# A Collaborative Localization Tolerant to Recognition Error by Double Check Particle Exchange 

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#### Abstract

Statistical algorithms for collaborative multi-robot localization have been proposed using particle filter. In these algorithms, with synchronizing each robot's belief or exchanging particle of robots each other, fast and accurate localizations are attained. These algorithms assume correctness of recognition of other robots, and influence of recognition error is not discussed. However, if the recognition of other robots is wrong, a large amount of error in localization may occur. This paper explains this problem. Furthermore, an algorithm for collaborative multi-robots localization is proposed to cope with this problem. In the proposed algorithm, particles in a robot are sent to other robots according to measurement result in the sending robot, at the same time, some particles remain in the sending robot. Received particles from other robots are evaluated using measurement result in the receiving robot. The proposed method is tolerant to recognition error by remaining particles and twice evaluations of exchanging particles in sending robot and receiving robot, and if there is no recognition error, the proposed method increases accuracy of estimation by these twice evaluations. These properties of the proposed method are argued mathematically. Simulation results show that mistakes of recognition of other robots does not cause serious problem in the proposed method.


Keywords: Multi-Robot Localization, Particle Filter, Localization, Collaborative Multi-Robot System

## I. INTRODUCTION

Localization of mobile robot using sensors is considered to be one of the most important problems in mobile robotics, and probabilistic methods have been proposed [1]. Probabilistic methods are expected to be robust for sensor noise or some inappropriate sensor information. The Kalman filter and Monte Carlo Localization (MCL) are widely used for probabilistic localization [1]. These are based on Markov localization [2],[3],[4],[5]. MCL uses particle filter that consists of possible positions of a robot [6],[7],[8].

In a multi-robot system, each robot can recognize other robots as mobile landmarks for localization and know relative positions of other robots by using sensors, and each robot can know the estimation results of locations in other robots by communication. In this situation, the accuracy of localization may increase and the calculation time may decrease by collaborating information of robots. For example, there are such collaborative methods by using geometrical group configuration of robots. Nakamura et al.[9] propose a localization method for mobile robots using geometrical constraints of observed robots and landmarks in the environment. Also, Kurazume et al. propose CPS (Cooperative Positioning System) [10]. In CPS, robots are divided into two groups, and while robots in one
group are moving, robots in another group are stationary as landmarks.

On the other hand, there are methods by unifying localizations of robots. In these methods, first, each robot localizes own position independently without information of other robots, and then localization information of robots are exchanged and unified. Using Kalman filter, Bahr et al. [11] propose a method for cooperative localization. This method combines multiple estimations. In general, Kalman filter assumes the distribution of noise to be Gaussian. In particle filter, there are few assumptions about distribution of noise. In this paper, robot localization problem using particle filter is discussed.

Fox et al.[12],[13] propose a method for collaborative multi-robot localization using particle filter. In this method, each robot's belief is synchronized whenever one robot detects another. With this collaboration, faster calculation and higher accuracy of localization is obtained. Gasparri et al. [14] propose another method, in which particles and sensor information are exchanged if these weights exceed a threshold.

In these probabilistic collaborative multilocalization algorithms using particle filter, recognition of other robots is assumed to be correct, and influence of recognition error of other robot is not discussed. However, recognition of other robots is difficult in some
cases. For example, in a case that a robot recognizes other robot using laser range sensors and shapes of robots are similar, it is difficult to distinguish robots. This may cause a serious problem for localization. This paper discusses this problem.

To cope with this problem, a new algorithm for probabilistic collaborative multi-robot localization is proposed. In the proposed algorithm, particles in a robot are sent to other robots according to measurement result in the sending robot, at the same time, some particles remain in the sending robot. Received particles from other robots are evaluated using measurement result in the receiving robot. With remaining particle, localization results are expected to be tolerant to recognition error of other robot, and with twice evaluation of exchanging particles in sending robot and receiving robot, high accuracy is expected. These properties of the proposed method are argued mathematically and confirmed by simulation in this paper.

