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Robust l_∞ Preview Control for Biped Walking Pattern Generation

Sadaaki Kunimatsu, Takahiro Fukuda, Makoto Kumon and Mitsuaki Ishitobi
Graduate School of Science and Technology, Kumamoto University
Kurokami 2-39-1, Kumamoto, 860-8555, JAPAN

Abstract

We propose robust l_∞ preview control for biped walking pattern generation. First, we state robust l_∞ preview control with robustness against initial value and apply it to the cart-table model. Next, we propose a method to reduce the conservativeness of the robust l_∞ preview control and an algorithm to solve it. Finally, the effectiveness of our proposed methods is illustrated by simulation.

1 Introduction

Recently, attention has been focused on the study of biped walking for life size humanoid robots. Generally, an approach to biped walking is the method of adjusting futural ZMP(Zero Moment Point) proposed in Reference [2]. ZMP changes as a step function when the humanoid robot shifts the way of supporting from one leg to both legs. Moreover, it is known that the change of ZMP generates after that of the center of gravity does. Therefore, we cannot use normal servo control in regarding ZMP as a step input. This problem is solved by applying the preview control with the performance index of l_2 norm [3] that use futural information of reference inputs [2]. However, since the performance index of l_2 norm minimizes the integral square of the error between references and outputs, the maximal value of the error is not always small. Therefore, there exists possibility of resulting in toppling. There are several researches using the maximal value as performance index [1, 4, 6]. We propoesed l_∞ preview control via LMI(Linear Matrix Inequality [1]) optimization problem by using l_∞ norm as the performance index [5]. Since l_∞ norm can evaluate ZMP error directly, it is expected that this approach prevents the humanoid robot from toppling. However, the upper bound of performance index of robust l_∞ preview control was conservative in spite of having good performance in simulation.

In this paper, we propose a method to reduce the conservativeness of the robust l_∞ preview control and an algorithm to solve it. Finally, the effectiveness of our proposed methods is illustrated by simulation.

2 Walking pattern generation based on cart-table model

We consider a cart-table model [2] as follows:

$$p = x_c - \frac{z_c}{g}\ddot{x}_c \tag{1}$$

where p is the position of ZMP, z_c is the height of the center of gravity, g is the gravity acceleration, x_c is the position of the cart. We define the input as the time derivative of the horizontal acceleration of the cart and construct a continuous-time state space system. With discretizing the system as the sampling time h, the discrete-time state space equation is transformed into the following equation.

$$x_{k+1} = Ax_k + Bu_k \tag{2a}$$
$$p_k = Cx_k \tag{2b}$$

where

$$A := \begin{bmatrix} 1 & h & \frac{h^2}{2} \\ 0 & 1 & h \\ 0 & 0 & 1 \end{bmatrix}, \quad B := \begin{bmatrix} \frac{h^3}{6} \\ \frac{h^2}{2} \\ h \end{bmatrix},$$
$$C := \begin{matrix} 1 & 0 & -\frac{z_c}{g} \end{matrix}$$

We define the error e_k between the reference ZMP p_k^{ref} and the measured ZMP p_k as follows:

$$e_k := p_k^{ref} - p_k \tag{3}$$

In order to eliminate the steady state error, we construct the error system with the first-order difference value of the state and the error as the state variable.

$$\bar{x}_{k+1} = \bar{A}\bar{x}_k + \bar{B}\Delta u_k + B_R \Delta p_{k+1}^{ref}, \tag{4a}$$
$$e_k = \bar{C}\bar{x}_k \tag{4b}$$

where

$$\bar{A} := \begin{bmatrix} I & -CA \\ 0 & A \end{bmatrix}, \quad \bar{B} := \begin{bmatrix} -CB \\ B \end{bmatrix},$$

$$B_R := \begin{bmatrix} I \\ 0 \end{bmatrix}, \quad \bar{C} := \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix},$$

