rescale time by $\tau=\sqrt{g/r_e}\,t$, (1) is rewritten as

$$\ddot{\theta}(\tau)=\frac{1}{\beta}\sin\theta(\tau) \tag{2}$$

Using the Lagrange Equation, we can also obtain the dynamic equation in the stance leg extension and shortening phase II to IV. Notice that the length r(τ) is a variable parameter when in the process of extension and shortening.

$$\ddot{\theta}(\tau)=\frac{r_e\sin\theta(\tau)}{r(\tau)}-\frac{2}{r(\tau)}\dot{r}(\tau)\dot{\theta}(\tau) \tag{3}$$

2. Transition in Inelastic Collision

In the collision, the geometric condition is met.

$$\theta_{\mathrm{I}}(n+1)=-(\varphi_0-\theta_{\mathrm{V}}(n)) \tag{4}$$

We assume the collision occurs instantaneously and there is no double support. According to the conservation of angular momentum, the new stance leg angular velocity is determined by

$$\omega_{\mathrm{I}}(n+1)=\omega_{\mathrm{V}}(n)\cos(\varphi_0) \tag{5}$$

(2) to (5) are the walking map **f** of this model. Note that the Poincare section is at the beginning of a step, and the walking map **f** maps the states in one section to the states in the next section.

IV. THE EQUIVALENT MODEL

1. Model Description

Because it is difficult to study the extension and shortening process analytically, we find an equivalent model which extends and shortens instantaneously. This equivalent model is shown in Fig. 1(c).

In this equivalent model, the stance leg extends from r_s to r_e instantaneously at the position where $\theta^*_{II}=\theta^*_{III}$ and then shortens instantaneously at the position where $\theta^{**}_{III}=\theta^*_{IV}$. So we have the conservation of angular momentum about the stance foot contact point, which leads to a discontinuous change in the angular velocity of the stance leg. From II^* to III^*, we have

$$\omega^*_{\mathrm{III}}=\beta^2\omega^*_{\mathrm{II}} \tag{6}$$

Similar to the process II^* to III^*, we can obtain the relationship between III^{**} and IV^*.

$$\omega^*_{\mathrm{IV}}=\frac{1}{\beta^2}\omega^{**}_{\mathrm{III}} \tag{7}$$

The equivalence we consider here is from the mechanical energy's point of view. In such a definition of equivalence, we must make sure the mechanical energy in instant II^* equals to the mechanical energy in instant II and the mechanical energy in instant III^* equals to that of instant III. The same goes for the shortening process. Here we only discuss how to get the position II^* according to II and III, which can be applied to the position IV^*.

From the mechanical energy equality, we have the following two equations:

$$\frac{1}{2}\omega_{\mathrm{II}}^2\beta+\cos\theta_{\mathrm{II}}=\frac{1}{2}\omega^{*2}_{\mathrm{II}}\beta+\cos\theta^*_{\mathrm{II}} \tag{8}$$

$$\frac{1}{2}\omega_{\mathrm{III}}^2+\cos\theta_{\mathrm{III}}=\frac{1}{2}\omega^{*2}_{\mathrm{III}}+\cos\theta^*_{\mathrm{III}} \tag{9}$$

Note that the kinetic energy has been rescaled by the dimensionless time τ.

From (6), (8) and (9), the instantaneously extension angle θ^*_{II} can be represented as:

$$\theta_{II}^{*}=\arccos\left(\frac{\frac{1}{2}\left(\omega_{III}^{2}-\beta^{4}\omega_{II}^{2}\right)+\cos\theta_{III}-\beta^{3}\cos\theta_{II}}{1-\beta^{3}}\right) \quad (10)$$

Using the same method, we can also get θ^*_{IV}.

$$\theta_{IV}^{*}=\arccos\left(\frac{\frac{1}{2}\left(\omega_{IV}^{2}-\beta^{4}\omega_{III}^{2}\right)+\cos\theta_{IV}-\beta^{3}\cos\theta_{III}}{1-\beta^{3}}\right) \quad (11)$$

The free swing phases and the collision rules are the same in these two models.