## II. OUTLINE OF SOME CONVENTIONAL ALGORITHMS FOR COLLABORATIVE MULTI-ROBOT LOCALIZATION

First, the outline of MCL without collaboration is presented [1]. Here, the only one robot is considered. MCL is a probabilistic method and uses particle filter to represent the probability of location of robot that is called belief. Belief is the probability $\operatorname{bel}\left(\mathbf{x}_{t}\right)=$ $p\left(\mathbf{x}_{t} \mid \mathbf{u}_{t}, \mathbf{z}_{t}\right)$, where $\mathbf{x}_{t}$ is the location of robot at the time $t$ and $\mathbf{x}_{t}=\left(x_{t}, y_{t}, \theta_{t}\right)$. $\mathbf{u}_{t}$ is input data such as behavior of robot, and $\mathbf{z}_{t}$ is measurement data by sensors of robot. Precisely, there are two types of belief, that is, prior belief and posterior belief. $\operatorname{bel}\left(\mathbf{x}_{t}\right)$ means posterior belief and prior belief is denoted by $\operatorname{bel}^{-}\left(\mathbf{x}_{t}\right)$.

Particle filter represents above probability distributions by particles. Current set $X_{t}$ of particles is obtained as follows. For each particle $\mathbf{x}_{t-1}^{(k)} \in X_{t-1}$ $(1 \leq k \leq M)$, another set $X_{t}^{-}$of particles is made according to the motion model $p\left(\mathbf{x}_{t} \mid \mathbf{u}_{t}, \mathbf{x}_{t-1}^{(k)}\right)$. $X_{t}^{-}$represents prior belief $\operatorname{bel}^{-}\left(\mathbf{x}_{t}\right)$, expressed by $X_{t}^{-} \approx \operatorname{bel}^{-}\left(\mathbf{x}_{t}\right)$. For each $\overline{\mathbf{x}}_{t}^{(k)} \in X_{t}^{-}(1 \leq k \leq M)$, a weight $w_{t}^{(k)}=p\left(\mathbf{z}_{t} \mid \overline{\mathbf{x}}_{t}^{(k)}\right)$ is calculated according to measurement model $p\left(\mathbf{z}_{t} \mid \mathbf{x}_{t}\right)$.

Finally, $M$ particles are selected randomly from $X_{t}^{-}$with probability proportional to its weight, and $X_{t}$ consists of these $M$ particles with $X_{t} \approx p\left(\mathbf{x}_{t} \mid \mathbf{u}_{t}, \mathbf{z}_{t}\right)$.

Fox et al. [13] have expanded algorithms for multirobot by using this MCL. They denote $N$ is the number
of robots, $d_{i}$ is the data gathered by robot $i$ and $\mathbf{x}_{n}$ is the location of robot $n \cdot d_{i}$ consists of odometry measurements, environmental measurements and detections by robot $i$. When robot $j$ recognizes other robot $i$, information about the location of robot $i$ relative to robot $j$ is sent to robot $i$ from robot $j$, and $\operatorname{bel}\left(\mathbf{x}_{i, t}\right)$ is calculated as below.

$$
\begin{align*}
& \operatorname{bel}\left(\mathbf{x}_{i, t}\right)=p\left(\mathbf{x}_{i, t} \mid d_{i, t}\right) p\left(\mathbf{x}_{i, t} \mid d_{j, t}\right) \\
& =p\left(\mathbf{x}_{i, t} \mid d_{i, t}\right) \int_{p\left(\mathbf{x}_{i, t} \mid \mathbf{x}_{j, t}, r_{j, t}\right) p\left(\mathbf{x}_{j, t} \mid d_{j, t-1}\right) d \mathbf{x}_{j, t},} \tag{1}
\end{align*}
$$

where $r_{j, t}$ is a detection variable showing relative position of robot $i$ from robot $j$.

Gasparri et al. have proposed another algorithm for collaborative multi-robot localization [14]. Particles in each robot are exchanged if these weights exceed some threshold, under the assumption that recognitions of robots are correct.

## III. THE PROBLEM OF RECOGNITION ERROR OF OTHER ROBOTS

The conventional collaborative multi-robot localization methods assume that recognition of a robot from other robot is correct. That is, the robot number of robot $i$ is recognized as $i$ correctly from robot $j$. In a multi-robot system, many robots with the same shape are used. In that case, recognition of a robot from other robot is difficult. In the case that initial positions of all robots are known, a robot can recognize other robots by tracing positions of other robots successively using laser sensors of the robot. However, if other two robots are very near, these robots cannot be distinguished by the robot and recognition of other robots may fail.