$$\bar{x}_k := \begin{bmatrix} e_k \\ \Delta x_k \end{bmatrix},$$

$$\Delta x_k := x_k - x_{k-1},$$

$$\Delta u_k := u_k - u_{k-1},$$

$$\Delta p_k^{ref} := p_k^{ref} - p_{k-1}^{ref}$$

Then, by using $\bar{x}_k$ and the reference up to N-step future, the preview control input Δu is given by

$$\Delta u_k = -F\bar{x}_k + \sum_{j=1}^{N} f_j \Delta p_{k+j}^{ref} \tag{5}$$

3 l preview control

In order to apply l_∞ control, we transform the system (4) into the following augmented error system with $\Delta p_{k+1}^{ref} \sim \Delta p_{k+N}^{ref}$ [3].

$$\tilde{x}_{k+1} = \tilde{A}\tilde{x}_k + \tilde{B}\Delta u_k \tag{6a}$$

$$\tilde{e}_k = \tilde{C}\tilde{x}_k \tag{6b}$$

where

$$\tilde{x}_k := \begin{bmatrix} \bar{x}_k \\ \Delta p_{k+1}^{ref} \\ \Delta p_{k+2}^{ref} \\ \vdots \\ \Delta p_{k+N}^{ref} \end{bmatrix}, \quad \tilde{A} := \begin{bmatrix} \bar{A} & B_R & 0 & \cdots & 0 \\ 0 & 0 & I & 0 & 0 \\ \vdots & \vdots & 0 & \ddots & 0 \\ \vdots & 0 & 0 & 0 & I \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$\tilde{B} := \begin{bmatrix} \bar{B} & 0 & \cdots & \cdots & 0 \end{bmatrix}^T,$$

$$\tilde{C} := \begin{bmatrix} \bar{C} & 0 & \cdots & 0 \end{bmatrix},$$

$$p_{k+N+\alpha}^{ref} := p_{k+N}^{ref}$$

The preview input Δu_k is also transformed as follows:

$$\Delta u_k = -\tilde{F}\tilde{x}_k \tag{7}$$

where $\tilde{F} := \begin{bmatrix} F & -f_1 & \cdots & -f_N \end{bmatrix}$. Here, we give an initial value defined by

$$\tilde{x}_0 := \begin{bmatrix} \bar{x}_0^T, \Delta p_1^{ref}, \Delta p_2^{ref}, \cdots, \Delta p_N^{ref} \end{bmatrix}^T$$

As performance index, we consider the l_∞ norm to evaluate the maximum of the error as follows:

$$\mathit{\Gamma}_\infty := \max_{k \geq 0} \left(\tilde{e}_k^T \tilde{e}_k \right)^{1/2} \tag{8}$$

Then we can obtain l_∞ preview controller minimizing the upper bound of the performance index (8) (See Reference [5]).

4 Robust l preview control

In order to avoid obstacles, it is necessary to change walking pattern of humanoid robots on the way. This means that the preview trajectory need be adjusted from the scheduled preview trajectory. The l_∞ preview control is dependent on the initial value and its influence is crucial. We consider l_∞ preview control with robustness against the initial value.

4.1 l preview control with robustness against initial value

Let us consider the set Ω defined by

$$\Omega(\tilde{x}_0) := \mathrm{Co}\{\tilde{x}_0^{[1]},\ \tilde{x}_0^{[2]},\ \cdots, \tilde{x}_0^{[M]}\} \tag{9}$$

where Co denotes the convex hull. If $\tilde{x}_0 \in \Omega$, then $\tilde{x}_0$ satisfies

$$\tilde{x}_0 = \sum_{i=1}^{M} \alpha_i \tilde{x}_0^{[i]} \tag{10}$$

where

$$\alpha_i \geq 0, \quad \sum_{i=1}^{M} \alpha_i = 1 \tag{11}$$

In order to evaluate the worst case of (8) for $\tilde{x}_0 \in \Omega$, we introduce new performance index defined by

$$\mathit{\Gamma}_\infty^{\max} := \max_{\tilde{x}_0 \in \Omega} \mathit{\Gamma}_\infty \tag{12}$$

Then we obtain the following lemma on the upper bound of $\mathit{\Gamma}_\infty^{\max}$.

