The Fourteenth International Symposium on Artificial Life and Robotics 2009 (AROB 14th '09),
B-Con Plaza, Beppu, Oita, Japan, February 5 - 7, 2009

Integrative Bayesian Model of Two Opposite Types of Sensory Adaptation

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Abstract

Adaptation is a fundamental property of human perception. Recently, it was found that there are two opposite types of adaptation to repetitive stimuli with temporal difference. In this paper, we construct an integrative model of adaptation. We model the perception as a Bayesian inference and also model the two types of adaptation as changes in the likelihood function and the prior distribution in the Bayesian inference. We examine our model analytically and show how the type of adaptation depends on model parameters.

Keywords: Bayesian inference, lag adaptation, Bayesian calibration, ventriloquism aftereffect

1 Introduction

Our surrounding world is constantly changing. Our perception has to deal with such changes in statistics of our surroundings, by adjusting the inner representations of those statistics. In addition to those changes in the outer world, there are also changes in our body. For example, when we injure our eyes or ears, our perception would be impaired, due to the change in the inner representation of the physically delivered stimulus in the brain. Such adaptation phenomena are the important aspects of human perception and they themselves are worth to be investigated. In addition to that, by investigating the properties of the adaptation of a particular type of perception or a motor system, its neural mechanism can often be deduced by psychophysical experiments and brain imaging experiments (e.g. [1]).

We showed in earlier works that the ventriloquism aftereffect, which is an adaptation phenomenon in audiovisual spatial perception, can be explained by updating the parameter that determines the mean value of the likelihood function that represents a noise distribution [2].

In the ventriloquism aftereffect, the repeated stimuli are perceived to be presented at the same place. This type of adaptation is also observed in the adaptation to audiovisual temporal difference, that is, the participants perceive the temporal difference in the adapting stimuli to be simultaneous [3]. This type of adaptation is call the "lag adaptation". However, recently, an opposite type of adaptation was found [4] in tactile temporal adaptation. They showed that adaptational effect was opposite to the lag adaptation, that is, the participants were more unlikely to perceive simultaneity for the repeatedly presented stimuli. They showed that the result could be explained by assuming that the participants had learned the prior distribution of stimulus timing. They called the adaptation "Bayesian calibration".

In our earlier work [2], adaptation was modeled as the update of the mean values of likelihood functions. Therefore, lag adaptation can be considered to be changes in the likelihood functions. On the other hand, as Miyazaki et al. showed in [4], Bayesian calibration can be considered to be changes in the prior distributions.

In this paper, we extend our earlier model [2] and investigate the interaction of these two types of adaptation and show what parameters determine the type of adaptation.

2 Integrative Bayesian model of adaptation

We consider an audiovisual localization task. We consider a task in which a pair of sound and light with spatial disparity is presented, and the participant determines which stimulus is presented at the right. If we plot the percentage of "sound right" response for various test stimulus disparities, we obtain a psychometric function. The center point of the psychometric function represents the disparity that the participants judged to be at the same location. During an adaptation period, stimuli with, in most experiments, constant disparity are repeated. The adapting stimuli during the adaptation period are not necessarily constant: they may be drawn from a probability distribution. Then we measure the psychometric function again. The type of adaptation is represented as the difference in the center point of the psychometric function between before and after adaptation period. If the center point is shifted toward the adapting stimuli, it is "lag adaptation" type, and if it is shifted opposite from the adapting stimuli, it is "Bayesian calibration" type.