In the method proposed by Fox et al., probabilities of localizations of robots are collected and multiplied and the probability of localization of a robot is obtained as (1). However, if recognition of a robot from another robot is wrong, collected probability from wrong recognized robot may be very small and the probability of localization of a robot is also very small. In (1), if robot $i^{\prime}$ is recognized as robot $i$ incorrectly, $\operatorname{bel}\left(\mathbf{x}_{i, t}\right)$ $=p\left(\mathbf{x}_{i, t} \mid d_{i, t}\right) p\left(\mathbf{x}_{i^{\prime}, t} \mid d_{j, t}\right)$ and this may become very small.

In the case that the recognized robot location is far from the correct robot location, the probability of localization may be almost zero. In this case, the probability of location cannot be calculated correctly and estimated location of a robot may be very far from the correct location and this causes a serious problem.

In the case that the recognized robot location is near to the correct robot location, the estimation error of
location is not very large. However, as the distance between the correct robot location and the wrong recognized robot is larger, the estimation error of location becomes large.

## IV. THE PROPOSED ALGORITHM

To cope with the problem described in the previous section, a new algorithm for collaborative multi-root localization is required. Recently the accuracy of laser range sensor is very high, therefore we assume that there is no error about measurements for relative position from a robot to another robot. Additionally, we assume that there is no communication delay between robots. In the proposed algorithm, particles of robots are exchanged. Some particles are selected according to their weights determined by perception using sensors. Selected particle are sent to another robot and weights of these particles are evaluated again by using sensors of another robot. These particles are evaluated twice at sending robot and receiving robot. Some particles remain in the sending robot. If detection of another robot is wrong, weight of received particle is very small, and detection error is not harmful for localization by using remained particles. If detection of another robot is correct, weight of particle is evaluated in two robots, and improvement of accuracy of localization is expected.

The number of robots is assumed to be $N$. Robot has a set of $M$ particles $X_{i, t}=\left\{\mathbf{x}_{i, t}^{(1)}, \mathbf{x}_{i, t}^{(2)}, \ldots, \mathbf{x}_{i, t}^{(M)}\right\}$ at the time $t$ which represents $\operatorname{bel}\left(\mathbf{x}_{i, t}\right)=p\left(\mathbf{x}_{i, t} \mid \mathbf{u}_{i, t}, \mathbf{z}_{i, t}\right)$, where $\mathbf{x}_{i, t}=\left(x_{i}, y_{i}, \theta_{i}\right)$.

In the same way, $X_{i, t}^{-}=\left\{\overline{\mathbf{x}}_{i, t}^{(1)} \overline{\mathbf{x}}_{i, t}^{(2)}, \ldots, \overline{\mathbf{x}}_{i, t}^{(M)}\right\}$ represents $\operatorname{bel}^{-}\left(\mathbf{x}_{i, t}\right)$, that is, $X_{i, t}^{-} \approx \operatorname{bel}^{-}\left(\mathbf{x}_{i, t}\right)$. A weight of $\overline{\mathbf{x}}_{i, t}^{(k)}$ is denoted by $w_{i, t}^{(k)}$, that is, $w_{i, t}^{(k)}=p\left(\mathbf{z}_{i, t} \mid \overline{\mathbf{x}}_{i, t}^{(k)}\right)$.

The relative position of robot $j$ from robot $i$ can be represented as $\mathbf{s}_{i j}=\left(x_{i j}, y_{i j}, \theta_{i j}\right)$, and ideally $\mathbf{x}_{j}=$ $\mathbf{x}_{i}+\mathbf{s}_{i j}$. In this paper, $\mathbf{s}_{i j}$ is assumed to be correct and $p\left(\mathbf{x}_{j} \mid \mathbf{x}_{i}, \mathbf{s}_{i j}\right)$ is assumed to be Dirac delta function, then

$$
p\left(\mathbf{x}_{j} \mid \mathbf{u}_{i}, \mathbf{z}_{i}, \mathbf{s}_{i j}\right)=\int \delta\left(\mathbf{x}_{j}-\left(\mathbf{x}_{i}+\mathbf{s}_{i j}\right)\right) p\left(\mathbf{x}_{i} \mid \mathbf{u}_{i}, \mathbf{z}_{i}\right) d \mathbf{x}_{i}
$$

$$
\begin{equation*}
=p\left(\mathbf{x}_{j}-\mathbf{s}_{i j} \mid \mathbf{u}_{i}, \mathbf{z}_{i}\right) \tag{2}
\end{equation*}
$$