Lemma 1 [5] *Assume that $\tilde{x}_0 \in \Omega$ in the system (6). The upper bound γ_∞ satisfying $\mathit{\Gamma}_\infty^{\max} \leq \gamma_\infty$ can be obtained by solving the following LMI optimization problem with respect to $Q > 0$, L and $\gamma_\infty > 0$. Then the optimal feedback gain of (7) is given by $\tilde{F} = LQ^{-1}$.*

$$\min_{Q,\, L} \gamma_\infty \tag{13}$$

subject to

$$\begin{matrix} -Q & (\tilde{A}Q - \tilde{B}L)^T \\ \tilde{A}Q - \tilde{B}L & -Q \end{matrix} < 0 \tag{14a}$$

$$\begin{bmatrix} -Q & \tilde{x}_0^{[i]} \\ (\tilde{x}_0^{[i]})^T & -\gamma_\infty \end{bmatrix} \le 0, \quad \text{for} \quad i = 1, 2, \cdots, M \tag{14b}$$

$$\begin{matrix} -Q & (\tilde{C}Q)^T \\ \tilde{C}Q & -\gamma_\infty I \end{matrix} \le 0 \tag{14c}$$

4.2 Reduction of conservativeness

Since the upper bound γ_∞ of $\Gamma_\infty^{\max}$ obtained by Lemma 1 becomes coservative [5], we consider reducing the conservativeness. Experientially, the l_∞ preview control has almost the maximal value Γ_∞ in time $k = 1$. We obtain the next theorem by using this fact.

Theorem 1 *Assume that $\tilde{x}_0 \in \Omega$ in the system (6) and $\|\tilde{e}_0\| \le \|\tilde{e}_1\|$. Then the upper bound γ_∞ satisfying $\Gamma_\infty^{\max} \le \gamma_\infty$ can be obtained by solving the following BMI(Bilinear Matrix Inequality) optimization problem with respect to $Q > 0$, $\tilde{F}$ and $\gamma_\infty > 0$.*

$$\min_{Q,\,\tilde{F}} \gamma_\infty \tag{15}$$

subject to

$$\begin{matrix} -Q & (\tilde{A}Q - \tilde{B}\tilde{F}Q)^T \\ \tilde{A}Q - \tilde{B}\tilde{F}Q & -Q \end{matrix} < 0 \tag{16a}$$

$$\begin{bmatrix} -Q & (\tilde{A} - \tilde{B}\tilde{F})\tilde{x}_0^{[i]} \\ (\tilde{x}_0^{[i]})^T(\tilde{A} - \tilde{B}\tilde{F})^T & -\gamma_\infty \end{bmatrix} \le 0$$
$$\text{for} \quad i = 1, 2, \cdots, M \tag{16b}$$

$$\begin{matrix} -Q & (\tilde{C}Q)^T \\ \tilde{C}Q & -\gamma_\infty I \end{matrix} \le 0 \tag{16c}$$

Proof: From (11) and (16b), we obtain

$$\begin{aligned} &\begin{matrix} -Q & \hat{x}_1 \\ (\hat{x}_1)^T & -\gamma_\infty \end{matrix} \\ &= \begin{bmatrix} -Q\sum_{i=1}^M \alpha_i & \sum_{i=1}^M \alpha_i \hat{x}_1^{[i]} \\ (\sum_{i=1}^M \alpha_i \hat{x}_1^{[i]})^T & -\gamma_\infty \sum_{i=1}^M \alpha_i \end{bmatrix} \\ &= \sum_{i=1}^M \left(\alpha_i \begin{bmatrix} -Q & (\tilde{A} - \tilde{B}\tilde{F})\hat{x}_0^{[i]} \\ (\hat{x}_0^{[i]})^T(\tilde{A} - \tilde{B}\tilde{F})^T & -\gamma_\infty \end{bmatrix} \right) \le 0 \end{aligned}$$

