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Data envelopment analysis for supply chain

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Abstract: DEA (Data Envelopment Analysis) is a method for evaluating management efficiency of DMU (Decision Making Unit). This paper proposes DEA model for supply chain management. Traditional study focuses on selection of partners and construction of supply chain. Therefore, this study would like to consider how to optimize supply chain itself for maximizing benefit by DEA. In addition, the significant matter is that supply chains sometimes have the unbalanced business processes. It means some particular DMUs on supply chain have superiority and keep the efficiency excessively. That is why the rest DMUs on supply chain need to operate in unfavorable condition. As a result, their operations affect to whole efficiency on supply chain badly. Thus, the proposed method introduces adjustment variable to calculate optimum operation as a supply chain. The utility and effectiveness of the proposed method are shown by numerical experiments.

Keywords: Data Envelopment Analysis, Linear Programming, Supply Chain Management, Decision Making Support**.**

I. INTRODUCTION

DEA (Data Envelopment Analysis) is a method for analyzing management efficiency of DMU (Decision Making Unit). This method assumes production activity of DMU as transformation process by focusing on the input and output data. Then the efficiency of the transformation is evaluated by efficiency value shown as θ . DEA introduces benchmarking method for measuring efficiency value. And the efficiency value of each DMU is calculated by relative comparison to DMU set which is under competition. DEA enables analysts to know the strength (or weakness) among the DMUs in same field or industry. In recent study, the model for supply chain is developing in order to extend applicability of DEA since consideration for collaboration among DMUs in different field or industry is needed [1]. This traditional study utilizes the aspect of evaluation method comes from DEA and proposes the way for choosing beneficial partners to construct supply chain effectively. Therefore, this paper complements the previous study through discussion how to optimize the supply chain after choosing the partners.

So this study proposes the method for optimizing the production activity on supply chain by adjusting input and output data. Then it is possible to regard the collaboration among multiple DMUs as continuous activity under the DEA analysis. Here it is defined that optimization is improvement of efficiency value on supply chain (collaborated DMUs).

II. PROBLEM SETTING

1. Supply chain

This study assumes that five different production activities (A~E) form one business and the network is constructed shown in Fig.1. Production activities of A and B in upper chain operate procurement of raw materials and its processing. Then processed goods are manufactured into the products in activity of C. Finally activities of D and E distribute the products to consumers. The study hereafter is expanded based on this business flow.

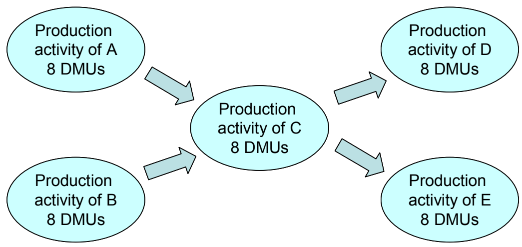

Fig.1. Supply Chain

There are multiple DMUs on each production activity and they are competitive. In case of manufacturing regarding automobile, products are manufactured from iron and electrical parts. Then consumers get the products through dealers or trade company.

2. Argument for applicability of DEA

Definition for input and output elements on each production activity are shown in Fig.2. Production activity of A has two inputs and one output, that of B has one input and two outputs, that of C has two inputs and one output, that of D has two inputs and one output, and that of E has one input and two outputs. The eight supply chains that have the same elements are prepared in order to carry out relative comparison.

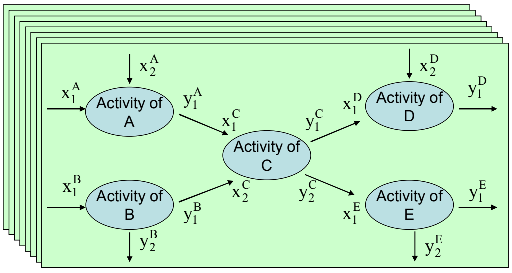

Fig.2. Elements of input and output

The definitions and conditions in this study are summarized as follows.

Condition 1:

There are seven competitors in each production activity (A~E) and their performances are evaluated individually based on their own production activity. Therefore, evaluation concerning one supply chain is judged by summation of five different efficiency values.

Condition 2:

The four arrows connecting between production activities shown in Fig. 2 signify the possible adjustment parts. So that analyst can adjust the amount of input and output. For instance, output y_1^A on production activity of A and input x_1^C on production activity of B are possible to be adjusted. And they have relationship that input x_1^C is decreased if output y_1^A is restricted.