In robot $i,\left(1-p_{0}\right) M /(N-1)$ particles $\left(0<p_{0}<1\right)$ are selected from $X_{i, t}^{-}$according to their weights. These particles are considered to represent $p\left(\mathbf{x}_{i, t} \mid \mathbf{u}_{i, t}, \mathbf{z}_{i, t}\right)$. For each selected particle $\overline{\mathbf{x}}_{i, t}^{(k)}$, $\hat{\mathbf{x}}_{j, t}^{(k)}=\overline{\mathbf{x}}_{i, t}^{(k)}+\mathbf{s}_{i j, t}$ is calculated. The set $\hat{X}_{i j, t}$ of these $\hat{\mathbf{x}}_{j, t}^{(k)}$ represents $p\left(\mathbf{x}_{j, t} \mid \mathbf{u}_{i, t}, \mathbf{z}_{i, t}, \mathbf{s}_{i j, t}\right)$. Then $\hat{X}_{i j, t}$ is sent to robot $j$ from robot $i$. In the same way, robot $i$ receives a set $\hat{X}_{j i, t}$ of particles from robot $j$. The number of received particles in robot $i$ is $\left(1-p_{0}\right) M$.

Whereas a set $R_{i}$ of $p_{0} M$ particles remains in robot $i$, which are selected from $X_{i, t}^{-}$according to their weight.

In robot $i$, a weight for each received particle is calculated according to the measurement model. Particles remained in $i$ and received particles are collected as the disjoint union : $X_{i, t}=R_{i} \cup \bigcup_{j \neq i} \hat{X}_{j i, t}$.
$M$ particles are resampled from $X_{i, t}$ according to its weight, and $\tilde{X}_{i, t}$ is produced. $\tilde{X}_{i, t}$ represents current position of robot $i$.

Probabilistic property of the proposed method will be discussed mathematically. In probabilistic expression,

$$
\begin{align*}
\tilde{X}_{i, t} \approx & \approx \eta\left[p_{0} \cdot p\left(\mathbf{x}_{i, t} \mid \mathbf{u}_{i, t}, \mathbf{z}_{i, t}\right)\right. \\
& \left.+\frac{1-p_{0}}{N-1} \sum_{j \neq i} p\left(\mathbf{x}_{i, t} \mid \mathbf{u}_{i, t}, \mathbf{z}_{i, t}\right) p\left(\mathbf{x}_{i, t} \mid \mathbf{u}_{j, t}, \mathbf{z}_{j, t} \mathbf{s}_{j i, t}\right)\right], \tag{3}
\end{align*}
$$

where $\eta$ is the normalization coefficient for probability distribution.

When abilities of sensors of robots are almost the same and relative position of robots $j$ 's from robot $i$ are estimated correctly, the second term of (3) have smaller standard deviation than that of the first term of the equation and the estimation of position of robot $i$ is expected to be more accurate than the original estimation.

For example, $\quad p\left(\mathbf{x}_{i, t} \mid \mathbf{u}_{i, t}, \mathbf{z}_{i, t}\right) \quad$ and $p\left(\mathbf{x}_{i, t} \mid \mathbf{u}_{j, t}, \mathbf{z}_{j, t} \mathbf{s}_{j i, t}\right)$ are assumed to be normal distribution with mean value being true value $\hat{\mathbf{x}}_{i, t}=\left(\hat{x}_{i}, \hat{y}_{i}, \hat{\theta}_{i}\right)$ for the location of robot $i$.
$p\left(\mathbf{x}_{i, t} \mid \mathbf{u}_{i, t}, \mathbf{z}_{i, t}\right)=\eta \exp \left[-\frac{1}{2}\left(\mathbf{x}_{i, t}-\hat{\mathbf{x}}_{i, t}\right) \Sigma_{i}^{-1}\left(\mathbf{x}_{i, t}-\hat{\mathbf{x}}_{i, t}\right)^{T}\right]$, (4) where $\eta$ is normalization coefficient and $\Sigma_{i}$ is the covariance matrix assumed to be a diagonal matrix $\Sigma_{i}=\operatorname{diag}\left\{\sigma_{i}^{2}, \sigma_{i}^{2}, \mu_{i}^{2}\right\} \quad$ with diagonal elements $\sigma_{i}^{2}, \sigma_{i}^{2}, \mu_{i}^{2}$, and
$p\left(\mathbf{x}_{i, t} \mid \mathbf{u}_{j, t}, \mathbf{z}_{j, t}, \mathbf{s}_{j i, t}\right)=\eta \exp \left[-\frac{1}{2}\left(\mathbf{x}_{i, t}-\hat{\mathbf{x}}_{i, t}\right) \sum_{j i}^{-1}\left(\mathbf{x}_{i, t}-\hat{\mathbf{x}}_{i, t}\right)^{T}\right]$, where $\eta$ is another normalization coefficient and $\Sigma_{j i}$ is the covariance matrix assumed to be $\Sigma_{j i}=\operatorname{diag}\left\{\sigma_{j i}^{2}, \sigma_{j i}^{2}, \mu_{j i}^{2}\right\}$.

