The Fifteenth International Symposium on Artificial Life and Robotics 2010 (AROB 15th '10),
B-Con Plaza, Beppu,Oita, Japan, February 4-6, 2010

Memory Capacity and Information Capacity of the Sparsely Encoded Associative Memory with Replacing Units

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Abstract: We introduce sparse encoding into the autoassociative memory model with replacing units. We search by computer simulation the optimal number of replacing units in two terms; the memory capacity and the information capacity of the network. We show that the optimal number of replacing units to maximize the memory capacity and the information capacity decreases as the firing ratio decreases, and that the difference of the memory capacity between sparse encoding and non-sparse encoding becomes small as the number of replacing units increases.

Keyword: autoassociative memory, sparse encoding, catastrophic forgetting, rebirth neuron, memory capacity, information capacity

1 Introduction

The associative memory model is one of neural network models made by imitating the form of memory in human brain, and consists of neurons and synapses.

Properties of an associative memory model largely depend on how items are encoded in pattern vectors to be stored. When most of the components of encoded patterns are 0 and only a small ratio of the components are 1, the encoding scheme is said to be sparse. Rolls(1987) observed that sparse encoding was realized in the hippocampus, and he proposed an associative memory model of the hippocampus[1]. Amari(1989) gave a mathematical analysis of associative memory models with sparse encoding[2]. He proved that the memory capacity (the maximum number of patterns to be stored in the network in the form of its equilibria) and the information capacity (the total amount of information stored) of the sparsely encoded associative memory model are much larger than the ordinary non-sparse encoding scheme. Moreover, he proved that the sparsely encoded associative memory model had a large basin of attraction around each memorized pattern, when and only when an activity (the number of excited components) control mechanism is attached to it.

If the number of memorized patterns surpasses the memory capacity, the network cannot recall any memorized pattern due to the overloading[3]. This phenomenon is called catastrophic forgetting.

Eriksson et al.(1998) discovered newborn neurons in the hippocampus, where the associative memory was considered to be realized[4]. Date and Kurata(2008) reported that the network, in which a fixed number of units are replaced by newborn ones while the model learns one pattern, can keep up memorized patterns studied recently without catastrophic forgetting[5]. They showed that the optimal number of replacing units to maximize the memory capacity is about 3, and that it is independent of the network size. Komatsu et al.(2009) analyzed the associative memory model with replacing units by utilizing statistical mechanics[6]. They showed that replacing 3.2 or more units could make the network avoid catastrophic forgetting and that replacing 6.9 units is optimal to maximize the memory capacity. The difference between the results of Date and Kurata(2008) and that of Komatsu et al.(2009) is due to the order of selection of units to be replaced. While Date and Kurata(2008) set units replaced in a fixed order repeatedly, Komatsu et al.(2009) set units replaced randomly with a certain probability.

Now we introduce sparse encoding into the associative memory model with replacing units. We search the optimal number of replacing units to maximize the memory capacity and the information capacity of the model by using computer simulation. The replacing order is the same as Date and Kurata(2008). We treat in the present paper only an autoassociative memory model, which recalls a memorized pattern from its noisy version. We show that the optimal number of replacing units decreases as the firing ratio decreases, and that increase of the number of replacing units reduces the difference of the memory capacity between sparse encoding and non-sparse encoding, but then makes the information capacity of sparse encoding smaller than that of non-sparse encoding.

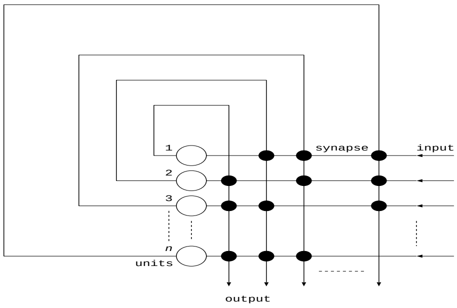


Fig. 1: The autoassociative memory model. The units are interconnected by the network of synapses with the synaptic strength w_{ij} from unit j to unit i. Each unit i has two states, $x_i = \pm 1$.