We formalize the optimal observer that uses Bayesian inference to estimate the true positions of stimuli. We assume that the observer can only observe noisy position of sound and light, denoted as y_A and y_V, respectively, that are deviated from the true positions of stimuli, denoted as x_A and x_V, respectively. The observer is assumed to determine estimators $\hat{x}_A$ and $\hat{x}_V$ from y_A and y_V by maximizing the posterior probability distribution $P(x_A, x_V|y_A, y_V)$. We assume independence between the auditory and visual noise. Then, from Bayes' theorem, it follows that

$$P(x_V, x_A|y_V, y_A) \propto P(y_A|x_A)P(y_V|x_V)P(x_A, x_V), \quad (1)$$

We model the adaptation by changing the mean values of the likelihood function [2], $P(y_A|x_A)$ and $P(y_V|x_V)$, and the prior distribution $P(x_A, x_V)$. We assume that the noise are Gaussian noise and the prior probability of x_A and x_V depends only on their difference $x_A - x_V$. Thus, we assume

$$P(y_V|x_V) = \frac{1}{\sqrt{2\pi}\sigma_V} \exp\left(-\frac{(y_V - x_V - \mu_V)^2}{2\sigma_V^2}\right), \quad (2)$$

$$P(y_A|x_A) = \frac{1}{\sqrt{2\pi}\sigma_A} \exp\left(-\frac{(y_A - x_A - \mu_A)^2}{2\sigma_A^2}\right), \quad (3)$$

$$P(x_A, x_V) = \frac{1}{\sqrt{2\pi}\sigma_p L} \exp\left(-\frac{(x_A - x_V - \mu_p)^2}{2\sigma_p^2}\right), \quad (4)$$

where μ_A, μ_V, and μ_p modifies represent the mean values of distributions.

We interpret the adaptational effect observed in psychophysical experiments as the false update of μ_A and μ_V due to the unnatural stimuli that the participants are exposed to, and the learning of μ_p of such unnatural stimuli. We assume that the real values of μ_A and μ_V are zero and unchanged from their initial values, and that the observer knows the other parameters like σ_A, σ_V, and σ_p. Quantities σ_p and μ_p can be controlled by the experimenter.

Each time the observer receives the adapting audiovisual stimuli, it estimates the corresponding parameters and updates its estimations on μ_V, μ_A, and μ_p based on observations and estimations. We denote these observer's estimations of μ_V, μ_A, and μ_p as $\hat{\mu}_V$, $\hat{\mu}_A$, and $\hat{\mu}_p$. The observer determines MAP estimators $\hat{x}_V$ and $\hat{x}_A$ from y_V and y_A, and updates $\hat{\mu}_V$, $\hat{\mu}_A$, and $\hat{\mu}_p$ as

$$\begin{aligned}
\hat{\mu}_A(t+1) &= (1-\alpha_A)\hat{\mu}_A(t) + \alpha_A(y_A - \hat{x}_A), \quad (5)\\
\hat{\mu}_V(t+1) &= (1-\alpha_V)\hat{\mu}_V(t) + \alpha_V(y_V - \hat{x}_V), \quad (6)\\
\hat{\mu}_p(t+1) &= (1-\alpha_p)\hat{\mu}_p(t) + \alpha_p(\widehat{x_A} - \hat{x}_V), \quad (7)
\end{aligned}$$

where $\hat{\mu}_A(t)$, $\hat{\mu}_V(t)$, and $\hat{\mu}_p(t)$ represent the observer's estimations at time t. Quantities α_A, α_V, and α_p determine the relative adaptation effect in each step, and are assumed to satisfy $0 \le \alpha_i \le 1$, where i represents each one of $\{A, V, p\}$. We assume that the initial values of $\hat{\mu}_A$ and $\hat{\mu}_V$ are their true values, that is, zero. We also assume that the initial value of $\hat{\mu}_p$ is zero.

3 Psychometric function

Here, we derive the dependency of the center point of a psychometric function on model parameters.