Then, $\quad p\left(\mathbf{x}_{i, t} \mid \mathbf{u}_{i, t}, \mathbf{z}_{i, t}\right) p\left(\mathbf{x}_{i, t} \mid \mathbf{u}_{j, t}, \mathbf{z}_{j, t}, \mathbf{s}_{j i, t}\right)=$ $\eta \exp \left[-\frac{1}{2}\left(\left(\mathbf{x}_{i, t}-\hat{\mathbf{x}}_{i, t}\right)\left(\Sigma_{i}^{-1}+\Sigma_{j i}^{-1}\right)\left(\mathbf{x}_{i, t}-\hat{\mathbf{x}}_{i, t}\right)^{T}\right)\right]$ and the covariance matrix is $\left(\Sigma_{i}^{-1}+\Sigma_{j i}^{-1}\right)^{-1}=$ $\operatorname{diag}\left\{\left(\sigma_{i}^{-2}+\sigma_{j i}^{-2}\right)^{-1},\left(\sigma_{i}^{-2}+\sigma_{j i}^{-2}\right)^{-1},\left(\mu_{i}^{-2}+\mu_{j i}^{-2}\right)^{-1}\right\} \quad$. Since $\left(\sigma_{i}^{-2}+\sigma_{j i}^{-2}\right)^{-1}<\sigma_{i}^{2}$, the standard deviation of the second term of (3) becomes smaller than that of the first term, and the localization is expected to be more accurate. Especially when $\sigma_{i} \approx \sigma_{i j}$ and $\mu_{i} \approx \mu_{i j}$, standard deviation becomes $\left(\sigma_{i}^{-2}+\sigma_{j i}^{-2}\right)^{-1 / 2} \approx \sigma_{i} / \sqrt{2}$.

Now the problem of the previous section is discussed in the proposed algorithm. Robot $i$ detects
another robot $j$ by using sensor. We consider the case that robot $j$ is recognized incorrectly as robot $j^{\prime}$.

Proposed method works well under these conditions. If there is a recognition error of robot as above, the corresponding summand in the second term of (3) becomes $p\left(\mathbf{x}_{i, t} \mid \mathbf{u}_{i, t}, \mathbf{z}_{i, t}\right) p\left(\mathbf{x}_{i, t} \mid \mathbf{u}_{j, t}, \mathbf{z}_{j, t}, \mathbf{s}_{j^{\prime}, t}\right)$ and may be very small. Then, remaining parts of the equation (in the case all recognitions are wrong, only the first term $\left.p\left(\mathbf{x}_{i, t} \mid \mathbf{u}_{i, t}, \mathbf{z}_{i, t}\right)\right)$ are not small and correct estimation is done from the remaining part. This can be seen as below. $p\left(\mathbf{x}_{i, t} \mid \mathbf{u}_{i, t}, \mathbf{z}_{i, t}\right)$ and $p\left(\mathbf{x}_{i, t} \mid \mathbf{u}_{j, t}, \mathbf{z}_{j, t}, \mathbf{s}_{j^{\prime}, t,}\right)$ are assumed to be normal distributions as above (4),(5). But in this case, from (2), $p\left(\mathbf{x}_{i, t} \mid \mathbf{u}_{j, t}, \mathbf{z}_{j, t}, \mathbf{s}_{j_{i j, t}}\right)=p\left(\mathbf{x}_{i}-\mathbf{s}_{j^{\prime}} \mid \mathbf{u}_{j}, \mathbf{z}_{j}\right)=$ $\eta \exp \left[-\frac{1}{2}\left(\mathbf{x}_{i, t}-\hat{\mathbf{x}}_{i j j^{\prime}, t}\right) \Sigma_{j i}^{-1}\left(\mathbf{x}_{i, t}-\hat{\mathbf{x}}_{i j j^{\prime}, t}\right)^{T}\right]$, where $\hat{\mathbf{x}}_{i j j^{\prime}, t}=$ $\hat{\mathbf{x}}_{i, t}-\left(\hat{\mathbf{x}}_{j, t}-\hat{\mathbf{x}}_{j^{\prime}, t}\right)=\left(\hat{x}_{i j j^{\prime}}, \hat{y}_{i j j^{\prime}}, \hat{\theta}_{i j j^{\prime}}\right)$. If $\Sigma_{j i} \approx \Sigma_{i}$, and $\Sigma_{i}$ is assumed as above,