The above means $\hat{x}_1^T Q^{-1} \hat{x}_1 \le \gamma_\infty$. By using this fact, (16a) and (16b), we obtain the following inequality.

$$\gamma_\infty^{-1} \hat{x}_k^T \hat{C}^T \hat{C} \hat{x}_k \le \hat{x}_k^T Q^{-1} \hat{x}_k \le \hat{x}_1^T Q^{-1} \hat{x}_1 \le \gamma_\infty$$

By $\|\tilde{e}_0\| \le \|\tilde{e}_1\|$, this implies $\Gamma_\infty^{\max} \le \gamma_\infty$. ♠

If the maximal value Γ_∞ is in time $k = 1$, the invariant ellipsoid is not conservative. Therefore, it is important to choose $\Omega(\tilde{x}_0)$ appropriately.

We apply the following LMI-based iterative algorithm to solve Theorem 1.

Iterative algorithm

Step1: Obtain an initial solution $\tilde{F}^{(0)}$ by Lemma 1.

Step2: Solve the follwoing two problems alternately when $k = 1, 2, \cdots$
a) Minimize $\gamma_\infty^{(k)}$ subject to (16) for given $\tilde{F}^{(k)}$ where $\gamma_\infty^{(k)} := \min_Q \gamma_\infty$.
b) Obtain $\tilde{F}^{(k+1)}$ minimizing $\hat{\gamma}_\infty^{(k+1)}$ subject to (16) for given $Q^{(k)}$ where $\hat{\gamma}_\infty^{(k+1)} := \gamma_\infty$.

Step3: Repeat Step2 until $|\gamma_\infty^{(k+1)} - \gamma_\infty^{(k)}| < \varepsilon$ for some $\varepsilon > 0$. ♠

By using the above algorithm, we obtain Theorem 2.

Theorem 2 *Given a certain initial solution of $\tilde{F}^{(0)}$, the above LMI-based iterative algorithm is always feasible, and the gain $\tilde{F}^{(k+1)}$ obtained in Step2-b satisfies*

$$\begin{aligned} \Gamma_\infty^{\max} \le \cdots \le \gamma_\infty^{(k+1)} \le \hat{\gamma}_\infty^{(k+1)} \le \gamma_\infty^{(k)} \\ \le \cdots \le \hat{\gamma}_\infty^{(1)} \le \gamma_\infty^{(0)} \end{aligned} \tag{17}$$

5 Simulation on biped walking pattern generation

We made simulation of shifting the reference ZMP into 0.2[m] with the control approach of Section 2. We give $z_c = 0.85$[m], $g = 9.81$[m/s^2] and $h = 0.04$[s] in this simulation. We also give $N = 50$ as the number of preview steps.

We state the case that the reference ZMP is changed in 2.8[s] from the scheduled preview trajectory. We show the simulation results of the trajectories of the reference and measured ZMPs and those of the reference ZMP and the center of gravity in Figure 1 and 2, respectively. We also show the magnified figure of Figure 1 in Figure 3. When we calculated the upper bound of the worst case of l_∞ norm by the approach of Lemma 1 and Theorem 2, we obtained $\gamma_\infty = 0.104$[m] and $\gamma_\infty = 0.0499$[m] for the following $\Omega(\tilde{x}_0)$, respectively.

$$\Omega(\tilde{x}_0) := \mathrm{Co}\{\tilde{x}_0^{[1]},\ \tilde{x}_0^{[2]},\ \tilde{x}_0^{[3]}, \tilde{x}_0^{[4]}\}$$

where $\tilde{x}_0^{[1]} := [e + \delta_1, \Delta x^{[1]} + \delta_2, \Delta x^{[2]}, \Delta x^{[3]}, \Delta p_1^{ref}, \cdots, \Delta p_N^{ref}]$, $\tilde{x}_0^{[2]} := [e + \delta_1, \Delta x^{[1]} - \delta_2/2, \Delta x^{[2]},$