In our model, the observer's task corresponds to judging the sign of $\hat{x}_A - \hat{x}_V$: if it is positive, sound is on the right. Therefore, the probability that the observer's response is "sound right" given a presented disparity $_x \equiv x_A - x_V$ is equivalent to $P(\hat{x}_A - \hat{x}_V > 0|x_A - x_V = \ _x)$. As we will show later, this probability distribution does not depend on the absolute values of x_A or x_V but only on their difference $_x$. Thus in our model, the psychometric function, denoted as $Psycho(\ _x)$, can be written as

$$Psycho(\ _x) = P(\hat{x}_A - \hat{x}_V > 0|x_A - x_V = \ _x). \quad (8)$$

In usual experiments, it is known that psychometric function can be approximated by a cumulative Gaussian distribution (e.g. [4]). Therefore, it can be written as

$$Psycho(\ _x) = \int_{-\infty}^{\Delta_x} d\ \ _x' N(\ _x'; \mu_{psycho}, \sigma_{psycho}^2), \quad (9)$$

where $N(x; \mu, \sigma^2)$ represents a normal probability distribution of x with mean μ and variance σ^2. Thus, by calculating $P(\hat{x}_A - \hat{x}_V > 0|x_A - x_V = \ _x)$ and comparing equations (8) and (9), we can determine how the center point of the psychometric function, i.e. μ_{psycho}, depends on the model parameters.

By substituting equations (2), (3), and (4) into equation (1) and maximizing it, we obtain

$$\hat{x}_A = \frac{1}{\sigma_{all}^2}\left(\left(\sigma_V^2+\sigma_p^2\right)(y_A-\hat{\mu}_A)+\sigma_A^2(y_V-\hat{\mu}_V)+\sigma_A^2\hat{\mu}_p\right), \tag{10}$$

$$\hat{x}_V = \frac{1}{\sigma_{all}^2}\left(\sigma_V^2(y_A-\hat{\mu}_A)+\left(\sigma_A^2+\sigma_p^2\right)(y_V-\hat{\mu}_V)-\sigma_V^2\hat{\mu}_p\right), \tag{11}$$

where $\sigma_{all}^2 \equiv \sigma_A^2+\sigma_V^2+\sigma_p^2$. From equations (10) and (11), we obtain

$$\hat{\Delta}_x = \frac{\sigma_p^2}{\sigma_{all}^2}(\Delta_y - \hat{\Delta}_\mu) + \frac{\sigma_A^2+\sigma_V^2}{\sigma_{all}^2}\hat{\mu}_p, \tag{12}$$

where $\hat{\Delta}_x \equiv \hat{x}_A - \hat{x}_V$, $\Delta_y \equiv y_A - y_V$, and $\hat{\Delta}_\mu \equiv \hat{\mu}_A - \hat{\mu}_V$.

Then we can calculate $P(\hat{\Delta}_x > 0 | \Delta_x)$ as follows:

$$P(\hat{\Delta}_x > 0|\Delta_x) = \int_{-\infty}^{\Delta_x} d\hat{\Delta}'_x \, N\left(\hat{\Delta}'_x; \hat{\Delta}_\mu - \frac{\sigma_A^2+\sigma_V^2}{\sigma_p^2}\hat{\mu}_p, \sigma_A^2+\sigma_V^2\right). \tag{13}$$

Thus, from equations (9) and (13), we obtain

$$\mu_{psycho} = \hat{\Delta}_\mu - \frac{\sigma_A^2+\sigma_V^2}{\sigma_p^2}\hat{\mu}_p, \tag{14}$$

$$\sigma_{psycho} = \sigma_A^2+\sigma_V^2. \tag{15}$$

Now that we know how μ_{psycho} depends on model parameters and $\hat{\mu}_A$, $\hat{\mu}_V$, and $\hat{\mu}_p$, next we must investigate the time course of these $\hat{\mu}$s during the adaptation period and their converging values. Thus, we can show how the type of adaptation is determined.

4 Analysis of the model behavior

It can be seen from equations (5), (6), and (7) that the update rules of $\hat{\mu}_A$, $\hat{\mu}_V$, and $\hat{\mu}_p$ are independent from each other given $\hat{x}_A$ and $\hat{x}_V$. However, because $\hat{x}_A$ and $\hat{x}_V$ depend on $\hat{\mu}_A(t)$, $\hat{\mu}_V(t)$, and $\hat{\mu}_p(t)$, the values of $\hat{\mu}$s are not independently changed.