2. The Equivalent Walking Map

In the equivalent model, the walking step can be divided as: free swing phase with leg length r_s from I to II^*, instantaneously extension phase from II^* to III^*, free swing phase with leg length r_e from III^* to III^{**}, instantaneously shortening phase from III^{**} to IV^*, and free swing phase with leg length r_s from IV^* to V.

The dynamic function from III^* to III^{**} is

$$\ddot{\theta}(\tau)=\sin\theta(\tau) \quad (12)$$

So (2), (6), (7) and (12) are the walking map in this equivalent model.

According to this function, the simulation results are shown in Fig. 2. The red dotted line is the limit cycle of the equivalent model in Fig. 1(c) while the black solid line is the limit cycle of the model in Fig. 1(b). These two limit cycles are the same expect in the extension and shortening phases, so we can say that these two models' extension and shortening have the equivalent effect.

V. ANALYSIS OF EQUIVALENT MODEL

1. Finding Fixed Point

In order to find the cyclic gait of this model, we need to find the fixed point of the walking map. The fixed point is an initial state which maps to itself in the Poincare section. The state in this model is the stance leg angle θ_I and stance leg rate ω_I. Since θ is a constant that depends on the inter-leg angle φ_0 in the instant after collision, there leaves only one state variable ω_I in Poincare section.

In free swing phases, we have the mechanical energy conservation as follows:

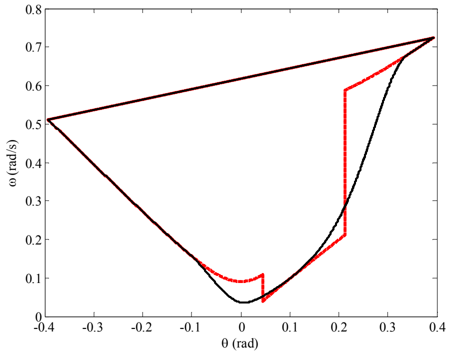

Fig.2. Limit cycles in phase space

$$\begin{cases} E_{I}=E_{II}=E_{II}^{*} \\ E_{III}^{*}=E_{III}^{**} \\ E_{IV}^{*}=E_{IV}=E_{V} \end{cases} \quad (13)$$

Together with the discontinuous changes of stance leg angular velocity in instantaneous extension, shortening and collision, we can obtain

$$\omega_I^2(n+1)=f(\omega_I^2(n)) \quad (14)$$
$$=\cos^2\varphi_0\omega_I^2(n)+\frac{2\cos^2\varphi_0(1-\beta^3)}{\beta^4}[\cos\theta_{II}^*(n)-\cos\theta_{IV}^*(n)]$$

To make it easy to analyse the fixed point, we take a new state variable $q=\omega_I^2$ into consideration, then according to the definition of a fixed point

$$q^f=f(q^f) \quad (15)$$

We have the analytical expression of fixed point.

$$q^f=\frac{2\cos^2\varphi_0(1-\beta^3)[\cos\theta_{II}^*-\cos\theta_{IV}^*]}{\beta^4(1-\cos^2\varphi_0)} \quad (16)$$

Once we have the expression of fixed point, we can calculate it with the parameters' values of the model. So the equivalent model is much easier to study.

2. Stability of Fixed Point

A. Local Stability

Once we have obtained a fixed point, we would like to know whether it is stable or not. The stable fixed point is what we want. The eigenvalues of the Jacobian matrix of the walking map can determine the local stability of the fixed point. If all eigenvalues are inside the unit cycle, which means a small disturbance will decay over time, then the fixed point is asymptotical stable, else if there is one eigenvalue outside the unit cycle, the fixed point is unstable.