2 Associative memory model

Let us consider a neural nework of n mutually connected formal neurons. We assume that all the neurons work synchronously at discrete-times $t = 1, 2, \cdots$. The associative memory model consists of memorizing process and recall process.

2.1 Memorizing process

The autoassociative memory model is a network whose connection weight matrix $W = \{w_{ij}\}$ is determined by

$$w_{ij} = \sum_{\mu=1}^{m} {s_i}^{(\mu)} {s_j}^{(\mu)}, \tag{1}$$

from m patterns $\boldsymbol{s}^{(\mu)} = ({s_1}^{(\mu)}, {s_2}^{(\mu)}, \cdots, {s_n}^{(\mu)})^{\mathrm{T}}, \mu = 1, 2, \cdots, m$, to be stored, where $s_i^{(\mu)}$ is the i-th component of $\boldsymbol{s}^{(\mu)}$. w_{ij} is the weight of connection from the j-th neuron to the i-th neuron and it is, therefore, symmetric. Regardless of this definition, w_{ii} is assumed to 0. This learning process is local; the increment for connection w_{ij} does not depend on the global structure of the state or past memories, but only on ${s_i}^{(\mu)}$ and ${s_j}^{(\mu)}$. It is fast, and does not need to learn each memory repeatedly.

This network now functions as an associative memory. For example, if started from an initial state which somewhat resembles state $\boldsymbol{s}^{(1)}$ and which resembles other $\boldsymbol{s}^{(\mu)} (\mu \neq 1)$ very little, the state will evolve to the state $\boldsymbol{s}^{(1)}$. The state $\boldsymbol{s}^{(1)}$ is evocable memory, and the system correctly reconstructs an entire memory from any initial partial information, as long as the partial information is sufficient to identify a single memory. Detailed properties of the collective operation of this network have been studied extensively[3].

2.2 Recall process

Let $\boldsymbol{x}(t) = (x_1(t), x_2(t), \cdots, x_n(t))^{\mathrm{T}}$ be a vector whose component $x_i(t)$ denotes the output of the i-th neuron. This vector is called the state vector of the network.

In our network of the associative memory model (Fig.1), each unit i has two states, and is described by variable $x_i(t) = \pm 1$. The instantaneous state of the system of n units can be thought of as an n-dimensional vector having components $x_i(t)$. The units are interconnected by a network synapses, with a synaptic strength w_{ij} from unit$j$ to unit i. The instantaneous output to unit i is

$$x_i(t+1) = \mathrm{sgn}(\sum_{j=1}^{n} w_{ij} x_j(t)), \tag{2}$$

where $x_j(t)$ is the present state ± 1 of unit j. This equation defines the state transition of the network from the state $x_i(t)$ at discrete-time t to the next state $x_i(t+1)$. The function $\mathrm{sgn}(u)$ denotes the unit signum function,

$$\mathrm{sgn}(u) = \begin{cases} 1, & u > 0, \\ -1, & u \leq 0. \end{cases} \tag{3}$$

A neuron is excited when a weighted sum of its inputs exceeds 0. A neuron emits output 1 when it is excited, and its output is -1 when it is not excited.

The state of the system changes in time; each unit i readjusts its state, setting $x_i(t) = \pm 1$ according to whether $\sum_{j=1}^{n} w_{ij} x_j(t)$, the input to i at this moment, is greater or less than 0. This algorithm defines the time evolution of the state of the system. For any symmetric connection matrix $\{w_{ij}\} : w_{ji}$, there are stable states of the network of units. Starting from any arbitrary initial state, the system reaches a stable state and cease to evolve.

2.3 Catastrophic forgetting

There is a critical memory capacity in the conventional associative memory model. The memory capacity for one-half coded random memories is given by Amit et al.(1985) as about $0.138n$, where n is the number of units[3]. If the number of memorized patterns surpasses the memory capacity, the network cannot recall any memorized patterns due to the overloading[3]. This phenomenon is called catastrophic forgetting. Fig.2 shows catastrophic forgetting; the network which consists of $n = 1000$ neurons could recall all the memorized patterns for $\mu < 140$, but at $\mu = 140$, forgetting started, and no memorized pattern could be recalled correctly after $\mu = 200$, i.e., the network could not recall even the most recent memory.