III. DEA AS AN EVALUATION METHOD

Though DEA has various models, this study employs RAM model [2]. This model is able to deal with surplus of input and lack of output together and show the improvement for ideal management states. Thus RAM model is suitable for our study since it can consider adjustment between DMUs on supply chains. RAM model is defined by following formula (1).

$$\begin{aligned}
\max \quad & 1-\frac{1}{m+s}\left\{\sum_{i=1}^{m}\frac{d_{xi}}{R_{xi}}+\sum_{r=1}^{s}\frac{d_{yr}}{R_{yr}}\right\} \\
\text{s.t.} \quad & \sum_{j=1}^{n} x_{ij}\lambda_j + d_{xi} = x_{ik} \quad (i=1,2,\cdots,m) \\
& \sum_{j=1}^{n} y_{rj}\lambda_j - d_{yr} = y_{rk} \quad (r=1,2,\cdots,s) \qquad (1) \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& \lambda_j \geq 0 \ (j=1,2,\cdots,n), \ d_{xi} \geq 0 \ (i=1,2,\cdots,m) \\
& d_{yr} \geq 0 \ (r=1,2,\cdots,s)
\end{aligned}$$

There are *n* evaluated DMUs. And they have *m* input elements and *s* output elements. Then maximization to objective function (surplus of input and lack of output) is carried out. That is to say, objective function indicates how far remarkable DMU is performing compared with efficient state. Moreover, scales of the data are controlled by Rx and Ry that are maximum value of input and output.

DMU is the state of efficient if all slacks are zero in RAM model. On the other hand, DMU is the states of inefficient if one or more slacks have value.

IV. DEA MODEL FOR ADJUSTING INPUT AND OUTPUT

1. Proposed model

This study defines the characters as follows in order to extend the RAM model to the proposed model.

・The number of supply chain: p

・The number of production activity: q

Then more detail definitions are set for production activity of h on k supply chain.

・The number of input element: m_h

・The number of output element: s_h

・The variable signifies the degree of reference to each DMU: λ_j^h $(j=1,2,\ldots,p)$

・The input and output data of DMU_k: (x_{ik}^h, x_{rk}^h)

・The variable signifies the slack of input element i: $d_{x_{ik}^h}$

・The variable signifies the slack of output element r: $d_{y_{rk}^h}$

・The adjustment variable of input element i: $I^+_{x_{ik}^h}, I^-_{x_{ik}^h}$

・The adjustment variable of output element r: $O^+_{y_{rk}^h}, O^-_{y_{rk}^h}$

The proposed method calculates efficiency of k supply chain by applying following formula (2).

$$\begin{aligned}
\max \quad & \sum_{h=1}^{q}\left\{1-\frac{1}{m_h+s_h}\left(\sum_{i=1}^{m_h}\frac{d_{x_{ik}^h}}{R_{x_{ik}^h}}+\sum_{r=1}^{s_h}\frac{d_{y_{rk}^h}}{R_{y_{rk}^h}}\right)\right\} \\
\text{s.t.} \quad & \sum_{j=1}^{p} x_{ij}^h\lambda_j^h + d_{x_{ik}^h} + I^+_{x_{ik}^h} - I^-_{x_{ik}^h} = x_{ik}^h \\
& (i=1,2,\ldots,m_h;\ h=1,2,\ldots,q) \\
& \sum_{j=1}^{p} y_{rj}^h\lambda_j^h - d_{y_{rk}^h} + O^+_{y_{rk}^h} - O^-_{y_{rk}^h} = y_{rk}^h \\
& (r=1,2,\ldots,s_h;\ h=1,2,\ldots,q) \\
& \sum_{j=1}^{p}\lambda_j^h = 1 \qquad (h=1,2,\ldots,q) \qquad (2) \\
& \lambda_j^h \geq 0 \qquad (j=1,2,\ldots,p;\ h=1,2,\ldots,q) \\
& d_{x_{ik}^h} \geq 0 \qquad (i=1,2,\ldots,m_h;\ h=1,2,\ldots,q) \\
& d_{y_{rk}^h} \geq 0 \qquad (r=1,2,\ldots,s_h;\ h=1,2,\ldots,q) \\
& I^+_{x_{ik}^h},\ I^-_{x_{ik}^h} \geq 0 \qquad (i=1,2,\ldots,m_h;\ h=1,2,\ldots,q) \\
& O^+_{y_{rk}^h}, O^-_{y_{rk}^h} \geq 0 \qquad (r=1,2,\ldots,s_h;\ h=1,2,\ldots,q)
\end{aligned}$$

The objective function in formula (2) indicates total efficiency of q DMUs form k supply chain.

2. Explanation of the proposed method

The proposed method analyzes improvement of efficiency as whole production activity by introducing adjustment variables between activities. However, this method can not obtain optimum solution without restriction concerning adjustment variables.