$$
\begin{aligned}
& p\left(\mathbf{x}_{i, t} \mid \mathbf{u}_{i, t}, \mathbf{z}_{i, t}\right) p\left(\mathbf{x}_{i, t} \mid \mathbf{u}_{j, t}, \mathbf{z}_{j, t}, \mathbf{s}_{j^{\prime} i, t}\right) \approx \\
& \eta \exp \left[-\frac{1}{2}\left\{\left(\left(x_{i}-\hat{x}_{i}\right)^{2}+\left(x_{i}-\hat{x}_{i j j^{\prime}}\right)^{2}\right) \sigma_{i}^{-2}+\left(\left(y_{i}-\hat{y}_{i}\right)^{2}+\right.\right.\right. \\
& \left.\left.\left.\left(y_{i}-\hat{y}_{i j j^{\prime}}\right)^{2}\right) \sigma_{i}^{-2}+\left(\left(\theta_{i}-\hat{\theta}_{i}\right)^{2}+\left(\theta_{i}-\hat{\theta}_{i j j^{\prime}}\right)^{2}\right) \mu_{i}^{-2}\right\}\right] \\
& \leq \eta \exp \left[-\frac{1}{4}\left\{\left(\hat{x}_{j^{\prime}}-\hat{x}_{j}\right)^{2} \sigma_{i}^{-2}+\left(\hat{y}_{j^{\prime}}-\hat{y}_{j}\right)^{2} \sigma_{i}^{-2}+\left(\hat{\theta}_{j^{\prime}}-\hat{\theta}_{j}\right)^{2} \mu_{i}^{-2}\right\}\right] .
\end{aligned}
$$

The last inequality is obtained from the inequality $\left(a^{2}+b^{2}\right) \geq \frac{1}{2}(a-b)^{2}$ for arbitrary real numbers $a, b$.

In the case that wrong recognized robot $j^{\prime}$ is apart from robot $j,\left(\hat{x}_{j^{\prime}}-\hat{x}_{j}\right)^{2},\left(\hat{y}_{j^{\prime}}-\hat{y}_{j}\right)^{2}$ becomes very large, hence $p\left(\mathbf{x}_{i, t} \mid \mathbf{u}_{i, t}, \mathbf{z}_{i, t}\right) p\left(\mathbf{x}_{i, t} \mid \mathbf{u}_{j, t}, \mathbf{z}_{j, t}, \mathbf{s}_{j j^{\prime}, t}\right)$ becomes very small.

## V. SIMULATION

The effectiveness of the proposed method is confirmed in this section by computer simulation. Each robot knows the precise locations of landmarks and makes a map of the field. Behaviors of each robot are assumed to be two : straight moving and pivoting (rotation in place). One of these two behaviors is selected randomly in each time. In this experiment, five robots are used.

The comparison result of the proposed method and conventional MCL without collaboration is shown in Fig. 1. The horizontal axis shows the number of particles and the vertical axis shows the estimation error between estimated location and true location. In this simulation, there assumes to be no recognition error of robots, and $p_{0}=0.85$. Simulations are executed with 150 trials. Lines without markers show the results of conventional estimation. Lines with markers show the results of the proposed method.


Fig.1. Estimation error and number of particles

Average of estimation error of the proposed method is better than that of conventional method. The proposed method needs less number of particles than conventional method for attaining the same average of estimation error. For example, seeing the horizontal line of estimation error 30 cm , the number of particles required for attaining the average of estimation error less than 30 cm is approximately 90 in the proposed method. However, that number is approximately 1050 in the conventional method.


Fig.2. Estimation error and recognition error

The relation of estimation error and recognition error probability is shown in Fig. 2. The number of particles is 320 and $p_{0}=0.85$ as before, and the number of trials is 150 . The recognition error probability is varied between 0.0 and 1.0 . As an implementation of recognition error of other robots at robot $i$ in the simulation, a wrong robot $j^{\prime}$ is selected randomly except robot $i$ and the correct robot
$j$. Particles for exchange are sent to robot $j^{\prime}$ instead of robot $j$.