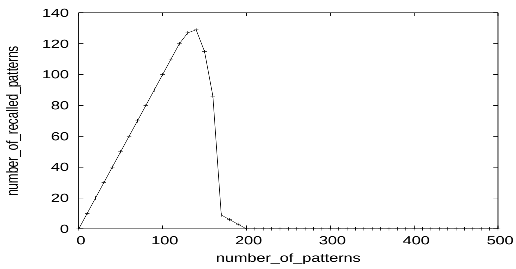


Fig. 2: Catastrophic forgetting on the conventional autoassociative memory model with 1000 neurons.

If the dynamics of weight connections of the network have decay or saturation, the catastrophic forgetting does not occur, and the network can keep recent memories[7]. But, as yet, there is no conclusive experimental evidence for the existence of such a system in human brain.

3 Rebirth neuron and the modeling; replacing units

Eriksson et al.(1998) discovered newborn neurons in the hippocampus, where the associative memory was considered to be realized[4]. Date and Kurata(2008) reported that the one-half coded associative memory network, in which a fixed number of units were replaced by newborn ones while learning one pattern, could keep up memory patterns studied recently without catastrophic forgetting[5]. This corresponds to resetting the connection weights $w_{ij} = w_{ji} = 0, j = 1, 2, \cdots, n$, for replaced neurons i. Units were replaced from the oldest one first, i.e., they were always replaced in the same order.

4 Sparse encoding

We consider the case where patterns to be stored are generated independently and randomly under the condition that they have a fixed activity. The encoding that the number of active components in $\boldsymbol{s}^{(\mu)}$ is negligibly small compared to n is said to be sparse. $\boldsymbol{s}^{(\mu)}$ are independent random vectors subject to a common probability distribution. More precisely, $\boldsymbol{s}^{(\mu)}$ are generated in such a manner that, $nq, (0 \leq q \leq 1)$ components, randomly chosen among n components, take $1-q$ and all the other components are put equal to $-q$.

Amari(1989) proved that the memory capacity C_{M} of the associative memory model increases as encoding sparser[2]. One pattern to be stored has an average information content $H(q)$ with the occurence probability q as below

$$H(q) = -q \log_2 q - (1-q) \log_2 (1-q). \qquad (4)$$

So, one sparsely encoded pattern $\boldsymbol{s}^{(\mu)}$ includes a smaller amount of information than non-sparse encoding case. However, Amari(1989) proved that the total amount of information stored in the network, or the information capacity increases as encoding is sparser[2]. This is due to the increase of the memory capacity in the sparse encoding case.

Here, We define the information capacity C_{I} of the network as below,

$$C_{\mathrm{I}} = H(q) C_{\mathrm{M}}. \qquad (5)$$

5 Simulation

5.1 Settings

Simulations were carried out on a computer for $n = 1000$ and 2000 by varying the number R of replacing units and the firing ratio q to maximize the memory capacity and the information capacity.

We applied equation(1) with replacing units to memorizing process. For simplicity, we assume that the total number of neurons does not change over time. Every time the network memorizes a new pattern, R neurons die and the same number of neurons are born. The number m of memorized patterns depends on the number R of replacing units as below,

$$m = \frac{n}{R}. \qquad (6)$$

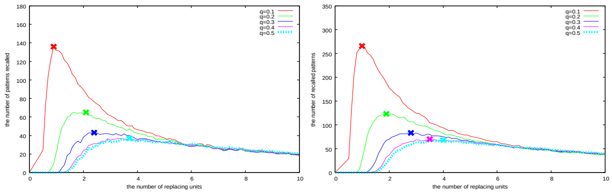
We also used a non-integer value for R. In this case, we define the number $r(t)$ of replacing units at t as below,

$$r(t) = \mathrm{int}(R(t+1)) - \mathrm{int}(Rt), \qquad (7)$$

where $\mathrm{int}(x)$ is the function which truncates a number after the decimal point. We used the memorized patterns for the initial states in recall process. Since the firing ratio calcurated with equation(3) couldn't keep the constant value q, the top nq neurons in descending order of the sum of weighted inputs were let to fire. The proximity of the state $\boldsymbol{s}^{(\mu)}$ of the memorized pattern and the state $\boldsymbol{x}^{(\mu)}$ of the recalled pattern was measured by the direction cosine