Then the proposed method assumes that adjustment variables $O^+_{y^h_{rk}} - O^-_{y^h_{rk}}$, $I^+_{x^{h+1}_{ik}} - I^-_{x^h_{ik}}$ are equal if output y^h_{rj} (production activity of h) is equal to input x^{h+1}_{ij} (production activity of h+1). Thus input of following production activity increases when previous output is increased. Therefore, it is possible to consider optimum input and output.

Moreover, the calculated adjustment variables can show direction for higher efficiency. For instance, the input about production activity of h should be decreased in case of $I^+_{x^{h+1}_{ik}} > 0$. On the other hand, the input should be increased in case of $I^-_{x^h_{ik}} < 0$ to optimize the efficiency of supply chain.

V. NUMERICAL EXPERIMENTS

1. Experimental conditions and data

There are two experiments in order to examine the utility of the proposed method which introduces adjustment variables for input and output. The first experiment is the case of introduction of adjustment variables and the second one is case of non-introduction of those variables. The experimental conditions are as follows to apply the proposed (2) formula to the supply chain model shown in the second chapter.

<Common condition in both experiments>

$p=8$, $q=5$, $m_1=2$, $s_1=1$, $m_2=1$, $s_2=2$, $m_3=2$, $s_3=2$, $m_4=2$, $s_4=1$, $m_5=1$, $s_5=2$

<Condition in experiment 1>

$I^+_{x^C_{1k}} - I^-_{x^C_{1k}} = O^+_{y^A_{1k}} - O^-_{y^A_{1k}}$, $I^+_{x^C_{2k}} - I^-_{x^C_{2k}} = O^+_{y^B_{1k}} - O^-_{y^B_{1k}}$,

$I^+_{x^D_{2k}} - I^-_{x^D_{2k}} = O^+_{y^C_{1k}} - O^-_{y^C_{1k}}$, $I^+_{x^E_{1k}} - I^-_{x^E_{1k}} = O^+_{y^C_{2k}} - O^-_{y^C_{2k}}$,

Other adjustment variables are equal to zero.

<Condition in experiment 2>

All adjustment variables ($I^+_{x^h_{ik}}, I^-_{x^h_{ik}}, O^+_{y^h_{rk}}, O^-_{y^h_{rk}}$) are equal to zero.

Experimental data are shown in Table 1.

Table 1-(a). Experimental data (supply chain 1~4)

Production activity	Input and Output	Supply chain			
		1	2	3	4
A	x_1^A	144	229	102	261
	x_2^A	50	100	98	177
	y_1^A	205	250	184	203
B	x_1^B	52	59	63	61
	y_1^B	110	186	124	93
	y_2^B	210	209	200	13
C	$x_1^C (=y_1^A)$	205	250	184	203
	$x_1^C (=y_1^B)$	110	186	124	93
	y_1^C	76	96	44	111
	y_2^C	144	80	83	61
D	x_1^D	148	126	101	36
	$x_2^D (=y_1^C)$	76	96	44	111
	y_1^D	195	92	199	207
E	$x_1^E (=y_2^C)$	144	80	83	61
	y_1^E	18	204	76	195
	y_2^E	176	129	133	182

Table 1-(b). Experimental data (supply chain 5~8)

Production activity	Input and Output	Supply chain			
		5	6	7	8
A	x_1^A	77	203	244	56
	x_2^A	131	106	58	164
	y_1^A	132	88	126	242
B	x_1^B	108	71	49	72
	y_1^B	177	144	41	61
	y_2^B	224	190	67	266
C	$x_1^C (=y_1^A)$	132	88	126	242
	$x_1^C (=y_1^B)$	177	144	41	61
	y_1^C	156	185	62	50
	y_2^C	173	218	88	107
D	x_1^D	94	13	105	101
	$x_2^D (=y_1^C)$	156	185	62	50
	y_1^D	186	33	200	153
E	$x_1^E (=y_2^C)$	173	218	88	107
	y_1^E	37	57	282	224
	y_2^E	187	175	78	90

Characteristics of experimental data are summarized. First of all, the structure of experimental data are revealed by looking at the result of non- adjustment (experiment 2). According to the Table 2., first, third, fifth, and eighth supply chains have efficiency value as one in production activity of A. And other supply chains are the state of inefficient (efficiency value is less than one). Especially, sixth supply chain has less efficiency that is 0.66. This supply chain has large input and less output compared with third supply chain. This is clearly proven by their input and output data that are $x_1^A=203$, $x_2^A=106$, $y_1^A=88$ (sixth supply chain) and $x_1^A=102$, $x_2^A=98$, $y_1^A=184$ (third supply chain). The results reflect productivity regarding each production activity well.

Next supply chains themselves are remarkable. In the first supply chain, production activities of A and B have efficiency value as one and the other production activities have efficiency value less than one. Therefore, upper activities (procurement or its processing) are the state of efficient and lower activities (manufacturing or distribution) are that of inefficient in this supply chain.