In Fig. 2, vertical axis shows estimation error and horizontal axis shows recognition error probability. The horizontal line with 31 cm shows the estimation error of conventional MCL without collaboration in the case that the number of particles is 320 .

When there is no recognition error of robots, the average of estimation error is 25 cm and minimum among all recognition error probability. When all recognitions of robots are wrong, the average of estimation error is 34 cm and maximum. Even if recognition error probability is $95 \%$, estimation error of the proposed method is better than that of conventional MCL without collaboration. This result shows that the proposed method improves estimation error of localization by collaboration, unless recognition error probability is almost $100 \%$.

These simulation results confirm the effectiveness of proposed method under the condition of existence of recognition error of robots.

## VI. CONCLUSION

In this paper, the problem of recognition error of robots in localization of multi-robots has been discussed. The discussion has shown that if the recognition of other robots is wrong, a large amount of error in localization may occur. Then, a new algorithm for collaborative multi-robot localization using particle filter has been proposed to cope with this problem. In the proposed method, particles in a robot are sent to other robots according to measurement result in the sending robot, at the same time, some particles remain in the sending robot. Received particles from other robots are evaluated by using measurement result in the receiving robot. The proposed method copes with recognition error by remaining particles in sending robot, and increases accuracy of estimation by twice evaluation of exchanging particles in sending robot and receiving robot.

By simulations, estimation errors of localization of the proposed method and conventional MCL without collaboration have been compared. From the simulation with recognition error, even if there are many recognition errors, estimation error of the proposed method is better than that of conventional MCL without collaboration. From the simulation without recognition error, the accuracy of localization of the proposed
method is better than that of the conventional method, if the numbers of particles are the same. In addition, the proposed method needs less number of particles than the conventional method for attaining the same accuracy of estimation, and this shows that the execution time of the proposed method can be faster than the conventional method with the same estimation error.

From these results, the proposed method is tolerant to recognition error and accurate for multi-robot localization.

## REFERENCES

[1] S. Thrun, W. Burgard and D. Fox (2005), Prob abilistic Robotics, MIT
[2] R. Simmons and S. Koenig (1995), Probabilistic Robot Navigation in Partially Observable Environm ents, Proc. IJCAI
[3] A. R. Cassandra et al. (1996), Acting under Un certainty: Discrete Bayesian Models for Mobile-Rob ot Navigation, Proc. IROS
[4] W. Burgard et al. (1996), Estimating the Absolu te Position of a Mobile Robot Using Position Proba bility Grids, Proc. AAAI, 2, pp.896-901
[5] D. Fox, W. Burgard and S. Thrun (1999), Mark ov Localization for Mobile Robots in Dynamic Env ironments, Journal of Artificial Intelligence Research, Vol. 11, pp.391-427.
[6] F. Dellaert et al. (1996), Monte Carlo Localizati on for Mobile Robots, Proc. ICRA
[7] D. Fox, W. Burgard and S. Thrun (1999), Mont e Carlo localization: Efficient Position Estimation fo r Mobile Robots, Proc. AAAI
[8] S. Thrun et al. (2001), Robust Monte Carlo Lo calization for Mobile Robots, Artificial Intelligence, Vol. 128, pp.99-141
[9] T. Nakamura et al. (2003), Self-Localization Me thod for Mobile Robots Using Multiple Omnidirecti onal Vision sensors, Journal of the Robotics Society of Japan, Vol. 21, No.1, pp.109-117
[10] R. Kurazume, S. Nagata and S. Hirose (1994), Cooperative Positioning with Multiple Robots, Proc. ICRA, Vol. 2, pp.1250-1257
[11] A. Bahr, M. R. Walter and J. J. Leonard (200 9), Consistent Cooperative Localization, Proc. ICRA, pp.3415-3422
[12] D. Fox et al. (1999), Collaborative Multi-Robo t Localization, Lecture Note in Computer Science, Vol. 1701/1999, pp.255-266
[13] D. Fox et al. (2000), A Probabilistic Approach to Collaborative Multi-Robot Localization, Autono mous Robots, Vol. 8, Issue 3, pp.325-344.
[14] A. Gassparri et al. (2008), A Fast Conjunctive Resampling Particle Filter for Collaborative MultiRobot Localization, Proc. AAMAS