$\Delta x^{[3]}, \Delta p_1^{ref}, \cdots, \Delta p_N^{ref}]$, $\tilde{x}_0^{[3]} := [e - \delta_1/2, \Delta x^{[1]} + \delta_2, \Delta x^{[2]}, \Delta x^{[3]}, \Delta p_1^{ref}, \cdots, \Delta p_N^{ref}]$, $\tilde{x}_0^{[4]} := [e - \delta_1, \Delta x^{[1]} - \delta_2, \Delta x^{[2]}, \Delta x^{[3]}, \Delta p_1^{ref}, \cdots, \Delta p_N^{ref}]$ and $e := 3.97 \times 10^{-3}$, $x^{[1]} := 7.28 \times 10^{-3}$, $\Delta x^{[2]} := 1.82 \times 10^{-2}$, $\Delta x^{[3]} := 8.78 \times 10^{-2}$, $\delta_1 := 5.00 \times 10^{-3}$, $\delta_2 := 5.00 \times 10^{-4}$, $\Delta p_6^{ref} = 0.1$, $\Delta p_i^{ref} = 0$ $(i = 1, \cdots, 50, i \neq 6)$. In Figures 1, 2 and 3, the solid line denotes the controller by Theorem 2, the dashed line does the controller by Lemma 1 and the dotted line does the reference ZMP with change in 2.8[s]. In Figure 3, the maximum of the error between the reference and measured ZMPs in using the the controller by Theorem 2 is smaller than that in using the controller by Lemma 1. Therefore, we see that the conservativeness is reduced.

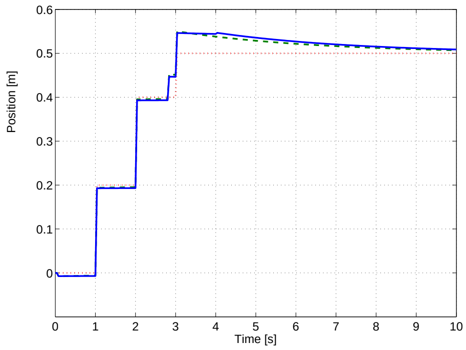

Figure 1: Trajectory of reference and measured ZMPs

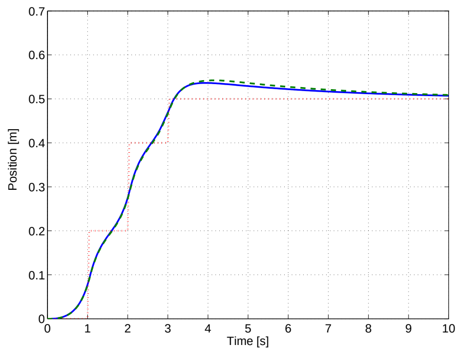

Figure 2: Trajectory of center of gravity

6 Conclusion

In this paper, we proposed the robust l_∞ preview control for biped walking pattern generation. First, we showed l_∞ preview control with robustness against initial value. Next, we proposed a method to reduce the conservativeness of Lemma 1 and an algorithm to solve it. Finally, we showed the effectiveness of our proposed method by simulation.

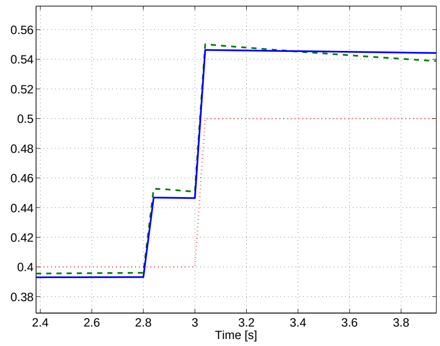

Figure 3: Trajectory of reference and measured ZMPs (Magnified figure of Fig. 1)

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