On the other hand, fourth supply chain is the opposite situation. Moreover, there is no case that all production activities (A~E) are the state of efficient in the prepared supply chains. That is why experimental data generate supply chains that have unbalanced production activities.

2. Experimental Result

The results regarding the efficiency value are as shown in Table 2.

Table 2-(a). Efficiency value (supply chain 1~3)

Production activity	Supply chain 1		2		3	
	Adjustment	—	Adjustment	—	Adjustment	—
A	1	1	1	0.80	1	1
B	1	1	1	1	0.98	0.87
C	0.90	0.77	0.83	0.50	0.85	0.63
D	0.53	0.67	0.48	0.65	0.71	1
E	0.57	0.57	0.69	0.90	0.53	0.74
Total	4.00	4.01	4.00	3.85	4.07	4.24

Table 2-(b). Efficiency value (supply chain 4~6)

Production activity	Supply chain 4		5		6	
	Adjustment	—	Adjustment	—	Adjustment	—
A	0.61	0.61	1	1	0.82	0.66
B	0.75	0.58	0.86	1	0.94	0.86
C	0.83	0.78	0.94	0.82	0.83	1
D	0.87	1	0.70	0.75	1	1
E	0.76	1	0.93	1	0.58	0.58
Total	3.82	3.97	4.43	4.57	4.17	4.1

Table 2-(c). Efficiency value (supply chain 7, 8)

Production activity	Supply chain 7		8	
	Adjustment	—	Adjustment	—
A	0.86	0.75	1	1
B	1	1	1	1
C	0.92	1	0.88	0.82
D	0.95	0.95	0.83	0.91
E	1	1	0.83	0.84
Total	4.73	4.7	4.54	4.57

3. Discussion

The power of adjustment variable is considered by the result in Table 2. Though production activities of A and B are the state of inefficient (A: 0.66, B: 0.86) with non-adjustment in sixth supply chain, the efficiency values are improved (A: 0.82, B: 0.94) with adjustment. However, efficiency value in production activity of C is decreased from 1.0 to 0.83. As a result, whole efficiency shown as total efficiency value is increased 4.1 to 4.17. It is revealed that introduction of adjustment variables works effectively. The result of slacks and adjustment variables is shown in Table 3. to explain the effect of the proposed method.

The values of slacks in Table 3. indicate surplus of input and lack of output and larger values mean less efficiency. Then output y_{23}^B in production activity of A and input x_{13}^C in production activity of C are remarkable. That is because the lack regarding y_{23}^B is decreased from 0.47 to 0 with adjustment. On the other hand, surplus regarding x_{13}^C is increased from 0 to 0.47. This is the situation that production activity of C receives the inefficient burden from that of A.

The adjustment variables work well in sixth supply chain since all five efficiency values are increased. However, some total efficiency value are decreased like eighth supply chain. The reason is that objective function of the proposed method might not be proper for the prepared supply chain network. Moreover, production activity of C plays an important role for optimization since it has four adjustment variables. Then total efficiency value of supply chains which have efficiency value as one with non-adjustment is increased by the proposed method. It means production activity of C has a function for receiving the burden from other activities when it is the state of efficient.

Table 3. Slacks and adjustment variables on sixth supply chain

Production activity	Input and Output	Slack adjustment	Slack —	Adjustment variable
A	x_{13}^A	0.23	0.23	—
	x_{23}^A	0.32	0.32	—
	y_{13}^A	0	0.47	−0.47
Efficiency value		0.82	0.66	
B	x_{13}^B	0.11	0.11	—
	y_{13}^B	0	0.23	−0.23
	y_{23}^B	0.07	0.07	—
Efficiency value		0.94	0.86	
C	x_{13}^C	0.47	0.00	−0.47
	x_{23}^C	0.23	0	−0.23
	y_{13}^C	0	0.00	0.00
	y_{23}^C	0	0.00	0.00
Efficiency value		0.83	1	
D	x_{13}^D	0.00	0	—
	x_{23}^D	0.00	0	0.00
	y_{13}^D	0.00	0	—
Efficiency value		1	1	
E	x_{13}^E	0.72	0.72	0.00
	y_{13}^E	0.49	0.49	—
	y_{23}^E	0.04	0.04	—
Efficiency value		0.58	0.58	

VI. CONCLUSION

This study has proposed DEA model which introduces adjustment variables in order to improve whole efficiency on supply chain. According to the numerical experiments, adjustment variables contribute for supply chain management since some of the supply chains get better efficiency. Moreover, the key DMU which softens and receives unbalanced operation is revealed. This benefit indicates the practical possibility of DEA for applying supply chain management more.

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