From the analytical expression of fixed point (14), we can calculate the eigenvalue of Jacobian matrix.

$$\lambda=\frac{\partial f}{\partial q^f}=\cos^2\varphi_0 \quad (17)$$

According to (5), we have $\cos\varphi_0 < 1$, so the fixed point is asymptotical stable. This can be confirmed by the simulation results, as shown in Fig. 3. It shows that an initial state near fixed point will always converge to the fixed point after several cycles. But this initial state cannot be too far away from the fixed point.

B. Global Stability

However, the eigenvalun of Jacobian matrix can only reflect the local stability. Wisse *et al* propose the basin of attraction to analyze the global stability [4]. The basin of attraction is an area where the initial states can lead to a stable walking instead of falling. Fig. 4 shows the basin of attraction with respect to different β.

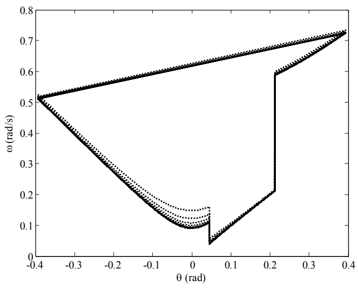

Fig.3. The convergence towards limit cycle

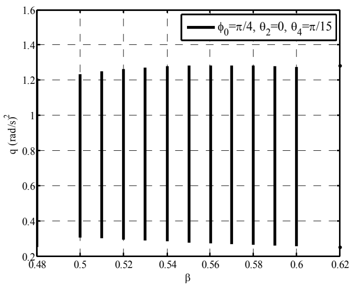

Fig.4. Basin of attraction

VI. MECHANICAL ENERGY

The model's total mechanical energy is the sum of kinetic energy and potential energy. It can be given by

$$E_i(n) = \frac{1}{2} m \dot{\theta}_i(t)^2 r_i^2 + mgr_i \cos\theta_i \tag{18}$$

Note that the time has been rescaled and leg length r is different in different phases.

We have energy conservation during free swing phases. That is (13). The energy changes during the instantaneously extension, instantaneously shortening and collision. For a fixed point, the complementary energy E_c is equal to the lost energy E_r. We know the energy is lost during collision, so it must be increased from instant II* to instant IV*. They can be expressed as

$$E_c = E_{\mathrm{IV}}^* - E_{\mathrm{II}}^* \tag{19}$$

According to the relationships about stance leg's angle, angular velocity and energy, we can obtain

$$E_c = mgr_e[\cos\theta_{\mathrm{II}}^* - \cos\theta_{\mathrm{IV}}^*](\frac{1-\beta^3}{\beta^2}) \tag{20}$$

$$E_r = mgr_e[\frac{1}{2}\beta^2(1-\cos^2\varphi_0)\omega_{\mathrm{I}}^2 + (\cos\theta_{\mathrm{II}}^* - \cos\theta_{\mathrm{IV}}^*)(\frac{1-\beta^3}{\beta^2})(1-\cos^2\varphi_0)] \tag{21}$$

From (20), we can see that the complementary energy only depends on the θ^*_{II}, θ^*_{IV}, β and r_e. These parameters are constant in the model, so the complementary energy is the same for different initial state. The lost energy depends on not only the above parameters, but also on the initial state ω^2_{I}. This means that there exists feedback in this model. When the initial state is larger than fixed point, the lost energy is larger than the complementary energy which makes the initial state in the next step smaller. Finally it will converge to the fixed point when the lost energy is equal to the complementary energy. When the initial state is smaller than fixed point, it will converge to the fixed point because the lost energy is smaller than the complementary energy. The energy feedback can also illustrate that the fixed point is stable.

VII. CONCLUSION

This paper has proposed a powered bipedal model walking on level ground. It has been proven that a passive dynamic walking model can walk down on a shallow slope without actuation or control because of the lost energy is recovered from the descent of center of mass. From this point of view, we study the up-and-down motion of the hip mass on stance-leg actuation. The simulation results of the model and its equivalent model show that there exist stable periodical gait. When the model has appropriate structure parameters, the complementary energy is a constant, which means there exist energy feedback.

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