By substituting equations (10) and (11) into equations (5), (6), and (7), we obtain

$$\begin{pmatrix}\hat{\mu}_A(t+1)\\ \hat{\mu}_V(t+1)\\ \hat{\mu}_p(t+1)\end{pmatrix} = \begin{pmatrix}1-a & a & -a\\ v & 1-v & v\\ -p & p & 1-p\end{pmatrix}\begin{pmatrix}\hat{\mu}_A(t)\\ \hat{\mu}_V(t)\\ \hat{\mu}_p(t)\end{pmatrix} + \begin{pmatrix}a\\ -v\\ p\end{pmatrix}\Delta_y, \tag{16}$$

where a,v, and p are defined by: $a \equiv \alpha_A\frac{\sigma_A^2}{\sigma_{all}^2}$, $v \equiv \alpha_V\frac{\sigma_V^2}{\sigma_{all}^2}$, and $p \equiv \alpha_p\frac{\sigma_p^2}{\sigma_{all}^2}$.

Although, in reality, y_A and y_V are determined randomly from trial to trial, we can pursue the average behavior of the model by fixing each of y_A and y_V to its mean value during the adaptation period. We validate this assumption later by numerical simulations. From equations (2), (3), and (4), and our assumption that the true values of μ_A and μ_V are zero, the mean value of $\Delta_y \equiv y_A - y_V$ is $\Delta_{adapt} \equiv \mu_p$. We use the notation Δ_{adapt} to avoid confusion of μ_p with $\hat{\mu}_p$.

With this assumption, we can solve equation (16) explicitly with respect to t, which yields

$$\begin{pmatrix}\hat{\mu}_A(t)\\ \hat{\mu}_V(t)\\ \hat{\mu}_p(t)\end{pmatrix} = \begin{pmatrix}\frac{a}{z}\Delta_y - \frac{a}{z}\Delta_y(1-z)^t\\ -\frac{v}{z}\Delta_y + \frac{v}{z}\Delta_y(1-z)^t\\ \frac{p}{z}\Delta_y - \frac{p}{z}\Delta_y(1-z)^t\end{pmatrix}, \tag{17}$$

where $z \equiv a + v + p$.

By definition, z satisfies $0 \leq z \leq 1$, and we omit the case $z = 0$, because it is the case where all αs are zero and the results are trivial. Then $(1-z)^t$ converges to zero. From equation (17), we can also see that the converging speed of all $\hat{\mu}$s are the same. Because α_i represents the degree of adaptation in each step, at first sight, it seems that the converging speed is different if α_i is different for different i. However, due to the interaction of all $\hat{\mu}$s, their converging speed are the same.

By substituting equation (17) into equation (14), after some calculations, we obtain

$$\mu_{psycho}(t) = \beta\Delta_{adapt} - \beta\Delta_{adapt}(1-z)^t, \tag{18}$$

where $\mu_{psycho}(t)$ is the center of the psychometric function measured with $\mu_A(t)$, $\mu_V(t)$, and $\mu_p(t)$, and β is defined as:

$$\beta \equiv \frac{1}{z\sigma_{all}^2}(\alpha_A\sigma_A^2 + \alpha_V\sigma_V^2 - \alpha_p(\sigma_A^2+\sigma_V^2)). \tag{19}$$

Thus, the direction of the shift in the center point of the psychometric function relative to Δ_{adapt} is determined by the sign of β.

In reality, in order to measure $\mu_{psycho}(t)$, test stimuli must be presented to the participant, and such stimuli must change $\hat{\mu}_A(t)$, $\hat{\mu}_V(t)$, and $\hat{\mu}_p(t)$, if presented too many times. Therefore, $\mu_{psycho}(t)$ can only be measured by conducting the whole experiment multiple times, with a small number of test stimuli, and averaging the results, like Miyazaki et al. did in [4].