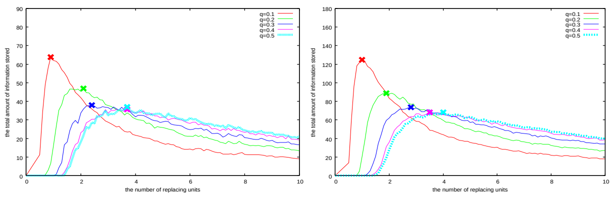
$$\cos\theta = \frac{\boldsymbol{s}^{(\mu)} \cdot \boldsymbol{x}^{(\mu)}}{\|\boldsymbol{s}^{(\mu)}\| \|\boldsymbol{x}^{(\mu)}\|}, \quad \mu = 1, 2, \cdots, m. \qquad (8)$$

We counted the number of successfully recalled memories in which the proximity was larger than 0.8. Because the system has the finite state transitions, the state is always supposed to reach an equilibrium or a periodical solution, and the period is known to be no more than 2. Here we assume that the system is forced to stop when the present state equals the second to last one in recall process.



(a)$n = 1000$ (b)$n = 2000$

Fig. 3: The results of simulation to maximize the memory capacity. The horizontal axes are the number R of replacing units. The vertical axes are the memory capacity $C_{\rm M}$. The 5 points represent the each optimal number in the firing ratio q from 0.1 to 0.5.



(a)$n = 1000$ (b)$n = 2000$

Fig. 4: The results of simulation to maximize the information capacity. The horizontal axes are the number R of replacing units. The vertical axes are the information capacity $C_{\rm I}$. The 5 points represent the each optimal number in the firing ratio q from 0.1 to 0.5.

5.2 Results

Fig.3 shows that the sparsely encoded associative memory model with a small number of replacing units for $R < 4$ had a larger memory capacity than the non-sparsely encoded one, i.e., $q = 0.5$, but there was little difference of the memory capacity between sparse encoding and non-sparse encoding after $R \approx 6$. It turned out that the optimal R to maximize the memory capacity decreased as the firing ratio decreased; $R \approx 1$ for $q = 0.1$, $R \approx 2$ for $q = 0.2$, $R \approx 2.6$ for $q = 0.3$, $R \approx 3.7$ for $q = 0.4$ or $q = 0.5$. As shown in Fig.3(a), $n = 1000$, and (b), $n = 2000$, the optimal R seemed to be independent of the size of the network.

Fig.4 shows that the sparsely encoded associative memory model with a small number before $R \approx 2$ had also a larger information capacity than the non-sparsely encoded one, but, after $R \approx 6$, the information capacity of sparse encoding became smaller than that of non-sparse encoding. It turned out that the optimal R to maximize the information capacity decreased as the firing ratio decreased, and it was about the same number as to maximize memory capacity. As shown in Fig.4(a) and (b), the optimal R also seemed to be independent of the size of the network.

6 Conclusion

We introduced sparse encoding into the associative memory model with replacing units. We reported the optimal number of replacing units to maximize the memory capacity and the information capacity of the model by using computer simulation. The sparsely encoded associative memory model with a small number of replaced units has also a larger memory capacity and a larger information capacity than the non-sparsely encoded one. We showed that the optimal number of replacing units decreases as the firing ratio decreases, and that increase of the number of replacing units reduces the difference of the memory capacity between sparse encoding and non-sparse encoding, but then makes the information capacity of sparse encoding smaller than that of non-sparse encoding. We also found that the optimal number of replaced units was independent of the size of the network.

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