5 Numerical simulations

In deriving equation (17), we assumed $_y$ was constant with respect to t and investigated the mean behavior of the model. Here, we validate this assumption using numerical simulations.

Parameter values were as follows: $\mu_p = 8$, $\sigma_A = 8$, $\sigma_V = 2.5$, $\sigma_p = 1$, $\alpha_A = 0.01$, $\alpha_V = 0.01$, and $\alpha_p = 0.005$. At each time step, we sampled x_A from a normal distribution with mean μ_p and variance σ_p^2, while x_V was fixed to 0. We also sampled y_A and y_V according to the noise distributions in equations(2) and (3). Then the model observer judged $\hat{x}_A$ and $\hat{x}_V$ based on equations (10) and (11) and updated $\hat{\mu}_A$, $\hat{\mu}_V$, and $\hat{\mu}_A$ according to equations (5), (6), and (7). This procedure was repeated 1000 times. We also investigated the time course of μ_{psycho}. At each time step, after updating all $\hat{\mu}$s, we measured the psychometric function using the updated $\hat{\mu}s$. We presented test stimuli with $_x$ from -30 to 30 with 1 step, each 1000 times. Then we calculated $\mu_{psycho}(t)$ by fitting the result to equation (9) by minimizing mean squared error.

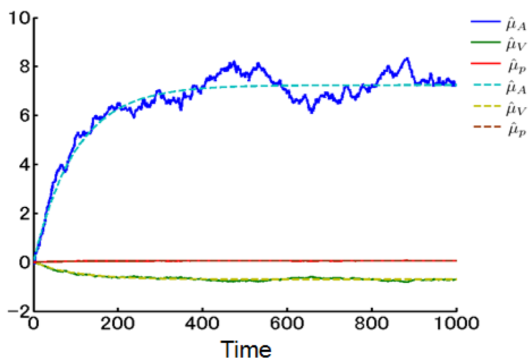

Figure 1: Time course of $\hat{\mu}_A$, $\hat{\mu}_V$, and $\hat{\mu}_A$. Solid lines show numerical simulation results and dashed lines show corresponding analytical results.

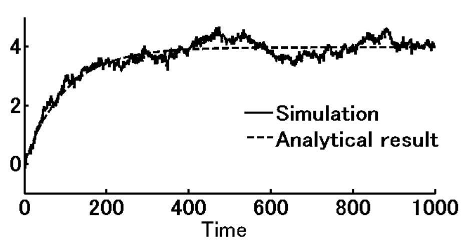

Figure 2: Time course of $\mu_{psycho}(t)$. The solid line shows numerical simulation results and the dashed line shows corresponding analytical result.

Figure 1 shows an example of the simulation result for the time course of $\hat{\mu}_A$, $\hat{\mu}_V$, and $\hat{\mu}_A$, together with the analytical results in equation (17). Figure 2 shows the simulation result for the time course of μ_{psycho}, together with the analytical results in equation (18). These figures clearly show that the analytical results in equations (17) and (18) correctly follow the average behavior of $\hat{\mu}$s or μ_{psycho}.

6 Conclusion

In this paper, we constructed an integrative Bayesian model of adaptation and investigated what factors determine the type of adaptation. We showed that the type of adaptation was determined by the sign of β defined in equation (19). Quantities σ_A, σ_V, and σ_p can be measured or adjusted experimentally. Therefore, according to our model, we might be able to control experimentally the type of adaptation by adjusting the parameters. However, it is not straightforward what determines the adaptation parameters α_A, α_V, and α_p. The investigation of the meaning of them remains as a future work.

Acknowledgments

This work was partially supported by Grant-in-Aid for JSPS Fellows from JSPS (20・8988) and Grant-in-Aid for Scientific Research on Priority Areas –System study on higher-order brain functions– from MEXT, Japan (17022012).

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